

# A Study of $s$ Star $p$ Star Homeomorphism in Topological Spaces

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## Abstract:

The present study aims to provide an overview and explore the new class of closed map and open map is termed as semi star pre star closed map (Briefly  $s^*p^*$ closed map), semi star pre star open map (Briefly  $s^*p^*$ open map) and some of its characterizations are studied. More over semi star pre star homeomorphism (Briefly  $s^*p^*$ Homeomorphism) in topological spaces is defined via  $s^*p^*$ open map and  $s^*p^*$ continuous map and to get a few of their specific features. Also, compared with existing one.

**Keywords:**  $s^*p^*$ Closed set,  $s^*p^*$ Open set,  $s^*p^*$ closed map,  $s^*p^*$  open map,  $s^*p^*$ Homeomorphism.

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## 1. Introduction

In Topology, the idea of homeomorphism is necessary. Homeomorphism is the process of continuously stretching and bending an object into a new shape. A homeomorphism is a function which is bijection between topological spaces so that the map and its inverse are both continuous.

S.R.Malghan explores as well as establishes the Generalized closed maps [1]. Benchalli [2] developed regular closed maps,  $rw$ -closed maps and  $\alpha rw$ -closed maps.  $rg\alpha$ -closed map also  $rg\alpha$ -open map was introduced by A.Vadivel and K.Vairamanikam [3].  $Sg$  homomorphism and  $gs$  homomorphism in topological spaces was studied by Devi et al [4]. Generalized homeomorphism were initially stated and examined by Maki et al. [5]. Rs Wali et.al [6] have introduced  $rgw\alpha$ -homeomorphism in topological spaces. Gnanambal [7] has defined  $gpr$ -closed maps and studied some of their unique features. D.Iyappan and N.Nagaveni [8] was delivered on  $sgb$ -continuous map,  $sgb$ -closed maps in topological space. Studies on generalization of homeomorphism in topological spaces introduced by N.Nagaveni [9]. T.Shyla Isac Mary and P.Thangavelu [10] studied and developed RPS-Homeomorphism in topological spaces. In topological spaces, A. Pushpalatha and K. Anitha [11] established properties of  $g^*s$ -closed set and A. Pushpalatha [12] looked on research on topological space generalizations of mapping. In topological spaces, generalizations of generalized closed sets and maps was developed by I.Arockiarani [13]. S.Sekar and B.Jothilakshmi [14] were created and explored the  $sg^*b$ -closed maps in topological spaces.

Fundamental topological ideas are explored in this paper with a special focus to the  $s$  star  $p$  star homeomorphism. The field of topological spaces and its fundamental properties are expected to be better understood with the development of  $s$  star  $p$  star homeomorphism ideas, which offer a new point of view.

In the present analysis, topological spaces are treated as TS,  $s^*p^*$ closed set as  $s^*p^*-C$  set,  $s^*p^*$  open set as  $s^*p^*-O$  set,  $s^*p^*$  homeomorphism as  $s^*p^*-H$ .

## 2. Objectives

This research aims to define and analyses  $s^*p^*$  homeomorphism in topological spaces in an exact way. Formulate a number of theorems that demonstrate these sets features and implications. Utilize the illustrations how these concepts are effective.

## 3. Preliminaries

**Definition: [15]** Let the TS be  $X$ . Let  $A \subseteq X$  is known as semi star pre star closed sets (Briefly  $s^*p^*$  closed sets) if  $scl$  of  $A$  is subset of  $U$  when  $A \subseteq U$  and  $U$  is pre semi open set.

The Complement of  $s^*p^*$ -C set is known as  $s^*p^*$ -O set.

**Definition:** The term  $s^*p^*$  continuous refers to a function  $f: X_1 \rightarrow X_2$  where each closed set in  $X_2$  has an inverse image that is also  $s^*p^*$ -C set in of every closed set in  $X_1$ .

## 4. $s^*p^*$ closed map

Using the basic ideas of  $s^*p^*$ -C sets, we bring about  $s^*p^*$ -C map in topological spaces in this part and go through some of its essential features.

**Definition 4.1:** A Function  $f: X_1 \rightarrow X_2$  is referred to be a  $s^*p^*$ -C ( $s^*p^*$ -O) map if all closed (open) set in  $X_1$  has an image is in  $s^*p^*$ -C ( $s^*p^*$ -O) set in  $X_2$ .

**Example 4.2:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \phi, \{r\}, \{t\}, \{r, t\}\}$  and  $\tau^c = \{X_1, \phi, \{s, t\}, \{r, s\}, \{s\}\}$ .  $\sigma = \{X_2, \phi, \{s\}, \{r, t\}\}$ ;  $\sigma^c = \{X_2, \phi, \{r, t\}, \{s\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{r\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = s$ ;  $f(s) = r$ ;  $f(t) = t$ . Then  $f$  is  $s^*p^*$ -C map because the image of closed map  $\{s\}$  in  $(X_1, \tau)$ ,  $f\{s\} = \{r\}$  is in  $s^*p^*$ -C set in  $X_2$ .

**Theorem 4.3:** A closed map is always a  $s^*p^*$ -C map.

**Proof:** Consider  $f: X_1 \rightarrow X_2$  be a closed map. Assume that  $V$  is in  $X_1$ . And it will be closed set. It Therefore, the image of  $V$  is in  $X_2$  and that is closed set. All closed set is known to be  $s^*p^*$ -C set. Subsequently,  $f(V)$  is  $s^*p^*$ -C set. Thus,  $f$  is a  $s^*p^*$ -C map.

This theorem reverse implication may not hold, as shown by the example that follows.

**Example 4.4:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \phi, \{r\}, \{s, t\}\}$  and  $\tau^c = \{X_1, \phi, \{s, t\}, \{r\}\}$ .  $\sigma = \{X_2, \phi, \{t\}, \{r, t\}\}$ ;  $\sigma^c = \{X_2, \phi, \{r, s\}, \{s\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{r\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = r$ ;  $f(s) = s$ ;  $f(t) = t$ . Hence  $f$  does not a closed map rather a  $s^*p^*$ -C map. Because the image of closed map  $\{a\}$  in  $(X_1, \tau)$ ,  $f\{r\} = \{r\}$  is not in closed set in  $X_2$  however it in  $s^*p^*$ -C set in  $X_2$ .

**Theorem 4.5:** Every map that belongs to pre- closed is  $s^*p^*$ -C map.

**Proof:** Let us consider  $f: X_1 \rightarrow X_2$  be a pre-closed map. Let us consider the closed set in  $X_1$  which is denoted by  $V$ . Thus, its image  $f(V)$  is pre closed set in  $X_2$ . Because each pre-closed set is  $s^*p^*$ -C set. So, the image of  $V$  is in  $s^*p^*$ -C set. Hence,  $f$  is a  $s^*p^*$ -C map.

Upcoming example shows the reverse of the earlier theorem never hold.

**Example 4.6:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \phi, \{r\}, \{s\}, \{r, s\}, \{s, t\}\}$  and  $\tau^c = \{X_1, \phi, \{s, t\}, \{r, t\}, \{t\}, \{r\}\}$ .  $\sigma = \{X_2, \phi, \{r\}, \{s\}, \{r, s\}\}$ ;  $\sigma^c = \{X_2, \phi, \{s, t\}, \{r, t\}, \{t\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{r\}, \{s\}\}$ . Pre-closed sets are  $\{X_2, \phi, \{s, t\}, \{r, t\}, \{t\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = s$ ;  $f(s) = t$ ;  $f(t) = r$ . Hence  $f$  is  $s^*p^*$ -C map. However, it is not pre-closed map. Because the closed map  $\{r\}$  in  $(X_1, \tau)$ , its image  $f(r) = \{s\}$  is not in pre- closed set in  $X_2$  but it in  $s^*p^*$ -C set in  $X_2$ .

**Theorem 4.7:** Every map which is  $g^*$ -closed map is also  $s^*p^*$ -C map.

**Proof:** Consider  $f: X_1 \rightarrow X_2$  be a  $g^*$ -closed map. Let us assume the closed set in  $X_1$  it is denoted by  $V$ . After that the image of  $V$  that is  $f(V)$  in  $X_2$  is  $g^*$ closed set. Each  $g^*$ closed set is  $s^*p^*$ -C set. Thus, the image of  $V$  is  $s^*p^*$ -C set. Hence, it is  $s^*p^*$ -C map.

Reverse implication of the previously stated theorem does not valid by the following example.

**Example 4.8:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \phi, \{r\}, \{t\}, \{r, t\}\}$  and  $\tau^c = \{X_1, \phi, \{s, t\}, \{r, s\}, \{s\}\}$ .  $\sigma = \{X_2, \phi, \{r\}, \{r, s\}\}$ ;  $\sigma^c = \{X_2, \phi, \{s, t\}, \{t\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{s\}\}$ .  $g^*$ closed sets are  $\{X_2, \phi, \{s, t\}, \{r, t\}, \{t\}\}$ . Assume  $f: X_1 \rightarrow X_2$  by  $f(r) = t$ ;  $f(s) = s$ ;  $f(t) = r$ . Hence  $f$  is not  $g^*$ closed map however it is in  $s^*p^*$ -C map. Hence the image of closed map  $\{r\}$  in  $(X_1, \tau)$ ,  $f\{r\} = \{s\}$  is not in  $g^*$  closed set in  $X_2$  but it in  $s^*p^*$ -C set in  $X_2$ .

**Theorem 4.9:**  $s^*p^*$ -C maps are all gpr closed maps.

**Proof:** Suppose  $f: X_1 \rightarrow X_2$  be a gpr closed map. Assume that  $V$  be in  $X_1$ . It is closed set. Then image of  $V$  that is  $f(V)$  is gpr closed set in  $X_2$ . Since any gpr-closed set is  $s^*p^*$ -C set thus image of  $V$  is gpr-closed set. So that  $f$  is  $s^*p^*$ -C map.

Upcoming Illustration shows the opposite of the previously stated theorem is not always right.

**Example 4.10:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \phi, \{s\}, \{t\}, \{r, s\}, \{s, t\}\}$  and  $\tau^c = \{X_1, \phi, \{r, t\}, \{r, s\}, \{t\}, \{r\}\}$ .  $\sigma = \{X_2, \phi, \{r\}, \{s\}, \{r, s\}\}$ ;  $\sigma^c = \{X_2, \phi, \{s, t\}, \{r, t\}, \{t\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{s\}\}$ . gpr closed sets of  $X_2$  are  $\{X_2, \phi, \{r, t\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = t$ ;  $f(s) = r$ ;  $f(t) = s$ . Hence  $f$  does not a gpr closed map rather than  $s^*p^*$ -C map. Consequently, the image of closed map  $\{r\}$  in  $(X_1, \tau)$ ,  $f\{r\} = \{s\}$  is not in gpr closed set in  $X_2$  but it in  $s^*p^*$ -C set in  $X_2$ .

**Theorem 4.11:** Every  $\alpha g$  closed map is a  $s^*p^*$ -C map.

**Proof:** Consider  $f: X_1 \rightarrow X_2$  as an  $\alpha g$  closed map. Take  $V$  in  $X_1$  be a closed set. Thus, its image is in  $\alpha g$  closed set in  $X_2$ . Because all  $\alpha g$  closed set is  $s^*p^*$ -C set, then the image of  $V$  is in  $s^*p^*$ -C in  $X_2$ . Hence,  $f$  is  $s^*p^*$ -C map.

Our next illustration reveals why the other side of the above-mentioned theorem may not be correct.

**Example 4.12:** Let  $X_1 = X_2 = \{q, r, s, t\}$ ;  $\tau = \{X_1, \phi, \{q\}, \{r\}, \{q, r\}, \{q, r, s\}\}$  and  $\tau^c = \{X_1, \phi, \{r, s, t\}, \{q, s, t\}, \{s, t\}, \{t\}\}$ .  $\sigma = \{X_2, \phi, \{q\}, \{s\}, \{t\}, \{q, s\}, \{q, t\}, \{s, t\}, \{q, s, t\}\}$ ;  $\sigma^c = \{X_2, \phi, \{r, s, t\}, \{q, r, t\}, \{q, r, s\}, \{r, t\}, \{r, s\}, \{q, r\}, \{r\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{q\}, \{s\}, \{t\}, \{q, s\}, \{q, t\}, \{s, t\}\}$ .  $\alpha g$  closed sets of  $X_2$  are  $\{X_2, \phi, \{r\}, \{q, r\}, \{r, t\}, \{q, r, t\}, \{q, r, t\}, \{r, s, t\}\}$ . Declare a Map  $f: X_1 \rightarrow X_2$  by  $f(q) = s$ ;  $f(r) = r$ ;  $f(s) = q$ ;  $f(t) = t$ . Hence  $f$  does not a  $\alpha g$  closed map rather than  $s^*p^*$ -C map. Because the image of closed map  $\{s, t\}$  in  $(X_1, \tau)$ ,  $f\{s, t\} = \{q, t\}$  is not in  $\alpha g$  closed set in  $X_2$  but it in  $s^*p^*$ -C set in  $X_2$ .

**Theorem 4.13:** All  $\omega$ -closed maps are  $s^*p^*$ -C map.

**Proof:** Given the idea and fact that any  $\omega$ -closed set is  $s^*p^*$ -C set, the proof is obvious.

As this next illustration clarifies, the opposite of given theorem is not valid.

**Example 4.14:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \phi, \{r\}, \{r, s\}\}$  and  $\tau^c = \{X_1, \phi, \{s, t\}, \{t\}\}$ .  $\sigma = \{X_2, \phi, \{r\}, \{r, t\}\}$ ;  $\sigma^c = \{X_2, \phi, \{s, t\}, \{s\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{t\}\}$ .  $\omega$  closed sets of  $X_2$  are  $\{X_2, \phi, \{s\}, \{r, s\}, \{s, t\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = s$ ;  $f(s) = r$ ;  $f(t) = t$ . Hence  $f$  is not  $\omega$  closed map but it is  $s^*p^*$ -C map. Because the image of closed map  $\{t\}$  in  $(X_1, \tau)$ ,  $f\{t\} = \{t\}$  is not in  $\omega$  closed set in  $X_2$  but it in  $s^*p^*$ -C set in  $X_2$ .

**Remark 4.15:** An upcoming example shows that  $g$ -closed map and  $s^*p^*$ -C map are not dependent.

Let  $X_1 = X_2 = \{q, r, s, t\}$ ;  $\tau = \{X_1, \phi, \{q\}, \{r\}, \{q, r\}, \{r, s\}, \{q, r, s\}\}$  and  $\tau^c = \{X_1, \phi, \{r, s, t\}, \{q, s, t\}, \{s, t\}, \{q, t\}, \{t\}\}$ .  $\sigma = \{X_2, \phi, \{r\}, \{t\}, \{r, t\}, \{r, s, t\}\}$ ;  $\sigma^c = \{X_2, \phi, \{q, s, t\}, \{q, r, s\}, \{q, s\}, \{q\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \phi, \{r\}, \{s\}, \{t\}, \{r, s\}, \{s, t\}\}$ .  $g$  closed sets of  $X_2$  are  $\{X_2, \phi, \{q\}, \{q, r\}, \{q, s\}, \{q, t\}, \{q, r, s\}, \{q, r, t\}, \{q, s, t\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(q) = r$ ;  $f(r) = q$ ;  $f(s) = s$ ;  $f(t) = t$ . Hence  $f$  does not a  $g$  closed map rather than  $s^*p^*$ -C map. When the image of closed map  $\{s, t\}$  in  $(X_1, \tau)$ ,  $f\{s, t\} = \{s, t\}$  is not in  $g$  closed set in  $X_2$  but it in  $s^*p^*$ -C set in  $X_2$ . Similarly,  $f$  is  $g$  closed but not

$s^*p^*$ -C map. Thus, the closed set  $\{r, s, t\}$  in  $(X_1, \tau)$ ,  $f\{r, s, t\} = \{q, s, t\}$  is in  $g$  closed set in  $X_2$  but not in  $s^*p^*$ -C set in  $X_2$ .

### 5. $s^*p^*$ - HOMEOMORPHISM

This portion deals with  $s^*p^*$ -Homeomorphism ( $s^*p^*$ -H) using  $s^*p^*$ -C maps and  $s^*p^*$ -O maps.

**Definition 5.1:** A Bijection maps  $f: X_1 \rightarrow X_2$  is known as  $s^*p^*$ Homeomorphism( $s^*p^*$ -H) if  $f$  and its inverse are  $s^*p^*$ continuous maps.

**Example 5.2:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \varphi, \{r\}, \{s\}, \{r, s\}\}$  and  $\tau^c = \{X_1, \varphi, \{s, t\}, \{r, t\}, \{t\}\}$ .  $s^*p^*$ -C sets of  $X_1$  are  $\{X_1, \varphi, \{r\}, \{s\}\}$ .  $\sigma = \{X_2, \varphi, \{s\}, \{t\}, \{r, s\}, \{s, t\}\}$ ;  $\sigma^c = \{X_2, \varphi, \{r, t\}, \{r, s\}, \{t\}, \{r\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \varphi, \{r\}, \{r, s\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = s$ ;  $f(s) = t$ ;  $f(t) = r$ . Here the inverse image of the closed set  $\{c\}$  in  $X_2$  is  $s^*p^*$ -C set in  $X_1$  and  $(f^{-1})^{-1}(t) = f(t) = \{r\}$  is in  $s^*p^*$ -C set in  $X_2$ . Hence,  $f$  and its inverse are  $s^*p^*$ continuous. Which leads to  $f$  is  $s^*p^*$ -H.

**Theorem 5.3:** All homeomorphism may be expressed as  $s^*p^*$ Homeomorphism.

**Proof:** A homeomorphism is defined as  $f: X_1 \rightarrow X_2$ . Consequently,  $f$  and its inverse are continuous and bijection. So that  $f$  is  $s^*p^*$ -H if any continuous function is  $s^*p^*$ continuous.

The subsequent example explains that the theorems in contradiction does not always valid.

**Example 5.4:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \varphi, \{t\}, \{r, t\}\}$  and  $\tau^c = \{X_1, \varphi, \{r, s\}, \{s\}\}$ .  $s^*p^*$ -C sets of  $X_1$  are  $\{X_1, \varphi, \{r\}\}$ .  $\sigma = \{X_2, \varphi, \{r\}, \{t\}, \{s, t\}, \{r, t\}\}$ ;  $\sigma^c = \{X_2, \varphi, \{s, t\}, \{r, s\}, \{r\}, \{s\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \varphi, \{s\}, \{t\}, \{s, t\}\}$ . Assume a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = r$ ;  $f(s) = t$ ;  $f(t) = s$ . Then  $f$  and its inverse are  $s^*p^*$ continuous. So,  $f$  does not a homeomorphism rather than a  $s^*p^*$ -H. Because the inverse image of closed map  $\{r, s\}$  in  $(X_1, \tau)$ ,  $(f^{-1})^{-1}(r, s) = f(r, s) = \{r, t\}$  is not in closed set  $X_2$ .

**Theorem 5.5:** Every  $\alpha$ -homeomorphism is a  $s^*p^*$ -Homeomorphism.

**Proof:** A  $\alpha$  homeomorphism is  $X_1 \rightarrow X_2$ . Following that  $f$  and its inverse are  $\alpha$ -continuous and  $f$  is bijection.  $f$  and its inverse are  $s^*p^*$  continuous obtained through each  $\alpha$ - continuous function that is  $s^*p^*$ -continuous. Accordingly,  $f$  is  $s^*p^*$ -Homeomorphism.

A Further illustration demonstrates that the previously stated theorems reversal is not generally correct.

**Example 5.6:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \varphi, \{r\}, \{s\}, \{r, s\}\}$  and  $\tau^c = \{X_1, \varphi, \{s, t\}, \{r, t\}, \{t\}\}$ .  $s^*p^*$ -C sets of  $X_1$  are  $\{X_1, \varphi, \{r\}, \{s\}\}$  and  $\alpha$ -closed sets of  $X_1$  are  $\{X_1, \varphi, \{r\}, \{s\}, \{t\}, \{s, t\}, \{r, t\}\}$ .  $\sigma = \{X_2, \varphi, \{s\}, \{t\}, \{r, s\}, \{s, t\}\}$ ;  $\sigma^c = \{X_2, \varphi, \{r, t\}, \{r, s\}, \{t\}, \{r\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \varphi, \{r\}, \{r, s\}\}$ .  $\alpha$ -closed sets of  $X_2$  are  $\{X_2, \varphi, \{r\}, \{s\}, \{r, s\}, \{r, t\}\}$ . Define a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = s$ ;  $f(s) = r$ ;  $f(t) = t$ . Here  $f$  is  $s^*p^*$ -H but not  $\alpha$ - homeomorphism because the inverse image of the closed set  $\{t\}$  in  $X_1$  and  $(f^{-1})^{-1}(t) = f(t) = \{t\}$  is not in  $\alpha$ - closed set in  $X_2$ .

**Theorem 5.7:** All  $g$ - homeomorphisms are equivalent to  $s^*p^*$ -Homeomorphism.

**Proof:** Consider a  $g$ - homeomorphism  $X_1 \rightarrow X_2$ . Thus,  $f$  and its inverse are  $g$  continuous as well as bijection.  $f$  and its inverse are  $s^*p^*$  continuous this comes from any  $g$ -continuous function being  $s^*p^*$ -continuous. Because of this  $f$  is  $s^*p^*$ Homeomorphism.

The reverse implications are not valid as demonstrated from the below example.

**Example 5.8:** Let  $X_1 = X_2 = \{r, s, t\}$ ;  $\tau = \{X_1, \varphi, \{t\}, \{r, t\}\}$  and  $\tau^c = \{X_1, \varphi, \{r, s\}, \{s\}\}$  and  $s^*p^*$ closed sets of  $X_1 = \{X_1, \varphi, \{r\}\}$ .  $g$  closed sets of  $X_1 = \{X_1, \varphi, \{s\}, \{r, s\}, \{s, t\}\}$ .  $\sigma = \{X_2, \varphi, \{r\}, \{t\}, \{s, t\}, \{r, t\}\}$ ;  $\sigma^c = \{X_2, \varphi, \{s, t\}, \{r, s\}, \{r\}, \{s\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \varphi, \{s\}, \{t\}, \{s, t\}\}$ .  $g$  closed sets of  $X_2$  are  $\{X_2, \varphi, \{s\}, \{t\}, \{s, t\}\}$ . Declare a Map  $f: X_1 \rightarrow X_2$  by  $f(r) = r$ ;  $f(s) = s$ ;  $f(t) = t$ . Hence  $f$  does not a  $g$ -homeomorphism rather than a  $s^*p^*$ -H. Because the inverse image of closed map  $\{s\}$  in  $(X_1, \tau)$ ,  $(f^{-1})^{-1}(r, s) = f(r, s) = \{r, s\}$  is not in  $g$ - closed set in  $X_2$ .

**Remark 5.9:** The example that comes next points out the independence of  $s^*p^*$ -H and  $\alpha g$ -homeomorphism.

**Example 5.10:** Let  $X_1 = X_2 = \{q, r, s, t\}$ ;  $\tau = \{X_1, \varphi, \{q\}, \{r\}, \{q, r\}, \{q, r, s\}\}$  and  $\tau^c = \{X_1, \varphi, \{r, s, t\}, \{q, s, t\}, \{s, t\}, \{t\}\}$ .  $s^*p^*$ -C sets of  $X_1$  are  $\{X_1, \varphi, \{q\}, \{r\}, \{t\}, \{r, t\}, \{q, t\}\}$ .  $\sigma = \{X_2, \varphi, \{q\}, \{s\}, \{t\}, \{q, s\}, \{q, t\}, \{s, t\}, \{q, s, t\}\}$  and  $\sigma^c = \{X_2, \varphi, \{r, s, t\}, \{q, r, t\}, \{q, r, s\}, \{r, t\}, \{r, s\}, \{q, r\}, \{r\}\}$  and  $s^*p^*$ -C sets of  $X_2$  are  $\{X_2, \varphi, \{q\}, \{s\}, \{t\}, \{q, s\}, \{q, t\}, \{s, t\}\}$ .  $\alpha g$  closed sets of  $X_2$  are  $\{X_2, \varphi, \{r\}, \{q, r\}, \{r, t\}, \{q, r, s\}, \{q, r, t\}, \{r, s, t\}\}$ . Assume a Map  $f: X_1 \rightarrow X_2$  by  $f(q) = q$ ;  $f(r) = t$ ;  $f(s) = r$ ;  $f(t) = s$ . Hence  $f$  does not a  $\alpha g$  homeomorphism rather than  $s^*p^*$ -H. Hence the image of closed set  $\{u\}$  in  $(X_1, \tau)$ ,  $(f^1)^{-1}(t) = f(t) = \{s\}$  is does not in  $\alpha g$  closed set in  $X_2$ . Also,  $f$  is  $\alpha g$  homeomorphism but not  $s^*p^*$ -H, consider the closed set  $\{q, s, t\}$  in  $(X_1, \tau)$ , then its inverse image  $(f^1)^{-1}(q, s, t) = f(q, s, t) = \{q, r, s\}$  is not in  $s^*p^*$ -C set in  $X_2$ .

## 6. Conclusion

In this paper, we derived unique features of  $s^*p^*$ closed map and even  $s^*p^*$ homeomorphism via  $s^*p^*$ continuous and many of the implications, relations and independence of relationship with few of the existing closed sets are studied.

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