

Continuous and Irresolute Maps Via δ -open Sets in Fermatean Fuzzy Topological Spaces And Application of MCDM Techniques

A. Vadivel¹, V. Sagunthaladevi² and S. Priya³

¹PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal - 637 002, India.

avmaths@gmail.com, av12582@annamalaiuniversity.ac.in

³Department of Mathematics, M.Kumarasamy College of Engineering, Karur - 639 113, India.

sagunthala98v@gmail.com

^{1,2,3}Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.

pre9433@gmail.com

Corresponding Author: S. Priya and V. Sagunthaladevi

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Abstract: In this paper, we develop the concept of Fermatean fuzzy (resp. δ , $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ & $\delta\beta$ or e^*)-continuity in Fermatean fuzzy topological spaces and specialize some of their basic properties with examples. Also, we discuss about properties and characterization of Fermatean fuzzy irresolute maps and application of Multiple Criteria Decision Making (MCDM) techniques to the real-world problem using a proposed entropy measure in Fermatean fuzzy topological spaces.

Keywords: $\mathfrak{F}\delta os$, $\mathfrak{F}\delta Cts$, $\mathfrak{F}\delta Irr$ and Fermatean fuzzy entropy measure.

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1 Introduction

Fuzzy sets were introduced Zadeh [15] in 1965. The fuzzy set concept was the basis of mathematical testing of the fuzzy concept that exists in our real world and the formation of new branches in mathematics. The fuzzy set concept corresponding to unexplained physical situations gives useful applications on many topics such as statistics, data processing and linguistics. A lot of research has been done on this subject since 1965. In 1968, Chang [6] defined the concept of fuzzy topological space and generalized some basic notions of topology such as open set, closed set, continuity and compactness to fuzzy topological spaces. The idea of intuitionistic fuzzy set was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2, 5]. Coker [7] initiated a study of intuitionistic fuzzy topological spaces. Later Yager [13] launched a non standard fuzzy set referred to as Pythagorean fuzzy set. Olgun et al., [9] defined a Pythagorean fuzzy topological spaces. Fermatean fuzzy sets proposed by Senapati and Yager in 2020 [10], can handle uncertain information more easily in the process of decision making. They defined basic operations over the Fermatean fuzzy sets. Hariwan Z. Ibrahim defined a Fermatean fuzzy topological spaces and the continuity of a function defined among Fermatean fuzzy topological spaces.

The aim of this paper is as follows. In Section 2, some basic definitions of fs 's, ifs 's, pfs 's and Fermatean fuzzy sets are briefly reviewed. In section 3 and 4, we develop the concept of some stronger

and weaker forms of Fermatean fuzzy continuous and irresolute maps in Fermatean fuzzy topological spaces and also specialized some of their basic properties with examples. In section 5, entropy measure was introduced by Zadeh [16] for classical fuzzy sets. Many authors developed and created for their version of entropy measure. Here we introduce entropy measure for Fermatean fuzzy sets and give an example for the decision making in real life problem. The paper is concluded in section 6.

2 Preliminaries

We recall some basic notions of fuzzy sets, *IFS*'s, *PFS*'s and *FFS*'s.

Definition 2.1 [15] *Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A: X \rightarrow [0,1]$. That is:*

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$ or $A = \left\{ \left(\frac{\mu_A(x)}{x} \right) \mid x \in X \right\}$, where the function $\mu_A(x): X \rightarrow [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A , where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0,1]$.

Let us consider two examples:

(i) all employees of *XYZ* who are over 1.8m in height; (ii) all employees of *XYZ* who are tall. The first example is a classical set with a universe (all *XYZ* employees) and a membership rule that divides the universe into members (those over 1.8m) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ . If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2 [1] *The intuitionistic fuzzy sets are defined on a non-empty sets X as objects having the form $I = \{ \langle x, \alpha_I(x), \beta_I(x) \rangle : x \in X \}$, where $\alpha_I(x): X \rightarrow [0,1]$ and $\beta_I(x): X \rightarrow [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set I ,*

respectively, and $0 \leq \alpha_I(x) + \beta_I(x) \leq 1$, for all $x \in X$.

Definition 2.3 [1, 2, 3, 4] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in X \}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle \mid x \in X \right\}$, where the functions $\mu_A(x): X \rightarrow [0,1]$ and $\lambda_A(x): X \rightarrow [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A , which is a subset of X , and for every $x \in X$: $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For each A in X : $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X . The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.

Example 2.1 Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{ \left\langle \frac{0.6, 0.1}{x} \right\rangle, \left\langle \frac{0.8, 0.1}{y} \right\rangle, \left\langle \frac{0.5, 0.3}{z} \right\rangle \right\}$, be the intuitionistic fuzzy set in X . The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3, \pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.

Definition 2.4 [12, 13, 14] Let X be a universal set. Then, a Pythagorean fuzzy set A , which is a set of ordered pairs over X , is defined by the following: $A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in X \}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle \mid x \in X \right\}$, where the functions $\mu_A(x): X \rightarrow [0,1]$ and $\lambda_A(x): X \rightarrow [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A , which is a subset of X , and for every $x \in X$, $0 \leq (\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.

Definition 2.5 [10] Let X be a universe of discourse. A Fermatean fuzzy set (\mathfrak{F} FS) F in X is an object having the form $F = \{ \langle x, \alpha_F(x), \beta_F(x) \rangle : x \in X \}$ where $\alpha_F(x): X \rightarrow [0,1]$ and $\beta_F(x): X \rightarrow [0,1]$, including the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\alpha_F(x)$ and $\beta_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F . For any \mathfrak{F} FS F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\alpha_F(x))^3 + (\beta_F(x))^3]}$ is identified as the degree of interminancy of x to F . In the interest of simplicity, we shall mention the symbol $F = (\alpha_F, \beta_F)$ for the \mathfrak{F} FS $F = \{ \langle x, \alpha_F(x), \beta_F(x) \rangle : x \in X \}$.

Definition 2.6 [10] Let $F = (\alpha_F, \beta_F)$, $F_1 = (\alpha_{F_1}, \beta_{F_1})$ and $F_2 = (\alpha_{F_2}, \beta_{F_2})$, be three Fermatean fuzzy sets (\mathfrak{F} FS's), then their operations are defined as follows:

1. $F_1 \cap F_2 = (\min\{\alpha_{F_1}, \alpha_{F_2}\}, \max\{\beta_{F_1}, \beta_{F_2}\})$.
2. $F_1 \cup F_2 = (\max\{\alpha_{F_1}, \alpha_{F_2}\}, \min\{\beta_{F_1}, \beta_{F_2}\})$.
3. $F^c = (\beta_F, \alpha_F)$.

Remark 2.1 If $\alpha_{F_1} = \alpha_{F_2}$ and $\beta_{F_1} = \beta_{F_2}$, then $F_1 = F_2$

Note that, for understanding the Fermatean fuzzy set better, we give an instance to illuminate the

understandability of the Fermatean fuzzy set. The point when someone needs will plan as much craving for the level for an alternative s_i on a criterion C_j , he might provide for the degree on which that alternative s_i fulfils those criteria C_j likewise 0.85, what is more correspondingly the elective s_i dissatisfies the criterion C_j similarly as 0.65. We can definitely get $0.85 + 0.65 = 1.5 > 1$, and therefore, it does not follow the condition of intuitionistic fuzzy sets. Also, we can get $(0.85)^2 + (0.65)^2 = 0.7225 + 0.4225 = 1.145 > 1$, which does not obey the constraint condition of Pythagorean fuzzy set. However, we can get $(0.85)^3 + (0.65)^3 = 0.614125 + 0.274625 = 0.88875 \leq 1$, which is good enough to apply the Fermatean fuzzy set to control it [10]. Throughout this paper, we use the notation $1_{\mathfrak{F}}$ for the Fermatean fuzzy subset $(1,0)$ and we use the notation $0_{\mathfrak{F}}$ for the Fermatean fuzzy subset $(0,1)$, that is, $\alpha_{1_{\mathfrak{F}}} = 1, \beta_{1_{\mathfrak{F}}} = 0, \alpha_{0_{\mathfrak{F}}} = 0, \beta_{0_{\mathfrak{F}}} = 1$. A Fermatean fuzzy subset \mathfrak{F} of a non-empty set X is a pair $(\alpha_{\mathfrak{F}}, \beta_{\mathfrak{F}})$ of a membership function $(\alpha_{\mathfrak{F}}(x): X \rightarrow [0,1])$ and a non-membership function $(\beta_{\mathfrak{F}}(x): X \rightarrow [0,1])$ with $(\alpha_{\mathfrak{F}}(x))^3 + (\beta_{\mathfrak{F}}(x))^3 = (\gamma_{\mathfrak{F}}(x))^3$ for any $x \in X$ where $\gamma_{\mathfrak{F}}(x): X \rightarrow [0,1]$ is a function which is called the strength of commitment at point x .

Definition 2.7 [8] *Let X be a non empty set and τ be a family of Fermatean fuzzy subsets of X . If*

1. $1_{\mathfrak{F}}, 0_{\mathfrak{F}} \in \tau$
2. for any $F_1, F_2 \in \tau$, we have $F_1 \cap F_2 \in \tau$,
3. for any $\{F_i\}_{i \in I} \subset \tau$, we have $\cup_{i \in I} F_i \in \tau$ where I is an arbitrary index set then τ is called a Fermatean fuzzy topology on X .

The pair (X, τ) is said to be a Fermatean fuzzy topological space. Each member of τ is called an Fermatean fuzzy oprn set. The complement of an Fermatean fuzzy open set is called a Fermatean fuzzy closed set.

Remark 2.2 [8] *As any Intuitionistic fuzzy subset or Pythagorean fuzzy subset of a set can be considered as Fermatean fuzzy subset, we observe that any Intuitionstic fuzzy topological space or Pythagorean fuzzy topological space is a Fermatean fuzzy topological space as well. On the other hand, it is obvious that a Fermatean fuzzy topological space need not be Intuitionistic fuzzy topological space and Pythagorean fuzzy topological space. Even an Fermatean fuzzy open set maybe neither an Intuitionistic fuzzy set nor Pythagorean fuzzy set.*

Example 2.2 [8] *Let $X = \{c_1, c_2\}$. Consider the following family Fermatean fuzzy subsets $\tau = \{1_{\mathfrak{F}}, 0_{\mathfrak{F}}, F_1, F_2\}$ where*

$$F_1 = \{\langle c_1, \alpha_{F_1}(c_1) = 0.4, \beta_{F_1}(c_1) = 0.6 \rangle, \langle c_2, \alpha_{F_1}(c_2) = 0.1, \beta_{F_1}(c_2) = 0.3 \rangle\} \text{ and}$$

$$F_2 = \{\langle c_1, \alpha_{F_2}(c_1) = 0.9, \beta_{F_2}(c_1) = 0.6 \rangle, \langle c_2, \alpha_{F_2}(c_2) = 0.2, \beta_{F_2}(c_2) = 0.3 \rangle\}.$$

Observe that (X, τ) is a Fermatean fuzzy topological space but (X, τ) is neither Intuitionistic fuzzy topological space nor Pythagorean fuzzy topological space.

Definition 2.8 [8] *Let (X, τ) be an $\mathfrak{F}Fts$ and $A = \{\langle a, \alpha_A(a), \beta_A(a) \rangle \mid a \in X\}$ be an $\mathfrak{F}Fs$ in X . Then the Fermatean fuzzy interior and the Fermatean fuzzy closure of A are denoted by $\mathfrak{F}Fint(A)$ and $\mathfrak{F}Fcl(A)$ and are defined as follows:*

$\mathfrak{F}int(A) = \cup \{G | G \text{ is a } \mathfrak{F}os \text{ and } G \subseteq A\}$ and $\mathfrak{F}cl(A) = \cap \{K | K \text{ is a } \mathfrak{F}cs \text{ and } A \subseteq K\}$. Also, it can be established that $\mathfrak{F}cl(A)$ is an $\mathfrak{F}cs$ and $\mathfrak{F}int(A)$ is an $\mathfrak{F}os$, A is an $\mathfrak{F}cs$ if and only if $\mathfrak{F}cl(A) = A$ and A is an $\mathfrak{F}os$ if and only if $\mathfrak{F}int(A) = A$. We say that A is \mathfrak{F} -dense if $\mathfrak{F}cl(A) = 1_{\mathfrak{F}}$.

Lemma 2.1 [8] For any Fermatean fuzzy set A in (X, τ) , we have $1_{\mathfrak{F}} - \mathfrak{F}int(A) = \mathfrak{F}cl(1_{\mathfrak{F}} - A)$ and $1_{\mathfrak{F}} - \mathfrak{F}cl(A) = \mathfrak{F}int(1_{\mathfrak{F}} - A)$.

Definition 2.9 [11] Let (X, τ) be an $\mathfrak{F}ts$ and A be an $\mathfrak{F}S$. Then A is said to be an Fermatean fuzzy (i) regular open set ($\mathfrak{F}ros$ in short) if $A = \mathfrak{F}int(\mathfrak{F}cl(A))$. (ii) regular closed set ($\mathfrak{F}rcs$ in short) if $A = \mathfrak{F}cl(\mathfrak{F}int(A))$. By Lemma 2.1, it follows that A is an $\mathfrak{F}ros$ iff \bar{A} is an $\mathfrak{F}rcs$.

Definition 2.10 [11] Let (X, τ) be an $\mathfrak{F}ts$ and $A = \{ \langle a, \alpha_A(a), \beta_A(a) \rangle | a \in X \}$ be an $\mathfrak{F}S$ in X . Then the δ -interior and the δ -closure of A are denoted by $\mathfrak{F}\delta int(A)$ and $\mathfrak{F}\delta cl(A)$ and are defined as follows. $\mathfrak{F}\delta int(A) = \cup \{G | G \text{ is an } \mathfrak{F}ros \text{ and } G \subseteq A\}$, $\mathfrak{F}\delta cl(A) = \cap \{K | K \text{ is an } \mathfrak{F}rcs \text{ and } A \subseteq K\}$.

Definition 2.11 [11] Let (X, τ) be an $\mathfrak{F}ts$ and $A = \{ \langle a, \alpha_A(a), \beta_A(a) \rangle | a \in X \}$ be an $\mathfrak{F}S$ in X . A set A is said to be \mathfrak{F}

1. δ -open set (briefly, $\mathfrak{F}\delta os$) if $A = \mathfrak{F}\delta int(A)$,
2. δ -pre open set (briefly, $\mathfrak{F}\delta P os$) if $A \subseteq \mathfrak{F}int(\mathfrak{F}\delta cl(A))$.
3. δ -semi open set (briefly, $\mathfrak{F}\delta S os$) if $A \subseteq \mathfrak{F}cl(\mathfrak{F}\delta int(A))$.
4. $\delta - \alpha$ open set or α -open set (briefly, $\mathfrak{F}\delta \alpha os$ or $\mathfrak{F}\alpha os$) if $A \subseteq \mathfrak{F}int(\mathfrak{F}cl(\mathfrak{F}\delta int(A)))$.
5. $\delta - \beta$ open set or e^* -open set (briefly, $\mathfrak{F}\delta \beta os$ or $\mathfrak{F}e^* os$) if $A \subseteq \mathfrak{F}cl(\mathfrak{F}int(\mathfrak{F}\delta cl(A)))$.
6. δ (resp. δ -pre, δ -semi, $\delta - \alpha$ and $\delta - \beta$) dense if $\mathfrak{F}\delta cl(A)$ (resp. $\mathfrak{F}\delta p cl(A), \mathfrak{F}\delta S cl(A), \mathfrak{F}\delta \alpha cl(A)$ and $\mathfrak{F}\delta \beta cl(A)$) = $1_{\mathfrak{F}}$.

The complement of an $\mathfrak{F}\delta os$ (resp. $\mathfrak{F}\delta P os$, $\mathfrak{F}\delta S os$, $\mathfrak{F}\delta \alpha os$ and $\mathfrak{F}\delta \beta os$) is called an $\mathfrak{F}\delta$ (resp. $\mathfrak{F}\delta P$, $\mathfrak{F}\delta S$, $\mathfrak{F}\delta \alpha$ and $\mathfrak{F}\delta \beta$) closed set (briefly, $\mathfrak{F}\delta cs$ (resp. $\mathfrak{F}\delta P cs$, $\mathfrak{F}\delta S cs$, $\mathfrak{F}\delta \alpha cs$ and $\mathfrak{F}\delta \beta cs$)) in X .

The family of all $\mathfrak{F}\delta os$ (resp. $\mathfrak{F}\delta cs$, $\mathfrak{F}\delta P os$, $\mathfrak{F}\delta P cs$, $\mathfrak{F}\delta S os$, $\mathfrak{F}\delta S cs$, $\mathfrak{F}\delta \alpha os$, $\mathfrak{F}\delta \alpha cs$, $\mathfrak{F}\delta \beta os$ and $\mathfrak{F}\delta \beta cs$) of X is denoted by $\mathfrak{F}\delta OS(X)$, (resp. $\mathfrak{F}\delta CS(X)$, $\mathfrak{F}\delta POS(X)$, $\mathfrak{F}\delta PCS(X)$, $\mathfrak{F}\delta SOS(X)$, $\mathfrak{F}\delta SCS(X)$, $\mathfrak{F}\delta \alpha OS(X)$, $\mathfrak{F}\delta \alpha CS(X)$, $\mathfrak{F}\delta \beta OS(X)$ and $\mathfrak{F}\delta \beta CS(X)$).

Definition 2.12 [11] Let (X, τ) be an $\mathfrak{F}ts$ and $A = \{ \langle a, \alpha_A(a), \beta_A(a) \rangle | a \in X \}$ be an $\mathfrak{F}S$ in X . Then the $\mathfrak{F}\delta$ -pre (resp. $\mathfrak{F}\delta$ -semi, $\mathfrak{F}\delta \alpha$ and $\mathfrak{F}\delta \beta$)-interior and the $\mathfrak{F}\delta$ -pre (resp. $\mathfrak{F}\delta$ -semi, $\mathfrak{F}\delta \alpha$ and $\mathfrak{F}\delta \beta$)-closure of A are denoted by $\mathfrak{F}\delta Pint(A)$ (resp. $\mathfrak{F}\delta Sint(A)$, $\mathfrak{F}\delta \alpha int(A)$ and $\mathfrak{F}\delta \beta int(A)$) and the $\mathfrak{F}\delta P cl(A)$ (resp. $\mathfrak{F}\delta S cl(A), \mathfrak{F}\delta \alpha cl(A)$ and $\mathfrak{F}\delta \beta cl(A)$) and are defined as follows:

$\mathfrak{F}\mathcal{F}\delta\mathcal{P}int(A)$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(A)$, $\mathfrak{F}\mathcal{F}\delta\alpha int(A)$ and $\mathfrak{F}\mathcal{F}\delta\beta int(A)$) $= \cup \{G | G \text{ in a } \mathfrak{F}\mathcal{F}\delta\mathcal{P}os \text{ (resp. } \mathfrak{F}\mathcal{F}\delta\mathcal{S}os, \mathfrak{F}\mathcal{F}\delta\alpha os \text{ and } \mathfrak{F}\mathcal{F}\delta\beta os) \text{ and } G \subseteq A\}$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cl(A)$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(A)$, $\mathfrak{F}\mathcal{F}\delta\alpha cl(A)$ and $\mathfrak{F}\mathcal{F}\delta\beta cl(A)$) $= \cap \{K | K \text{ is an } \mathfrak{F}\mathcal{F}\delta\mathcal{P}cs \text{ (resp. } \mathfrak{F}\mathcal{F}\delta\mathcal{S}cs, \mathfrak{F}\mathcal{F}\delta\alpha cs, \mathfrak{F}\mathcal{F}\delta\beta cs) \text{ and } A \subseteq K\}$.

3 Fermatean fuzzy δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-continuous mappings

In this section, we introduce Fermatean fuzzy δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-continuous mappings and discuss some of their properties.

Definition 3.1 Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s. Then a function $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ is said to be a Fermatean fuzzy δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$) continuous (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{C}ts$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{C}ts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}\mathcal{C}ts$, $\mathfrak{F}\mathcal{F}\delta\alpha\mathcal{C}ts$ and $\mathfrak{F}\mathcal{F}\delta\beta\mathcal{C}ts$)) function if $h_{\mathfrak{F}}^{-1}(G)$ is $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\delta\alpha o$ & $\mathfrak{F}\mathcal{F}\delta\beta o$) set in X_1 for all $\mathfrak{F}\mathcal{F}o$ set G in X_2 .

Lemma 3.1 Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a function. Then the following statements hold.

1. If S and T are $\mathfrak{F}\mathcal{F}s$'s of X_1 such that $S \subseteq T$, then $h_{\mathfrak{F}}(S) \subseteq h_{\mathfrak{F}}(T)$.
2. If S and T are $\mathfrak{F}\mathcal{F}s$'s of X_2 such that $S \subseteq T$, then $h_{\mathfrak{F}}^{-1}(S) \subseteq h_{\mathfrak{F}}^{-1}(T)$.

Lemma 3.2 Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a function. If S is a $\mathfrak{F}\mathcal{F}s$ of X_1 and T is a $\mathfrak{F}\mathcal{F}s$ of X_2 . Then

1. $h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(S)) \subseteq S$
2. $h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(S)) = S \Leftrightarrow h_{\mathfrak{F}}$ is surjective.
3. $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(S)) \supseteq S$
4. $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(S)) = S$ whenever $h_{\mathfrak{F}}$ is injective.

Theorem 3.1 Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$, then

- (i) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\mathcal{C}ts$.
- (ii) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{C}ts$.
- (iii) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}\mathcal{C}ts$.
- (iv) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\delta\beta\mathcal{C}ts$.
- (v) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\delta\beta\mathcal{C}ts$.
- (vi) Every $\mathfrak{F}\mathcal{F}\delta\alpha\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{C}ts$.
- (vii) Every $\mathfrak{F}\mathcal{F}\delta\alpha\mathcal{C}ts$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}\mathcal{C}ts$.

But not converse.

Proof. (i) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta\mathcal{C}ts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta o$ set is $\mathfrak{F}\mathcal{F}o$ s, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{C}ts$ function.

(ii) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta Cts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta o$ set is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$ function.

(iii) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta Cts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta o$ set is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ function.

(iv) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set is $\mathfrak{F}\mathcal{F}\delta\beta os$, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\beta o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta Cts$ function.

(v) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ set is $\mathfrak{F}\mathcal{F}\delta\beta os$, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\beta o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta Cts$ function.

(vi) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta\alpha Cts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\alpha o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta\alpha o$ set is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$ function.

(vii) Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta\alpha Cts$. Let S be a $\mathfrak{F}\mathcal{F}o$ set in (X_2, τ_2) . Then $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\alpha o$ set in (X_1, τ_1) . Since every $\mathfrak{F}\mathcal{F}\delta\alpha o$ set is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$, $h_{\mathfrak{F}}^{-1}(S)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in (X_1, τ_1) . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ function.

Remark 3.1 The following Figure shows the relations among the different types of Fermatean fuzzy δ continuous mappings that were studied in this section.

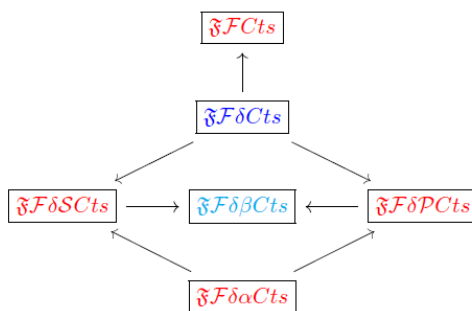


Figure : $\mathfrak{F}\mathcal{F}\delta Cts$ mappings in $\mathfrak{F}\mathcal{F}ts$

Example 3.1 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1 and A_2 are defined as

$$\alpha_{A_1}(a) = 0.4, \beta_{A_1}(a) = 0.1, \alpha_{A_1}(b) = 0.6, \beta_{A_1}(b) = 0.3;$$

$$\alpha_{A_2}(a) = 0.9, \beta_{A_2}(a) = 0.2, \alpha_{A_2}(b) = 0.6, \beta_{A_2}(b) = 0.3;$$

Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta\beta Cts$) but not $\mathfrak{F}\mathcal{F}\delta Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$). Since, A_2 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_2) = A_2$ is not $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$) set in X_1 .

Example 3.2 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}Fs$'s A_1, A_2 and A_3 are defined as

$$\alpha_{A_1}(a) = 0.2, \beta_{A_1}(a) = 0.8, \alpha_{A_1}(b) = 0.3, \beta_{A_1}(b) = 0.7;$$

$$\alpha_{A_2}(a) = 0.1, \beta_{A_2}(a) = 0.9, \alpha_{A_2}(b) = 0.1, \beta_{A_2}(b) = 0.9;$$

$$\alpha_{A_3}(a) = 0.2, \beta_{A_3}(a) = 0.8, \alpha_{A_3}(b) = 0.4, \beta_{A_3}(b) = 0.6;$$

Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3\}$ be a $\mathfrak{F}Fts$ on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}F\delta SCts$ (resp. $\mathfrak{F}F\delta PCts$) but not $\mathfrak{F}F\delta Cts$. Since, A_1 is a $\mathfrak{F}Fo$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_1) = A_1$ is not $\mathfrak{F}F\delta o$ set in X_1 .

Example 3.3 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}Fs$'s A_1, A_2 and A_3 are defined as

$$\alpha_{A_1}(a) = 0.2, \beta_{A_1}(a) = 0.6, \alpha_{A_1}(b) = 0.3, \beta_{A_1}(b) = 0.5;$$

$$\alpha_{A_2}(a) = 0.6, \beta_{A_2}(a) = 0.2, \alpha_{A_2}(b) = 0.5, \beta_{A_2}(b) = 0.3;$$

$$\alpha_{A_3}(a) = 0.6, \beta_{A_3}(a) = 0.6, \alpha_{A_3}(b) = 0.5, \beta_{A_3}(b) = 0.5;$$

Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3\}$ be a $\mathfrak{F}Fts$ on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}F\delta SCts$ (resp. $\mathfrak{F}F\delta PCts$) but not $\mathfrak{F}FaCts$. Since, A_3 is a $\mathfrak{F}Fo$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_3) = A_3$ is not $\mathfrak{F}Fa o$ set in X_1 .

Example 3.4 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}Fs$'s $A_1, A_2, A_3, A_4, B_1, B_2, B_3$ and B_4 are defined as

$$\alpha_{A_1}(a) = 0.4, \beta_{A_1}(a) = 0.6, \alpha_{A_1}(b) = 0.5, \beta_{A_1}(b) = 0.5;$$

$$\alpha_{A_2}(a) = 0.6, \beta_{A_2}(a) = 0.4, \alpha_{A_2}(b) = 0.6, \beta_{A_2}(b) = 0.4;$$

$$\alpha_{A_3}(a) = 0.7, \beta_{A_3}(a) = 0.3, \alpha_{A_3}(b) = 0.6, \beta_{A_3}(b) = 0.4;$$

$$\alpha_{A_4}(a) = 0.4, \beta_{A_4}(a) = 0.6, \alpha_{A_4}(b) = 0.4, \beta_{A_4}(b) = 0.6;$$

$$\alpha_{B_1}(a) = 0.2, \beta_{B_1}(a) = 0.8, \alpha_{B_1}(b) = 0.4, \beta_{B_1}(b) = 0.6;$$

$$\alpha_{B_2}(a) = 0.1, \beta_{B_2}(a) = 0.9, \alpha_{B_2}(b) = 0.3, \beta_{B_2}(b) = 0.7;$$

$$\alpha_{B_3}(a) = 0.9, \beta_{B_3}(a) = 0.1, \alpha_{B_3}(b) = 0.7, \beta_{B_3}(b) = 0.3;$$

$$\alpha_{B_4}(a) = 0.2, \beta_{B_4}(a) = 0.8, \alpha_{B_4}(b) = 0.3, \beta_{B_4}(b) = 0.7;$$

Let $\tau_1 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, B_1, B_2, B_3, B_4\}$ are $\mathfrak{F}Fts$'s on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}F\delta\beta Cts$ but not $\mathfrak{F}F\delta PCts$. Since, B_4 is a $\mathfrak{F}Fo$ set in X_2 but $h_{\mathfrak{F}}^{-1}(B_4) = B_4$ is not $\mathfrak{F}F\delta Po$ set in X_1 .

Theorem 3.2 Let (X_1, τ_1) & (X_2, τ_2) be a $\mathfrak{F}Fts$'s. A mapping $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ satisfies the following conditions are equivalent.

1. $h_{\mathfrak{F}}$ is $\mathfrak{F}F\delta\beta Cts$.
2. The inverse $h_{\mathfrak{F}}^{-1}(K)$ of all $\mathfrak{F}Fcs$ set K in X_2 is $\mathfrak{F}F\delta\beta cs$ in X_1 .

Proof. The proof is directly, from $h_{\mathfrak{F}}^{-1}(\overline{K}) = \overline{h_{\mathfrak{F}}^{-1}(K)}$ for all $\mathfrak{F}Cs$ K of X_2 .

Theorem 3.3 Let (X_1, τ_1) & (X_2, τ_2) be a $\mathfrak{F}Ts$'s. A mapping $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ satisfies the following conditions are hold.

- (i) $h_{\mathfrak{F}}(\mathfrak{F}\delta\beta cl(L)) \subseteq \mathfrak{F}\delta cl(h_{\mathfrak{F}}(L))$, for all $\mathfrak{F}Cs$ L in X_1 .
- (ii) $\mathfrak{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(K)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(K))$, for all $\mathfrak{F}Cs$ K in X_2 .

Proof. (i) Since $\mathfrak{F}\delta cl(h_{\mathfrak{F}}(L))$ is a $\mathfrak{F}\delta Cs$ in X_2 and $h_{\mathfrak{F}}$ is $\mathfrak{F}\delta\beta Cts$, then $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(h_{\mathfrak{F}}(L)))$ is $\mathfrak{F}\delta\beta Cs$ in X_1 . Now, since $L \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(h_{\mathfrak{F}}(L)))$, $\mathfrak{F}\delta\beta cl(L) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(h_{\mathfrak{F}}(L)))$. Therefore, $h_{\mathfrak{F}}(\mathfrak{F}\delta\beta cl(L)) \subseteq \mathfrak{F}\delta cl(h_{\mathfrak{F}}(L))$.

(ii) By replacing L with K in (i), we obtain $h_{\mathfrak{F}}(\mathfrak{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(K))) \subseteq \mathfrak{F}\delta cl(h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(K))) \subseteq \mathfrak{F}\delta cl(K)$. Hence, $\mathfrak{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(K)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(K))$.

Remark 3.2 Let (X_1, τ_1) & (X_2, τ_2) be a $\mathfrak{F}Ts$'s. Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a mapping. If $h_{\mathfrak{F}}$ is $\mathfrak{F}\delta\beta Cts$, then

- 1. $h_{\mathfrak{F}}(\mathfrak{F}\delta\beta cl(L))$ is not necessarily equal to $\mathfrak{F}\delta cl(h_{\mathfrak{F}}(L))$ where $L \in X_1$.
- 2. $\mathfrak{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(K))$ is not necessarily equal to $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(K))$ where $K \in X_2$.

Example 3.5 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}S$'s $A_1, A_2, A_3, A_4, B_1, B_2, B_3$ and B_4 are defined as

$$\begin{aligned} \alpha_{A_1}(a) &= 0.4, \beta_{A_1}(a) = 0.6, \alpha_{A_1}(b) = 0.5, \beta_{A_1}(b) = 0.5; \\ \alpha_{A_2}(a) &= 0.6, \beta_{A_2}(a) = 0.4, \alpha_{A_2}(b) = 0.6, \beta_{A_2}(b) = 0.4; \\ \alpha_{A_3}(a) &= 0.7, \beta_{A_3}(a) = 0.3, \alpha_{A_3}(b) = 0.6, \beta_{A_3}(b) = 0.4; \\ \alpha_{A_4}(a) &= 0.4, \beta_{A_4}(a) = 0.6, \alpha_{A_4}(b) = 0.4, \beta_{A_4}(b) = 0.6; \\ \alpha_{B_1}(a) &= 0.2, \beta_{B_1}(a) = 0.8, \alpha_{B_1}(b) = 0.4, \beta_{B_1}(b) = 0.6; \\ \alpha_{B_2}(a) &= 0.1, \beta_{B_2}(a) = 0.9, \alpha_{B_2}(b) = 0.3, \beta_{B_2}(b) = 0.7; \\ \alpha_{B_3}(a) &= 0.9, \beta_{B_3}(a) = 0.1, \alpha_{B_3}(b) = 0.7, \beta_{B_3}(b) = 0.3; \\ \alpha_{B_4}(a) &= 0.2, \beta_{B_4}(a) = 0.8, \alpha_{B_4}(b) = 0.3, \beta_{B_4}(b) = 0.7; \end{aligned}$$

Let $\tau_1 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, B_1, B_2, B_3, B_4\}$ and $\tau_2 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3, A_4\}$ are $\mathfrak{F}Ts$'s on X and let

$h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\delta\beta Cts$, [(i)]

1. $h_{\mathfrak{F}}(\mathfrak{F}\delta\beta cl(A_1)) = A_1$. But $\mathfrak{F}\delta cl(h_{\mathfrak{F}}(A_1)) = A_1^c$. Thus $h_{\mathfrak{F}}(\mathfrak{F}\delta\beta cl(A_1)) \neq \mathfrak{F}\delta cl(h_{\mathfrak{F}}(A_1))$.

2. $\mathfrak{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(A_1)) = A_1$. But $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(A_1)) = A_1^c$. Thus $\mathfrak{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(A_1)) \neq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\delta cl(A_1))$.

Theorem 3.4 Let (X_1, τ_1) & (X_2, τ_2) be a \mathfrak{F} Fts's. Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a mapping. If $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta$ Cts, then $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L)) \subseteq \mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L))$, for all $\mathfrak{F}\mathcal{F}$ s L in X_2 .

Proof. If $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta$ Cts and $L \subseteq X_2$. $\mathfrak{F}\mathcal{F}\delta\text{int}(L)$ is $\mathfrak{F}\mathcal{F}\delta$ os in X_2 and hence, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L))$ is $\mathfrak{F}\mathcal{F}\delta\beta$ os in X_1 . Therefore $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L))$. Also, $\mathfrak{F}\mathcal{F}\delta\text{int}(L) \subseteq L$, implies that $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L)) \subseteq h_{\mathfrak{F}}^{-1}(L)$. Therefore $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L))) \subseteq \mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L))$. That is $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L)) \subseteq \mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L))$.

Conversely, let $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L)) \subseteq \mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L))$ for all subset L of X_2 . If L is $\mathfrak{F}\mathcal{F}\delta$ o in X_2 , then $\mathfrak{F}\mathcal{F}\delta\text{int}(L) = L$. By assumption, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(L)) \subseteq \mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L))$. Thus $h_{\mathfrak{F}}^{-1}(L) \subseteq \mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L))$. But $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(L)$. Therefore $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(L)) = h_{\mathfrak{F}}^{-1}(L)$. That is, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\delta\beta$ o in X_1 , for all $\mathfrak{F}\mathcal{F}\delta$ os L in X_2 . Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta$ Cts in X_1 .

Remark 3.3 Let (X_1, τ_1) & (X_2, τ_2) be a \mathfrak{F} Fts's. Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a mapping. If $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta$ Cts, then $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(K))$ is not necessarily equal to $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(K))$ where $K \in X_2$.

Example 3.6 In Example 3.5, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta\beta$ Cts. Then $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(A_1)) = A_1$. But $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(A_1)) = A_1^c$. Thus $\mathfrak{F}\mathcal{F}\delta\beta\text{int}(h_{\mathfrak{F}}^{-1}(A_1)) \neq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\text{int}(A_1))$.

Remark 3.4 Theorems 3.2, 3.3, 3.4 and Remarks 3.2, 3.3 are true for $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ os and $\mathfrak{F}\mathcal{F}\delta\alpha$ os.

4 Fermatean fuzzy δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-irresolute maps

In this section, we introduce the concept of Fermatean fuzzy irresoluteness called Fermatean fuzzy (resp. δ , $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$)-irresolute maps by using $\mathfrak{F}\mathcal{F}\mathcal{S}$ os (resp. $\mathfrak{F}\mathcal{F}\delta$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ os, $\mathfrak{F}\mathcal{F}\delta\alpha$ os and $\mathfrak{F}\mathcal{F}\delta\beta$ os)'s and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve respective open sets.

Definition 4.1 A map $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ is said to be Fermatean fuzzy (resp. δ , $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$)-irresolute (in short, $\mathfrak{F}\mathcal{F}$ Irr (resp. $\mathfrak{F}\mathcal{F}\delta$ Irr, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ Irr, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ Irr, $\mathfrak{F}\mathcal{F}\delta\alpha$ Irr and $\mathfrak{F}\mathcal{F}\delta\beta$ Irr)) map if $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\mathcal{S}$ os (resp. $\mathfrak{F}\mathcal{F}\delta$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ os, $\mathfrak{F}\mathcal{F}\delta\alpha$ os and $\mathfrak{F}\mathcal{F}\delta\beta$ os) in (X_1, τ_1) for each $\mathfrak{F}\mathcal{F}\mathcal{S}$ os (resp. $\mathfrak{F}\mathcal{F}\delta$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ os, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ os, $\mathfrak{F}\mathcal{F}\delta\alpha$ os and $\mathfrak{F}\mathcal{F}\delta\beta$ os) K of (X_2, τ_2) .

Theorem 4.1 Let (X_1, τ_1) & (X_2, τ_2) be a \mathfrak{F} Fts's. Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a mapping. Then the following statements are hold for $\mathfrak{F}\mathcal{F}$ Ts, but not conversely.

- (i) Every $\mathfrak{F}\mathcal{F}$ Irr map is a $\mathfrak{F}\mathcal{F}\mathcal{S}$ Cts.
- (ii) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ Irr map is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ Cts.
- (iii) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ Irr map is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$ Cts.
- (iv) Every $\mathfrak{F}\mathcal{F}\delta\alpha$ Irr map is a $\mathfrak{F}\mathcal{F}\delta\alpha$ Cts.
- (v) Every $\mathfrak{F}\mathcal{F}\delta\beta$ Irr map is a $\mathfrak{F}\mathcal{F}\delta\beta$ Cts.

Proof. (i) Consider a $\mathfrak{F}\mathcal{F}$ Irr map $h_{\mathfrak{F}}$ and a $\mathfrak{F}\mathcal{F}$ os K in X_2 . As each $\mathfrak{F}\mathcal{F}$ os is a $\mathfrak{F}\mathcal{F}\mathcal{S}$ os, K is a

$\mathfrak{F}Sos$ in X_2 . By presumption, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}Sos$ in X_1 . Thus $h_{\mathfrak{F}}$ is a $\mathfrak{F}Scts$ map.

(ii) Consider a $\mathfrak{F}\delta S Irr$ map $h_{\mathfrak{F}}$ and a $\mathfrak{F}\delta os$ K in X_2 . As each $\mathfrak{F}\delta os$ is a $\mathfrak{F}os$ and $\mathfrak{F}\delta S os$, K is a $\mathfrak{F}\delta os$ and $\mathfrak{F}\delta S os$ in X_2 . By presumption, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\delta S os$ in X_1 . Thus $h_{\mathfrak{F}}$ is a $\mathfrak{F}\delta Scts$ map.

(iii) Consider a $\mathfrak{F}\delta P Irr$ map $h_{\mathfrak{F}}$ and a $\mathfrak{F}\delta os$ K in X_2 . As each $\mathfrak{F}\delta os$ is a $\mathfrak{F}os$ and $\mathfrak{F}\delta P os$, K is a $\mathfrak{F}\delta os$ and $\mathfrak{F}\delta P os$ in X_2 . By presumption, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\delta P os$ in X_1 . Thus $h_{\mathfrak{F}}$ is a $\mathfrak{F}\delta Pcts$ map.

(iv) Consider a $\mathfrak{F}\delta\alpha Irr$ map $h_{\mathfrak{F}}$ and a $\mathfrak{F}\delta os$ K in X_2 . As each $\mathfrak{F}\delta os$ is a $\mathfrak{F}os$ and $\mathfrak{F}\delta\alpha os$, K is a $\mathfrak{F}\delta os$ and $\mathfrak{F}\delta\alpha os$ in X_2 . By presumption, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\delta\alpha os$ in X_1 . Thus $h_{\mathfrak{F}}$ is a $\mathfrak{F}\delta\alpha cts$ map.

(v) Consider a $\mathfrak{F}\delta\beta Irr$ map $h_{\mathfrak{F}}$ and a $\mathfrak{F}\delta os$ K in X_2 . As each $\mathfrak{F}\delta os$ is a $\mathfrak{F}os$ and $\mathfrak{F}\delta\beta os$, K is a $\mathfrak{F}\delta os$ and $\mathfrak{F}\delta\beta os$ in X_2 . By presumption, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\delta\beta os$ in X_1 . Thus $h_{\mathfrak{F}}$ is a $\mathfrak{F}\delta\beta cts$ map.

Example 4.1 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}S$'s A_1, A_2, B_1 and B_2 are defined as

$$\alpha_{A_1}(a) = 0.2, \beta_{A_1}(a) = 0.7, \alpha_{A_1}(b) = 0.1, \beta_{A_1}(b) = 0.8;$$

$$\alpha_{A_2}(a) = 0.3, \beta_{A_2}(a) = 0.6, \alpha_{A_2}(b) = 0.4, \beta_{A_2}(b) = 0.5;$$

$$\alpha_{B_1}(a) = 0.1, \beta_{B_1}(a) = 0.9, \alpha_{B_1}(b) = 0.2, \beta_{B_1}(b) = 0.9;$$

$$\alpha_{B_2}(a) = 0.2, \beta_{B_2}(a) = 0.3, \alpha_{B_2}(b) = 0.4, \beta_{B_2}(b) = 0.7;$$

Let $\tau_1 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2\}$ and $\tau_2 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, B_1, B_2\}$ are $\mathfrak{F}S$'s on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}Scts$ (resp. $\mathfrak{F}\delta Pcts$) but not $\mathfrak{F}Irr$ (resp. $\mathfrak{F}\delta P Irr$). Since, A_2^c is a $\mathfrak{F}S$ o (resp. $\mathfrak{F}\delta P$ o) set in X_2 but $h_{\mathfrak{F}}^{-1}(A_2^c) = A_2^c$ is not $\mathfrak{F}S$ o (resp. $\mathfrak{F}\delta P$ o) set in X_1 .

Example 4.2 Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}S$'s $A_1, A_2, A_3, A_4, B_1, B_2, B_3$ and B_4 are defined as

$$\alpha_{A_1}(a) = 0.4, \beta_{A_1}(a) = 0.6, \alpha_{A_1}(b) = 0.5, \beta_{A_1}(b) = 0.5;$$

$$\alpha_{A_2}(a) = 0.6, \beta_{A_2}(a) = 0.4, \alpha_{A_2}(b) = 0.6, \beta_{A_2}(b) = 0.4;$$

$$\alpha_{A_3}(a) = 0.7, \beta_{A_3}(a) = 0.3, \alpha_{A_3}(b) = 0.6, \beta_{A_3}(b) = 0.4;$$

$$\alpha_{A_4}(a) = 0.4, \beta_{A_4}(a) = 0.6, \alpha_{A_4}(b) = 0.4, \beta_{A_4}(b) = 0.6;$$

$$\alpha_{B_1}(a) = 0.2, \beta_{B_1}(a) = 0.8, \alpha_{B_1}(b) = 0.4, \beta_{B_1}(b) = 0.6;$$

$$\alpha_{B_2}(a) = 0.1, \beta_{B_2}(a) = 0.9, \alpha_{B_2}(b) = 0.3, \beta_{B_2}(b) = 0.7;$$

$$\alpha_{B_3}(a) = 0.9, \beta_{B_3}(a) = 0.1, \alpha_{B_3}(b) = 0.7, \beta_{B_3}(b) = 0.3;$$

$$\alpha_{B_4}(a) = 0.2, \beta_{B_4}(a) = 0.8, \alpha_{B_4}(b) = 0.3, \beta_{B_4}(b) = 0.7;$$

Let $\tau_1 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, B_1, B_2, B_3, B_4\}$ are \mathfrak{F} Ts 's on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ but not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$. Since, A_4 is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_1 .

Example 4.3 Let $X_1 = X_2 = X = \{a, b\}$ and the \mathfrak{F} S's $A_1, A_2, A_3, A_4,$ and A_5 are defined as

$$\alpha_{A_1}(a) = 0.2, \beta_{A_1}(a) = 0.8, \alpha_{A_1}(b) = 0.4, \beta_{A_1}(b) = 0.6;$$

$$\alpha_{A_2}(a) = 0.1, \beta_{A_2}(a) = 0.9, \alpha_{A_2}(b) = 0.3, \beta_{A_2}(b) = 0.3;$$

$$\alpha_{A_3}(a) = 0.9, \beta_{A_3}(a) = 0.1, \alpha_{A_3}(b) = 0.7, \beta_{A_3}(b) = 0.3;$$

$$\alpha_{A_4}(a) = 0.2, \beta_{A_4}(a) = 0.8, \alpha_{A_4}(b) = 0.3, \beta_{A_4}(b) = 0.7;$$

$$\alpha_{A_5}(a) = 0.6, \beta_{A_5}(a) = 0.4, \alpha_{A_5}(b) = 0.6, \beta_{A_5}(b) = 0.4;$$

Let $\tau_1 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3\}$ are \mathfrak{F} Ts 's on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\alpha Cts$ but not $\mathfrak{F}\mathcal{F}\delta\alpha Irr$. Since, A_5 is a $\mathfrak{F}\mathcal{F}\delta\alpha o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_5) = A_5$ is not $\mathfrak{F}\mathcal{F}\delta\alpha o$ set in X_1 .

Example 4.4 Let $X_1 = X_2 = X = \{a, b\}$ and the \mathfrak{F} S's $A_1, A_2, A_3, A_4, B_1, B_2, B_3$ and B_4 are defined as

$$\alpha_{A_1}(a) = 0.2, \beta_{A_1}(a) = 0.8, \alpha_{A_1}(b) = 0.4, \beta_{A_1}(b) = 0.6;$$

$$\alpha_{A_2}(a) = 0.1, \beta_{A_2}(a) = 0.9, \alpha_{A_2}(b) = 0.3, \beta_{A_2}(b) = 0.7;$$

$$\alpha_{A_3}(a) = 0.9, \beta_{A_3}(a) = 0.1, \alpha_{A_3}(b) = 0.7, \beta_{A_3}(b) = 0.3;$$

$$\alpha_{A_4}(a) = 0.2, \beta_{A_4}(a) = 0.8, \alpha_{A_4}(b) = 0.3, \beta_{A_4}(b) = 0.7;$$

$$\alpha_{B_1}(a) = 0.4, \beta_{B_1}(a) = 0.6, \alpha_{B_1}(b) = 0.5, \beta_{B_1}(b) = 0.5;$$

$$\alpha_{B_2}(a) = 0.6, \beta_{B_2}(a) = 0.4, \alpha_{B_2}(b) = 0.6, \beta_{B_2}(b) = 0.4;$$

$$\alpha_{B_3}(a) = 0.7, \beta_{B_3}(a) = 0.3, \alpha_{B_3}(b) = 0.6, \beta_{B_3}(b) = 0.4;$$

$$\alpha_{B_4}(a) = 0.4, \beta_{B_4}(a) = 0.6, \alpha_{B_4}(b) = 0.4, \beta_{B_4}(b) = 0.6;$$

Let $\tau_1 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}}, 1_{\mathfrak{F}}, B_1, B_2, B_3, B_4\}$ are \mathfrak{F} Ts 's on X and let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta Cts$ but not $\mathfrak{F}\mathcal{F}\delta\beta Irr$. Since, A_4^c is a $\mathfrak{F}\mathcal{F}\delta\beta o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_4^c) = A_4^c$ is not $\mathfrak{F}\mathcal{F}\delta\beta o$ set in X_1 .

Definition 4.2 A \mathfrak{F} Ts (X_1, τ_1) is known as a Fermatean fuzzy $\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $\delta\mathcal{P}U_{\frac{1}{2}}$, $\delta\alpha U_{\frac{1}{2}}$ and $\delta\beta U_{\frac{1}{2}}$) (in short, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}U_{\frac{1}{2}}$, $\mathfrak{F}\mathcal{F}\delta\alpha U_{\frac{1}{2}}$ and $\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$))-space, if each $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$, $\mathfrak{F}\mathcal{F}\delta\alpha os$ and $\mathfrak{F}\mathcal{F}\delta\beta os$) in X is $\mathfrak{F}\mathcal{F}os$ in X_1 .

Theorem 4.2 Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Irr$, $\mathfrak{F}\mathcal{F}\delta\alpha Irr$ and $\mathfrak{F}\mathcal{F}\delta\beta Irr$) map. Then $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Cts$ map if X is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}U_{\frac{1}{2}}$, $\mathfrak{F}\mathcal{F}\delta\alpha U_{\frac{1}{2}}$ and $\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$).

$\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$ -space.

Proof.

(i) Consider a $\mathfrak{F}\mathcal{F}os$ K in X_2 . Then K is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ in X_2 . Therefore $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ in X_1 . Since X_1 is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}U_{\frac{1}{2}}$ -space, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}os$ in X_1 . Hence $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Cts$ map.

(ii) Consider a $\mathfrak{F}\mathcal{F}os$ K in X_2 . Then K is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$ in X_2 . Therefore $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$ in X_1 . Since X_1 is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}U_{\frac{1}{2}}$ -space, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}os$ in X_1 . Hence $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Cts$ map.

(iii) Consider a $\mathfrak{F}\mathcal{F}os$ K in X_2 . Then K is a $\mathfrak{F}\mathcal{F}\delta\alpha os$ in X_2 . Therefore $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta\alpha os$ in X_1 . Since X_1 is a $\mathfrak{F}\mathcal{F}\delta\alpha U_{\frac{1}{2}}$ -space, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}os$ in X_1 . Hence $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Cts$ map.

(iv) Consider a $\mathfrak{F}\mathcal{F}os$ K in X_2 . Then K is a $\mathfrak{F}\mathcal{F}\delta\beta os$ in X_2 . Therefore $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta\beta os$ in X_1 . Since X_1 is a $\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$ -space, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}os$ in X_1 . Hence $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Cts$ map.

Theorem 4.3 Let $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ and $g_{\mathfrak{F}}: (X_2, \tau_2) \rightarrow (X_3, \tau_3)$ be $\mathfrak{F}\mathcal{F}\delta Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S} Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P} Irr$, $\mathfrak{F}\mathcal{F}\delta\alpha Irr$ and $\mathfrak{F}\mathcal{F}\delta\beta Irr$) maps, then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_3, \tau_3)$ is a $\mathfrak{F}\mathcal{F}\delta Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S} Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P} Irr$, $\mathfrak{F}\mathcal{F}\delta\alpha Irr$ and $\mathfrak{F}\mathcal{F}\delta\beta Irr$) map.

Proof. Consider a $\mathfrak{F}\mathcal{F}\delta os$ K in X_3 . So $g_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta os$ in X_2 . As $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta Irr$ map, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(K))$ is a $\mathfrak{F}\mathcal{F}\delta os$ in X_1 . Thus $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta Irr$ map. The other cases are similar.

Theorem 4.4 Consider a $\mathfrak{F}\mathcal{F}\delta Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S} Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P} Irr$, $\mathfrak{F}\mathcal{F}\delta\alpha Irr$ and $\mathfrak{F}\mathcal{F}\delta\beta Irr$) map $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ and a $\mathfrak{F}\mathcal{F}\delta Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S} Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P} Cts$, $\mathfrak{F}\mathcal{F}\delta\alpha Cts$ and $\mathfrak{F}\mathcal{F}\delta\beta Cts$) map $g_{\mathfrak{F}}: (X_2, \tau_2) \rightarrow (X_3, \tau_3)$. Then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_3, \tau_3)$ is a $\mathfrak{F}\mathcal{F}\delta Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S} Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P} Cts$, $\mathfrak{F}\mathcal{F}\delta\alpha Cts$ and $\mathfrak{F}\mathcal{F}\delta\beta Cts$) map.

Proof. Consider a $\mathfrak{F}\mathcal{F}os$ K in X_3 . So $g_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta os$ in X_2 . As $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta Irr$ map, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(K))$ is a $\mathfrak{F}\mathcal{F}\delta os$ in X_1 . Thus $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta Cts$ map. The other cases are similar.

Theorem 4.5 Consider a map $h_{\mathfrak{F}}: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ from a $\mathfrak{F}\mathcal{F}ts$ X_1 into a $\mathfrak{F}\mathcal{F}ts$ X_2 . The following are equivalent if X_1 and X_2 are $\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$ -spaces.

- (i) $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta\beta Irr$ map.
- (ii) $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta\beta cs$ in X_1 for every $\mathfrak{F}\mathcal{F}\delta\beta cs$ K in X_2 .
- (iii) $\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}^{-1}(K)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K))$ for every $\mathfrak{F}\mathcal{F}s$ K of X_2 .

Proof. (i) \rightarrow (ii): Consider a $\mathfrak{F}\mathcal{F}\delta\beta cs$ K in X_2 . It follows K^c is a $\mathfrak{F}\mathcal{F}\delta\beta os$ in X_2 . As $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta Irr$, $h_{\mathfrak{F}}^{-1}((K)^c)$ is a $\mathfrak{F}\mathcal{F}\delta\beta os$ in X_1 . We know that $h_{\mathfrak{F}}^{-1}((K)^c) = (h_{\mathfrak{F}}^{-1}(K))^c$. Hence $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}\delta\beta cs$ in X_1 .

(ii) \rightarrow (iii): Consider a $\mathfrak{F}\mathcal{F}s$ K in X_2 and $K \subseteq \mathfrak{F}\mathcal{F}\delta\beta cl(K)$. Then $h_{\mathfrak{F}}^{-1}(K) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\beta cl(K))$. Since $\mathfrak{F}\mathcal{F}\delta\beta cl(K)$ is a $\mathfrak{F}\mathcal{F}\delta\beta cs$ in X_2 . By presumption, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\beta cl(K))$ is a $\mathfrak{F}\mathcal{F}\delta\beta cs$ in X_1 .

Also, as X_1 is $\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$ -space, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\beta cl(K))$ is a $\mathfrak{F}\mathcal{F}\delta\beta cs$ in X_1 .

(iii) \rightarrow (i): Consider a $\mathfrak{F}\mathcal{F}\delta\beta cs$ K in X_2 . As X_2 is $\mathfrak{F}\mathcal{F}\delta\beta U_{\frac{1}{2}}$ -space, K is $\mathfrak{F}\mathcal{F}cs$ in X_2 and $\mathfrak{F}\mathcal{F}cl(K) = K$. Thus $h_{\mathfrak{F}}^{-1}(K) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\delta\beta cl(K)) \supseteq \mathfrak{F}\mathcal{F}\delta\beta cl(h_{\mathfrak{F}}^{-1}(K)) = \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}^{-1}(K))$. But clearly $h_{\mathfrak{F}}^{-1}(K) \subseteq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}^{-1}(K))$. Therefore $\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}^{-1}(K)) = h_{\mathfrak{F}}^{-1}(K)$. It follows $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}cs$ and so it is a $\mathfrak{F}\mathcal{F}\delta\beta cs$ in X_1 . Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\beta irr$ map. The proof is similar for other cases of $\mathfrak{F}\mathcal{F}\delta os$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ and $\mathfrak{F}\mathcal{F}\delta\alpha os$.

5 Application

Entropy as a measure of fuzziness was first proposed by Zadeh [16]. Later many mathematicians defined several entropy measures. In this section, we focus on defining an entropy measure for $\mathfrak{F}fs$ that connects the degree of membership and non-membership. As an example, we have applied the proposed entropy measure in the field of decision making.

Definition 5.1 Let $A = \{ \langle x, \alpha_A(x), \beta_A(x) | x \in X \rangle$ be a $\mathfrak{F}fs$ in U . The new entropy measure for A denoted by $\varepsilon_{\mathfrak{F}fs}(A)$, is a function, $\varepsilon_{\mathfrak{F}fs}: \tau_{\mathfrak{F}fs}(U) \rightarrow [0,1]$ and is defined as $\varepsilon_{\mathfrak{F}fs}(A) = 1 - \frac{1}{n} \sum_{i=1}^n (\alpha_A - \beta_A)^2$; forevery $x_i \in A$, where $\tau_{\mathfrak{F}fs}(U)$ denote the family of all $\mathfrak{F}fs$'s on U .

Example 5.1 Consider an example of a shopping experience with different items. The pandemic situation of COVID-19 has broadened the doorstep of our shopping experience. Nowadays we depend on different methods of shopping like online (purchasing through internet, often through the websites or Apps), In-store (Visiting physically), Mobile (Using mobile to browse and purchase) shopping. Based on the reviews and ratings, we will find out the most reliable method of shopping for a specific item using the Fermatean fuzzy entropy measure.

Table 1. Ratings of products based on the different shopping ways.

	Gadgets (a)	Gold jewellery (b)	Food Products(c)	Cloths (d)
Online (1)	$\langle 1, a; 0.6, 0.3 \rangle$	$\langle 1, b; 0.8, 0.7 \rangle$	$\langle 1, c; 0.7, 0.3 \rangle$	$\langle 1, d; 0.5, 0.3 \rangle$
In-store (2)	$\langle 2, a; 0.9, 0.4 \rangle$	$\langle 2, b; 0.4, 0.7 \rangle$	$\langle 2, c; 0.7, 0.8 \rangle$	$\langle 2, d; 0.2, 0.8 \rangle$
Mobile (3)	$\langle 3, a; 0.6, 0.8 \rangle$	$\langle 3, b; 0.2, 0.1 \rangle$	$\langle 3, c; 0.9, 0.2 \rangle$	$\langle 3, d; 0.1, 0.5 \rangle$

Clearly, all values in the Table 1 are $\mathfrak{F}Fs$'s. Now we calculate the $\varepsilon_{\mathfrak{F}Fs}$ of each value. Table 2. Entropy measure of each Shopping for the different purchase.

	Gadgets (a)	Gold jewellery (b)	Food Products(c)	Cloths (d)
Online (1)	0.91	0.99	0.84	0.96
In-store (2)	0.75	0.91	0.99	0.64
Mobile (3)	0.96	0.99	0.51	0.84

From Table 2, it is clear that $\varepsilon_{\mathfrak{F}\mathcal{F}_S}(1, c) < \varepsilon_{\mathfrak{F}\mathcal{F}_S}(1, a) < \varepsilon_{\mathfrak{F}\mathcal{F}_S}(1, d) \leq \varepsilon_{\mathfrak{F}\mathcal{F}_S}(1, b)$

Similarly

$$\varepsilon_{\mathfrak{F}\mathcal{F}_S}(2, c) < \varepsilon_{\mathfrak{F}\mathcal{F}_S}(2, b) \leq \varepsilon_{\mathfrak{F}\mathcal{F}_S}(2, a) < \varepsilon_{\mathfrak{F}\mathcal{F}_S}(2, d)$$

$$\varepsilon_{\mathfrak{F}\mathcal{F}_S}(3, b) < \varepsilon_{\mathfrak{F}\mathcal{F}_S}(3, a) \leq \varepsilon_{\mathfrak{F}\mathcal{F}_S}(3, d) \leq \varepsilon_{\mathfrak{F}\mathcal{F}_S}(3, c)$$

It is clear that People are most like to shop the online and Mobile shopping for food products and In-store shopping was buy a cloths.

6 Conclusion

In this paper, $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}Cts$, $\mathfrak{F}\mathcal{F}\delta\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$, $\mathfrak{F}\mathcal{F}\delta\alpha Cts$, and $\mathfrak{F}\mathcal{F}\delta\beta Cts$ respective irresolute map is defined using $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$, $\mathfrak{F}\mathcal{F}\delta\alpha o$ and $\mathfrak{F}\mathcal{F}\delta\beta o$ set and its properties are analyzed with the examples. Then Fermatean fuzzy continuous maps are compared with other generalized Fermatean fuzzy continuous maps. Also we extended the concept of Fermatean fuzzy irresolute maps in Fermatean fuzzy topological spaces using $\mathfrak{F}\mathcal{F}o$ sets. Some examples and basic relationships between the mappings were also discussed. In future, these can be extended to Fermatean fuzzy open, closed, homeomorphism and contra maps. Application for MCDM to the real world problem was solved with the proposed entropy measure. In future, MCDM to the real world problem can be developed to the $\mathfrak{F}\mathcal{F}ts$.

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