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An Exponential Class Estimator of Mean in the Presence of Correlated Measurement

Error under Systematic Sampling Technique

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Abstract:

In this manuscript, an exponential class of estimators is proposed to estimate the average of the population by using auxiliary variables in the presence of measurement error (me) as well as correlated measurement error (cme). The estimation for the average is done for the systematic sampling technique. The impact me and cme on the mse of the estimators is obtained in terms of mean square error (mse) and bias. The mean square error is also obtained for the ratio, product and regression estimator under correlated measurement error. To validate the results of the theoretical findings simulation studies is done by using R programming.

Keywords: Mean, Mean Square Error, Bias, Systematic Sampling, Measurement Error, Correlated Measurement Error.

Introduction

Cochran (1977), Murthy (1967), Sukhatme et. al. (1984) have described all the literatures and theory about sampling techniques. Systematic sampling can be used where the population are natural population, for instances selection of every nth person visiting to a shopping mall, selection of a person from the list of any city or place, selection of fields from any geographical location. Cochran(1946) provided certain situations where systematic sampling is more efficient in comparison to simple random sampling and stratified sampling. Gautschi (1957), Meadow (1949, 1953) are legends in the literature of systematic sampling. Lahiri (1954), and Williams (1956) have done remarkable work in the systematic sampling.

Auxiliary information is used to obtain better efficiency in terms of precision and to reduce the cost of survey. For instance, in the estimation of the crop, temperature fertilizer, irrigation can be considered as amount auxiliary information. To estimate the income of any place, the expenditure and different sources of income can be considered as auxiliary variable. Murthy (1964) addressed the conditions under which the estimations techniques ratio, product and unbiased estimators can be efficiently used. Use of auxiliary variable in context of systematic sampling was done by Swain (1964). As he shown that systematic sampling is very efficient for estimation of volume of timber where auxiliary information can be taken as area of the leaf or the tree's girth as suggested by in introducing ratio estimator for systematic sampling. The product estimator for systematic sampling was introduce by Shukla (1971).

Measurement error is the contamination commingled in data during the survey or during the compilation of the data. Shalabh (1997) addressed the impact of measurement error in ratio estimation for simple random sampling. Singh et. al. (2019) proposed an efficient variant of the ratio, product and mean estimator in the presence of measurement error. The assumption of measurement error in the previous literature of the study variable Y and auxiliary variable X are uncorrelated. The measurement errors of the study variable Y and auxiliary variable X may be correlated with each other as the same person or same instrument are used to collect the information for both variables. Shalabh and Tsai (2017) first introduce the correlated measurement error with reference to ratio, product regression estimator. Further Singh and Vishwakarma (2019) studied about measurement error.

Singh and Vishwakarma (2020) introduce measurement error in the context of systematic sampling. They provided mean square error for ratio estimator, product estimator and regression estimator in the presence of measurement error for systematic sampling. In pre- existing literature, only Singh and Vishwakarma (2020) have addressed the measurement error in context of systematic sampling. Also, no any studied is done for correlated measurement error in context of systematic sampling.

By considering, the wide applicability of systematic sampling and following above literature, in this manuscript, a well-known exponential estimator is proposed to obtain the effect of measurement error as well as correlated measurement error on mean square error. The mean square error is also derived for well-known namely ratio and product estimator in the presence of correlated measurement error under systematic sampling.

For systematic sampling, the population of size N is divided into k intervals such that $N = nk$. Sample of size n is selected through systematic sampling. As first unit is selected at random from the first k units. If the first unit is the ith unit of the first k units the second unit of the sample is the $i + k$th unit of the second k units. Similarly, other units of the samples are selected. After sampling of n units are done through systematic sampling we observe the study and auxiliary variables. It is considered that a situation where each data of the study and auxiliary variable are observed with error. Let us assume (x_{ij}, y_{ij}) be the observed values and their true values are (X_{ij}, Y_{ij}) where the subscript ij represents jth unit of the ith interval and $i = 1,2, \dots, k$ and $j = 1,2, \dots, n$. As the observed values are with errors they can be represented in the following form, $x_{ij} = X_{ij} + V_{ij}$ and $y_{ij} = Y_{ij} + U_{ij}$ and (U, V) represent the errors. The errors (U, V) are normally distributed with mean zero and variance (σ_U^2, σ_V^2).

Let us assume that the error variables U and V are correlated to each other and a they are uncorrelated to all the combination with the study and auxiliary variables X and Y. From the before mentioned assumptions we obtain $Cov(X, U) = Cov(Y, U) = Cov(X, V) = Cov(Y, V) = 0$ and

$Cov(U,V) = Cov(X,Y) \neq 0$. Let μ_{Xsy}, μ_{Ysy} be the population mean of the auxiliary and study variable respectively and let $\sigma^2_{Ysy}, \sigma^2_{Xsy}$ be the population variance of the study and auxiliary variable respectively. Let ρ be the correlation coefficient between the study and auxiliary variable. Let the sample means of the true values of the study auxiliary and study variable be $\bar{Y}_{sy}, \bar{X}_{sy}$ respectively. The unbiased estimators of the population of mean of the study and auxiliary variables are the sample means of the observed data.

The unbiased estimators of the population are as follows,

$$\hat{\mu}_{Ysy} = \bar{y}_{ksy} = \frac{1}{n}\sum_{i=1}^{n} y_{ij}\,, \quad i = 1,2,\ldots,k. \tag{1.1}$$

$$\hat{\mu}_{Xsy} = \bar{x}_{ksy} = \frac{1}{n}\sum_{i=1}^{n} x_{ij}\,, \quad i = 1,2,\ldots,k. \tag{1.2}$$

To obtain the bias and variance we can write the error term ε_X, ε_Y as follows,

$$\varepsilon_X = \frac{\bar{x}_{sy} - \mu_{xsy}}{\mu_{xsy}}\,, \qquad \varepsilon_Y = \frac{\bar{y}_{sy} - \mu_{Ysy}}{\mu_{Ysy}}$$

As $\bar{y}_{sy}$ and $\bar{x}_{sy}$ are unbiased estimators of $\hat{\mu}_{Ysy}$ and $\hat{\mu}_{Xsy}$ respectively we can write

$$E(\varepsilon_Y) = E(\varepsilon_X) = 0, \qquad \bar{U}_{i.} = \bar{U}_{sy} = \frac{1}{n}\sum_{j=1}^{n} U_{ij},$$

$$E(\varepsilon_X^2) = \frac{1}{\mu_{Xsy}^2}\{\sigma^2_{Xsy} + \sigma^2_{Vsy}\}, \qquad E(\varepsilon_Y^2) = \frac{1}{\mu_{Ysy}^2}\{\sigma^2_{Ysy} + \sigma^2_{Usy}\},$$

$$E[\varepsilon_X \varepsilon_Y] = \frac{\rho\sigma_{Xsy}\sigma_{Ysy}}{\mu_{Ysy}\mu_{Xsy}}\,, \qquad R = \frac{\mu_{Ysy}}{\mu_{Xsy}}\,, \qquad E(\varepsilon_X^* \varepsilon_Y^*) = \frac{(\rho_{YX}\sigma_{Xsy}\sigma_{Ysy} + \rho_{UV}\sigma_{Usy}\sigma_{Vsy})}{\mu_{Ysy}\mu_{Xsy}},$$

$$\sigma^2_{Ysy} = \frac{1}{k}\sum_{i=1}^{k}(\bar{Y}_{ksy} - \mu_{Ysy})^2, \qquad \sigma^2_{Xsy} = \frac{1}{k}\sum_{i=1}^{k}(\bar{X}_{ksy} - \mu_{Xsy})^2,$$

$$\sigma^2_{Usy} = \frac{1}{k}\sum_{i=1}^{k}(\bar{U}_{i.})^2\,, \qquad \sigma^2_{Vsy} = \frac{1}{k}\sum_{i=1}^{k}(\bar{V}_{i.})^2,$$

2. Pre-Existing Estimators in the Presence of measurement error

The mean estimator in the presence of measurement error is $\bar{y}_{sy}$. The variance in the presence of measurement error is given as,

$$V(\bar{y}_{sym}) = \sigma^2_{Ysym} = \frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{sy} - \mu_{Ysy}\right)^2 = \frac{1}{k}\sum_{i=1}^{k}\left(\bar{Y}_{sy} + \bar{U}_{sy} - \mu_{Ysy}\right)^2 \tag{2.1}$$

$$V(\bar{y}_{sym}) = \sigma^2_{Ysy} + \sigma^2_{Usy} \tag{2.2}$$

Ratio estimator in the presence of measurement error is defined as,

$$\bar{y}_{Rsym} = \bar{y}_{sy}\frac{\mu_{Xsy}}{\bar{x}_{sy}} \tag{2.3}$$

The mean square error of ratio estimator in the presence of measurement error is obtained as,

$$bias(\bar{y}_{Rsym}) = \frac{1}{\mu_{Ysy}}\left\{R^2\left(\sigma^2_{Xsy} + \sigma^2_{Vsy}\right) - \rho R\sigma_{Xsy}\sigma_{Ysy}\right\} \tag{2.4}$$

$$(\bar{y}_{Rsym} - \mu_{Ysy})^2 = \mu_{Ysy}^2 \,(\varepsilon_Y^2 + \varepsilon_X^2 - 2\varepsilon_Y\varepsilon_X) \tag{2.5}$$

$$MSE(\bar{y}_{Rsym}) = [\sigma_{Ysy}^2 + \sigma_{Usy}^2 + R^2(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) - 2\rho R\sigma_{Ysy}\sigma_{Xsy}]. \tag{2.6}$$

The bias in the presence of correlated measurement error is derived as,

$$bias\left(\overline{y^*}_{Rsym}\right) = \frac{1}{\mu_{Ysy}}\{R^2(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) - R(\rho\sigma_X\sigma_Y + \rho_{UV}\sigma_U\sigma_V\} \tag{2.7}$$

The mean square error in the presence of correlated measurement error is derived as

$$MSE\left(\overline{y^*}_{Rsym}\right) = \left[\sigma_{Ysy}^2 + \sigma_{Usy}^2 + R^2(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) - 2R(\rho\sigma_{Xsy}\sigma_{Ysy} + \rho_{UV}\sigma_{Usy}\sigma_{Vsy})\right] \tag{2.8}$$

The product estimator in the presence of measurement error is

$$\bar{y}_{Psym} = \bar{y}_{sy}\frac{\bar{x}_{sy}}{\mu_{Xsy}} \tag{2.9}$$

The bias of the estimator is obtained as

$$bias(\bar{y}_{Psym}) = \left(\frac{\rho\sigma_{Ysy}\sigma_{Xsy}}{\mu_{Xsy}}\right) \tag{2.10}$$

The mean square error in the presence of uncorrelated measurement error is derived as

$$MSE(\bar{y}_{Psym}) = (\sigma_{Ysy}^2 + \sigma_{Usy}^2 + R^2(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) + 2\rho R\sigma_{Ysy}\sigma_{Xsy}). \tag{2.11}$$

The mean square error in the presence of correlated measurement error is derived as

$$MSE\left(\overline{y^*}_{Psym}\right) = (\sigma_{Ysy}^2 + \sigma_{Usy}^2 + R^2(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) + 2R(\rho\sigma_{Xsy}\sigma_{Ysy} + \rho_{UV}\sigma_{Usy}\sigma_{Vsy}). \tag{2.12}$$

The difference estimator in the presence of measurement error is

$$\bar{y}_{dsym} = \bar{Y}_{sy} + b(\mu_{Xsy} - \bar{X}_{sy}) \tag{2.13}$$

$$V(\bar{y}_{dsym}) = [(\sigma_{Ysy}^2 + \sigma_{Usy}^2) + b^2(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) - 2b\rho\sigma_{Ysy}\sigma_{Xsy}]. \tag{2.14}$$

After differentiating the above equation we get,

$$\frac{d(V(\bar{y}_{dsym}))}{db} = [2b(\sigma_{Xsy}^2 + \sigma_{Vsy}^2) - 2\rho\sigma_{Ysy}\sigma_{Xsy}] \tag{2.15}$$

Equating the above equation to zero gives,

$$b = \frac{\rho\sigma_{Ysy}\sigma_{Xsy}}{(\sigma_{Xsy}^2 + \sigma_{Vsy}^2)} \tag{2.16}$$

Now substituting the value of b, the minimum variance is obtained as

$$V(\bar{y}_{dsym}) = \left[(\sigma_{Ysy}^2 + \sigma_{Usy}^2) - \frac{\rho^2\sigma_{Ysy}^2\sigma_{Xsy}^2}{(\sigma_{Xsy}^2 + \sigma_{Ysy}^2)}\right] \tag{2.17}$$

3. 1. Proposed Class of Estimators

The proposed class of estimators in the presence of measurement error is

$$\hat{\mu}_{YREsym} = \bar{y}_{sy} exp\left(\frac{\mu_{Xsym} - \bar{x}_{sy}}{\mu_{Xsym} + \bar{x}_{sy} + 2K}\right)$$
(3.1.1)

$$MSE(\hat{\mu}_{YRsym}) = \left[(\sigma_{Ysy}^2 + \sigma_{Usy}^2) + \frac{(\sigma_{Ysy}^2 + \sigma_{Usy}^2)}{4(\mu_{Xsym} + K^2)} - \frac{\rho^2 \sigma_{Ysy}^2 \sigma_{Xsy}^2}{(\mu_{Xsym} + K^2)(\sigma_{Xsy}^2 + \sigma_{Vsy}^2)}\right] \quad (3.1.2)$$

In order to obtain the min. MSE differentiate (3.1.2) w.r.t K and equate it to zero

$$\widehat{\frac{d}{dK}(\mu_{YREsym})} = 0 \quad (3.1.3)$$

$$K = -\mu_{Xsym} + \frac{\mu_{Ysy}}{2} - (\sigma_{Ysy}^2 + \sigma_{Usy}^2)/S_{xy} \quad (3.1.4)$$

After substituting the value of K mean square error of the estimators we can write from equation (3.1.2) as

$$min. MSE(\hat{\mu}_{Ysym}) = (\sigma_{Ysy}^2 + \sigma_{Usy}^2) - \frac{\rho^2 \sigma_{Ysy}^2 \sigma_{Xsy}^2}{(\sigma_{Xsy}^2 + \sigma_{Vsy}^2)} \quad (3.1.5)$$

3.2. Bias and Mean Square Error for the correlated measurement error

The minimum mean square error of the proposed class of estimator in the presence of correlated measurement error is obtained as

$$MSE(\hat{\mu}_{Ysym}) = (\sigma_{Ysy}^2 + \sigma_{Usy}^2) - \frac{(\rho \sigma_{Xsy} \sigma_{Ysy} + \rho_{UV} \sigma_{Vsy} \sigma_{Usy})^2}{(\sigma_{Xsy}^2 + \sigma_{Vsy}^2)} \quad (3.2.1)$$

5.1. Simulation Study

In order to show the efficiency of the estimators and to show the impact of uncorrelated and correlated measurement error on the mse simulation study is conducted by using R studio. A data matrix is generated using multivariate normal distribution on the auxiliary variable X, study variable Y and the error variables U and V and the four variables X, Y, U and V are with mean $(\mu_{Xsy}, \mu_{Ysy}, 0,0)$.

The covariance matrix is:

$$\begin{pmatrix} S_Y^2 & \rho S_X S_Y & 0 & 0 \\ \rho S_X S_Y & S_X^2 & 0 & 0 \\ 0 & 0 & S_U^2 & \rho_{UV} S_U S_V \\ 0 & 0 & \rho_{UV} S_U S_V & S_V^2 \end{pmatrix}$$

Here

S_Y^2 and S_X^2 is the standard deviation of the study variable Y and X..

S_U^2 and S_V^2 is the standard deviation of the measurement error U associated with study variable Y sample and V is associated with the auxiliary variable X.

ρ is the correlation coefficient between the study variable Y and auxiliary variable X.

ρ_{UV} is the correlation coefficient between the measurement errors of U and V the study variable Y and auxiliary variable X.

From the simulation by using R studio , data is generated for multivariate normal distribution with mean vector for $(\mu_{Xsy}, \mu_{Ysy}, 0, 0)$ = (50,60,0,0) and $S_X = 35, S_Y = 25$. From the generated data 10, systematic samples has been taken.

The statistics is calculated for the generated data set. The Mean square error is defined as the expectation of the squared difference between the value that is estimated and true value. The mean squared error for the proposed class of estimators is obtained and compared with the mean square error for the ratio and product estimator for different combinations of

$\rho_{XY} = (0.95, 0.55, -0.95, -0.55)$ and $\rho_{UV} = (0.95, 0.55, 0.0, -0.95, -0.55)$.

To illustrate the performance of the estimator under measurement error and correlated measurement error 6 tables is created.

Table 1: MSE and PRE of various estimators of μ_{Ysy} for $\rho_{UV} = (0.95, 0.55, 0.0, -0.95, -0.55)$ and $\rho_{XY} = (0.95, 0.55, -0.95, -0.55)$ when $(\sigma_U^2, \sigma_V^2) = (0, 0)$ and $(\sigma_Y^2, \sigma_X^2) = (15, 20)$ and $N = 1000$ and $n = 10$.

σ_U^2	σ_V^2	ρ_{XY}	ρ_{UV}	MSE			var
				$(\hat{\mu}_{sym})$	$\hat{\mu}_{Rsym}$	$\hat{\mu}_{Psym}$	$\bar{y}_{sym}$
0	0	.95	0.95	0.791	1.038	24.263	8.187
			0.55	0.705	0.844	31.948	7.74
			0.0	0.703	0.836	23.633	7.313
			-.95	1.908	2.405	61.18	19.565
			-.55	0.742	1.143	35.259	7.611
		.55	0.95	6.82	11.467	38.284	9.964
			0.55	10.63	18.439	60.889	15.25
			0.0	16.29	16.449	48.846	23.367
			-.95	12.82	14.865	50.588	18.387
			-	13.32	19.251	65.774	19.1

			0.55				
		-.95	0.95 0.55 0.0 -.95 -.55	0.791 1.688 1.08 1.473 1.417	32.482 62.741 40.936 55.217 49.023	0.838 1.697 1.081 1.475 1.534	8.112 17.315 11.081 15.103 14.53
		-.55	0.95 0.55 0.0 -.95 -.55	11.02 5.724 15.76 7.554 5.466	34.034 20.841 66.635 32.194 31.118	11.203 6.267 19.386 9.36 9.408	15.803 8.206 22.607 10.83 7.837

Table 2: MSE and PRE of various estimators of μ_{Ysy} for ρ_{UV} $(0.95, 0.55, 0.0, -0,95, -0.55)$ and $\rho_{XY} = (0.95, 0.55, -0.95, -0.55)$ when $(\sigma_U^2, \sigma_V^2) = (2,2)$ and $(\sigma_Y^2, \sigma_X^2) = (15,20)$ and $N = 1000$ and $n = 10$.

σ_U^2	σ_V^2	ρ_{XY}	ρ_{UV}	MSE			Var
				$\hat{\mu}_{sym}$	$\hat{\mu}_{Rsym}$	$\hat{\mu}_{Psym}$	$\bar{y}_{sym}$
2	2	0.95	.95 .55 0.0 -.95 -.55	2.965 3.092 4.693 6.66 4.039	3.036 3.544 4.913 7.346 4.313	107.252 66.777 75.321 62.727 35.695	30.411 21.80 24.37 17.74 10.54
		0.55	0.95 0.55 0.0 -.95	11.651 11.436 10.895 9.606 8.599	11.707 12.912 12.726 15.486 15.453	38.672 43.573 35.056 35.752 40.151	17.807 16.396 14.067 11.413 10.847

			-.55				
		-.95	0.95 0.55 0.0 -.95 -.55	6.633 4.687 3.442 2.591 2.243	53.846 56.463 53.616 90.902 44.812	6.648 5.388 3.56 2.686 2.247	18.864 14.792 14.799 26.138 13.082
		-.55	0.95 0.55 0.0 -.95 -.55	16.095 13.339 15.91 7.796 17.332	61.465 58.549 84.222 45.465 88.096	22.308 20.043 27.213 12.467 25.926	20.596 17.613 21.92 11.749 24.841

Table 3: MSE and PRE of various estimators of μ_{Ysy} for $\rho_{UV} = (0.95, 0.55, 0.0, -0, 95, -0.55)$ and $\rho_{XY} = (0.95, 0.55, -0.95, -0.55)$ when $(\sigma_U^2, \sigma_V^2) = (4, 4)$ and $(\sigma_Y^2, \sigma_X^2) = (15, 20)$ and $N = 1000$ and $n = 10$

σ_U^2	σ_V^2	ρ_{XY}	ρ_{UV}	MSE			var
				$\hat{\mu}_{sym}$	$\hat{\mu}_{Rsym}$	$\hat{\mu}_{Psym}$	$\bar{y}_{sym}$
4	4	0.95	0.95 0.55 0.0 -.95 -.55	2.353 4.445 7.395 7.514 8.056	2.643 4.465 8.427 8.225 10.334	77.652 61.691 33.187 56.347 22.252	23.588 19.296 11.521 16.952 9.331
		0.55	0.95 0.55 0.0 -.95 -.55	22.598 10.359 19.769 19.429 22.739	26.067 12.593 19.964 28.441 33.259	94.832 42.14 45.97 48.519 62.502	33.61 14.65 25.237 20.859 25.083
		-.95	0.95 0.55	17.371 16.009 11.014	62.62 65.209 63.253	20.624 18.873 11.674	23.431 23.091 21.304

			0.0 -.95 -.55	1.594 3.777	56.864 42.463	1.644 3.934	16.255 12.257
		-.55	0.95 0.55 0.0 -.95 -.55	20.826 15.327 8.238 8.888 12.416	53.655 39.353 36.754 53.181 60.757	30.339 30.452 15.683 11.712 18.679	22.576 15.583 9.943 15.076 17.367

Table 4: MSE and PRE of various estimators of $\boldsymbol{\mu_{Ysy}}$ for $\boldsymbol{\rho_{UV} = (0.95, 0.55, 0.0, -0, 95, -0.55})$ and$\boldsymbol{\rho_{XY} = (0.95, 0.55, -0.95, -0.55)}$ when $\boldsymbol{(\sigma_U^2, \sigma_V^2) = (0, 0)}$ and $\boldsymbol{(\sigma_Y^2, \sigma_X^2) = (25, 35)}$ and $\boldsymbol{N = 1000}$ and $\boldsymbol{n = 10}$.

σ_U^2	σ_V^2	ρ_{XY}	ρ_{UV}	MSE			var
				$\hat{\mu}_{sym}$	$\hat{\mu}_{Rsym}$	$\hat{\mu}_{Psym}$	$\bar{y}_{sym}$
0	0	.95	.95 .55 0.0 -.95 -.55	1.99 1.14 3.206 2.117 1.752	2.105 1.187 3.393 2.202 1.904	81.594 46.199 131.50 85.741 73.051	20.40 11.69 32.87 21.71 17.96
		0.55	.95 .55 0.0 -.95 -.55	14.18 15.74 18.69 77.203 24.151	17.988 19.387 18.115 10.442 36.072	61.942 66.648 57.044 35.662 122.664	20.34 22.56 12.46 34.625
		- 0.95	.95 .55 0.0 -.95	4.102 2.013 1.27 1.607 4.866	143.92 671.11 453.32 863.693	4.352 2.115 1.398 1.634 4.889	42.07 20.64 13.02 16.47

			-.55		189.115		49.90
		-.55	.95 .55 0.0 -.95 -.55	8.069 14.068 7.148 24.73 8.322	37.133 52.502 51.253 107.91 644.93	10.791 15.654 16.881 31.339 13.389	11.56 20.16 10.24 35.45 11.93

Table 5: MSE and PRE of various estimators of μ_{Ysy} for $\rho_{UV} = (0.95, 0.55, 0.0, -0,95, -0.55)$ and $\rho_{XY} = (0.95, 0.55, -0.95, -0.55)$ when $(\sigma_U^2, \sigma_V^2) = (2, 2)$ and $(\sigma_Y^2, \sigma_X^2) = (25, 35)$ and $N = 1000$ and $n = 10$.

σ_U^2	σ_V^2	ρ_{XY}	ρ_{UV}	MSE			var
				$\hat{\mu}_{sym}$	$\hat{\mu}_{Rsym}$	$\hat{\mu}_{Psym}$	$\bar{y}_{sym}$
2	2	0.95	0.95 0.55 0.0 -.95 -.55	1.987 4.224 6.209 8.832 5.389	2.298 4.474 6.763 8.997 7.071	85.301 78.166 121.738 127.646 108.037	20.348 24.923 31.228 40.796 24.903
		0.55	0.95 0.55 0.0 -.95 -.55	6.515 12.925 5.413 13.497 23.983	12.478 14.524 11.423 33.11 26.047	44.475 48.501 33.232 80.573 54.299	9.974 18.447 7.405 17.034 28.124
		-.95	0.95 0.55 0.0 -.95 -.55	10.439 7.815 6.019 2.819 5.886	85.623 144.45 103.521 119.038 160.013	10.506 8.982 6.103 3.808 5.886	30.379 35.953 28.986 26.76 44.336

		-.55	0.95 0.55 0.0 -.95 -.55	12.365 22.751 13.301 18.897 22.275	40.978 84.452 53.519 82.443 74.537	21.946 27.465 17.381 22.246 23.301	13.634 30.828 17.971 28.326 31.935

Table 6: MSE and PRE of various estimators of μ_{Ysy} for $\rho_{UV} = (0.95, 0.55, 0.0, -0,95, -0.55)$ and $\rho_{XY} = (0.95, 0.55, -0.95, -0.55)$ when $\left(\sigma_U^2, \sigma_V^2\right) = (4,4)$ and $\left(\sigma_Y^2, \sigma_X^2\right) = (25,35)$ and $N = 1000$ and $n = 10$.

σ_U^2	σ_V^2	ρ_{XY}	ρ_{UV}	MSE			var
				$\hat{\mu}_{sym}$	$\hat{\mu}_{Rsym}$	$\hat{\mu}_{Psym}$	$\bar{y}_{sym}$
4	4	0.95	0.95 0.55 0.0 -.95 -.55	2.179 2.738 14.226 14.163 13.767	2.191 3.255 15.702 17.719 15.059	79.529 48.267 153.798 55.637 61.066	21.996 11.824 42.31 19.349 22.006
		0.55	0.95 0.55 0.0 -.95 -.55	23.383 20.011 26.354 31.146 14.102	34.379 24.771 32.019 40.656 22.02	130.356 84.505 92.99 83.963 48.613	35.727 28.564 34.714 35.522 16.444
		-0.95	0.95 0.55 0.0 -.95 -.55	15.811 11.338 5.359 2.522 4.189	58.113 79.016 74.701 82.203 101.412	18.213 12.265 6.132 3.128 4.311	21.947 24.529 19.226 19.12 26.803
		-0.55	0.95 0.55 0.0 -.95 -.55	17.646 14.695 17.007 19.215 8.308	62.357 70.549 70.806 113.737 54.996	39.282 28.031 22.684 29.443 19.39	18.682 18.347 23.137 29.856 11.378

Results from the tables:

From the table 1 for $\rho_{XY} = 0.95$ and $\rho_{UV} = (0.95, 0.55, 0.0, -0.95, -0.55)$ as the correlation between the study variable Y and auxiliary variable X is positively correlated the ratio estimator performs well. As observed from the table when $\rho_{XY} = 0.95$ the mse of the ratio estimator is very less as compared to the mse of product estimator but the mean square error for the proposed class of estimators is less than that of the ratio estimator. From the table 1, for $\rho_{XY} = 0.55$ and $\rho_{UV} = (0.95, 0.55, 0.0, -0.95, -0.55)$ as the correlation between the study variable Y and auxiliary variable X is positively correlated the ratio estimator performs well. As observed from the table when $\rho_{XY} = 0.55$ the mse of the ratio estimator is very less as compared to the product estimator but the mse for the proposed class of estimators is less than that of the ratio estimator.

From the table 1 for $\rho_{XY} = -0.95$ and $\rho_{UV} = (0.95, 0.55, 0.0, -0.95, -0.55)$ as the correlation between the study variable Y and auxiliary variable X is negatively correlated the product estimator performs well. As observed from the table when $\rho_{XY} = -0.95$ the mean square error of the product estimator is very less as compared to mse of the ratio estimator but the mse for the proposed class of estimators is less than that of the product estimator.

From the table 1 for $\rho_{XY} = -0.55$ and $\rho_{UV} = (0.95, 0.55, 0.0, -0.95, -0.55)$ as the correlation between the study variable Y and auxiliary variable X is negatively correlated the product estimator performs well. As observed from the table when $\rho_{XY} = -0.55$ the mse of the product estimator is very less as compared to the mean square error of ratio estimator but the mean square error for the proposed class of estimators is less than that of the product estimator. The above results are observed when auxiliary variable X, study variable Y and the error variables U and V are with mean $(\mu_{Xsy}, \mu_{Ysy}, 0, 0)$ and the population variance of the study variable and auxiliary variable X, $Y$are $(15, 20)$ and the variance of the measurement errors U and V are taken as $(0,0), (2,2)$ and $(4,4)$. A similar pattern is observed as described above is followed when the population variance of the study variable and auxiliary variable X, Y are increased to (25,35) and for the same values of variance of the measurement errors U and V.

Conclusions

In this manuscript, the effect of measurement error and correlated measurement error on the exponential estimator, under systematic sampling technique to estimate the population mean of the study variable Y by using the auxiliary information has been studied. The observation is assumed to be recorded with some errors. Also, the error present in study and auxiliary variable are assumed to be correlated. From the simulation study, it is observed that proposed method of estimation is more efficient than ratio and product and mean estimator . From simulation study, it is concluded that the mse in the presence of measurement error as well as in the presence of correlated measurement error is always high. Due to simplicity, systematic sampling has wide applicability and thus, the proposed method of estimation can be applied to many real-life events under systematic sampling survey where the data are prone to recorded with measurement error and as well as correlated measurement error. The limitation of the study is that the method of estimation is derived for systematic sampling.

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