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On Functional Identities Involving n-Derivations in Semiprime Rings

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Abstract: This paper investigates n-additive maps, known as symmetric n-derivations whose trace satisfies certain algebraic identities. We establish various results that enhance our understanding of these maps, notably their occurrence in semiprime rings and the behavior of rings under particular algebraic identities involving nonzero ideal. Furthermore, we prove that the semiprime ring possesses a nonzero central ideal under certain algebraic conditions.

Keywords: derivation, symmetric n-derivation, semiprime ring, ideal.

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1. Introduction:

Throughout, S will be an associative ring with $Z(S)$ as its center. A ring S is said to be prime if $\sigma S\varsigma = \{0\}$ implies that either $\sigma = 0$ or $\varsigma = 0$ and semiprime if $\sigma S\sigma = \{0\}$ implies that $\sigma = 0$ where $\sigma, \varsigma \in S$. The notion $[\sigma, \varsigma]$ and $\sigma \circ \varsigma$ denote the commutator $\sigma\varsigma - \varsigma\sigma$ and the anti-commutator $\sigma\varsigma + \varsigma\sigma$, respectively, for any $\sigma, \varsigma \in S$. A ring S is said to be n-torsion free if $nx = 0$ implies that $x = 0$ for all $x \in S$. If S is $n!$-torsion free, then it is m-torsion free for every divisor m of $n!$. An additive mapping $D: S \to S$ is called a derivation if $D(\sigma\varsigma) = D(\sigma)\varsigma + \sigma D(\varsigma)$ holds for all $\sigma, \varsigma \in S$. To expand the theory of derivation, Maksa [1] introduced the concept of symmetric bi-derivations on rings, which Vukman carried out a more extensive investigation[see in [2],[3]]. A bi-additive map $D: S \times S \to S$ is said to be a bi-derivation if $D(\sigma\sigma', \varsigma) = D(\sigma, \varsigma)\sigma' + \sigma D(\sigma', \varsigma)$, $D(\sigma, \varsigma\varsigma') = D(\sigma, \varsigma)\varsigma' + \varsigma D(\sigma, \varsigma')$ hold for any $\sigma, \sigma', \varsigma, \varsigma' \in S$. The aforementioned conditions are identical if D is also a symmetric map, that is, if $D(\sigma, \varsigma) = D(\varsigma, \sigma)$ for every $\sigma, \varsigma \in S$. In this case, D is referred to as a symmetric bi-derivation on S. Several authors have explored symmetric bi-derivations on rings (see[4],[5],[6]]) and brought about insightful conclusions. The study of tri-derivation was initiated by Ozturk [7], in which he proved various results. Advancing this idea, Park [8] introduced the concept of permuting n-derivation as follows: Let $n \geq 2$ be a fixed integer, and $S^n = \underbrace{S \times S \times \cdots \times S}_{n-times}$.

A map $D: S^n \to S$ is said to be symmetric (permuting) if $D(\sigma_1, \sigma_2, ..., \sigma_n) = D(\sigma_{\varphi(1)}, \sigma_{\varphi(2)}, ..., \sigma_{\varphi(n)})$ for all permutations $\varphi(t) \in S_n$ and $\sigma_j \in S$ where $j = 1, 2, ..., n.$ Let $n \geq 2$ be a fixed integer. An n-additive mapping (*i.e.*, additive in each argument) $D: S^n \to S$ is called an n-derivation on S if the relation:

$$D(\sigma_1\sigma_1', \sigma_2, ..., \sigma_n) = D(\sigma_1, \sigma_2, ..., \sigma_n)\sigma_1' + \sigma_1 D(\sigma_1', \sigma_2, ..., \sigma_n),$$
$$D(\sigma_1, \sigma_2\sigma_2', ..., \sigma_n) = D(\sigma_1, \sigma_2, ..., \sigma_n)\sigma_2' + \sigma_2 D(\sigma_1, \sigma_2', ..., \sigma_n),$$
$$D(\sigma_1, \sigma_2, ..., \sigma_n\sigma'_n) = D(\sigma_1, \sigma_2, ..., \sigma_n)\sigma'_n + \sigma_n D(\sigma_1, \sigma_2, ..., \sigma'_n)$$

hold for all $\sigma_j, \sigma'_j \in S,\ \mathrm{j} = 1, 2, ..., n$.Many authors have examined numerous algebraic identities involving the traces of n-derivations and have uncovered several insightful results (see[9],[10],[11],[12],[13]]). A symmetric 2-derivation is a bi-derivation, a symmetric 3-derivation (or tri-derivation) on rings, and a 1-derivation is clearly a derivation.

Posner's finding [14] showed that if a prime ring has a nonzero centralizing derivation, it becomes commutative. This led to the study of centralizing maps on prime rings. Since then, researchers like Brešar [15], Deng-Bell [16], Lanski [17], and Vukman [18] have made important contributions. Vukman derived conclusions about the trace of symmetric bi-derivations in prime rings (see [19,20] for more details). In [21], Ashraf formulated corresponding results for semiprime rings. Moreover, Ashraf et al. [22, 23] derived the commutativity of rings allowing n-derivations whose traces adhere to particular polynomial conditions. Several authors have explored different identities related to traces of bi-derivations and n-derivations, yielding numerous intriguing findings (see [24-26] and references therein).
The main objective of this paper is to establish comparable results on the permutation of n-derivations within the semiprime rings. In particular, we examine the structure of semiprime rings and identify the forms of mappings (traces of n-derivations) that satisfy to specific functional identities. More precisely, we establish that: For a fixed integer $n \geq 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits a nonzero symmetric n-derivation $D: S^n \to S$ with trace $\delta: S \to S$ then S has a nonzero central ideal.

2. Preliminary Results:

The following supporting implications are crucial to derive the primary results of this articlewe begin by presenting several well-established results.

Lemma 2.1. [8] Let n be a fixed positive integer and S an $n!$-torsion free ring. Assume that $x_1, x_2, \ldots, x_n \in S$ satisfy $\omega x_1 + \omega^2 x_2 + \cdots + \omega^n x_n = 0$ (or $\in Z(S)$) for $\omega = 1, 2, ..., n$. Then, $x_j = 0$ (or $\in Z(S)$) for $j = 1, 2, ..., n.$

Lemma 2.2. [[6], Lemma 3] Let S be a 2-torsion free semiprime ring and M be a nonzero ideal of S. If $[M,M]\subseteq Z(S)$, then S contains a nonzero central ideal.

Lemma 2.3. [[6], Lemma 4] Let S be a 2-torsion free semiprime ring and M be a nonzero ideal of S. If $M\circ M\subseteq Z(S)$, then S contains a nonzero central ideal.

We start our investigation of symmetric n-derivations with the following result.

3. Main Results:

Theorem 3.1. For a fixed integer $n\geq 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits a nonzero symmetric n-derivation $D: S^n \to S$ with trace $\delta: S\to S$ satisfying $\delta(\sigma)\delta(\varsigma)\pm\sigma\varsigma\in Z(S)$ for all $\sigma,\varsigma\in M$, then S has a nonzero central ideal.

Proof: It is assumed that

$$\delta(\sigma)\delta(\varsigma)\pm\sigma\varsigma\in Z(S) \text{ for all } \sigma,\varsigma\in M. \tag{3.1}$$

Replacing ς by $\varsigma+mk$ for $k\in M$ in (3.1) and $1\leq m\leq n-1$, we have

$$\delta(\sigma)(\delta(\varsigma)+\delta(mk)+\sum_{t=1}^{n-1}{}^{n}C_t \ D(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times},\underbrace{mk,...,mk}_{t-times}))\pm\sigma\varsigma\pm\sigma mk\in Z(S) \text{ for all } \sigma,\varsigma,k\in M.$$

(3.2)

With the given condition, we arrive at

$$\delta(\sigma)\sum_{t=1}^{n-1}{}^{n}C_t \ D(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times},\underbrace{mk,...,mk}_{t-times})\in Z(S) \text{ for all } \sigma,\varsigma,k\in M.$$

Using Lemma2.1, we see that

$$n\delta(\sigma)D(\varsigma,...,\varsigma,k)\in Z(S) \text{ for all } \sigma,\varsigma,k\in M.$$

Since S is $n!$-torsion free, we have

$$\delta(\sigma)D(\varsigma,...,\varsigma,k)\in Z(S) \text{ for all } \sigma,\varsigma,k\in M. \tag{3.3}$$

Writing ς in place of k in (3.3), we get

$$\delta(\sigma)\delta(\varsigma)\in Z(S) \text{ for all } \sigma,\varsigma\in M. \tag{3.4}$$

By utilizing the hypothesis, we obtain that

$$\sigma\varsigma\in Z(S) \text{ for all } \sigma,\varsigma\in M. \tag{3.5}$$

Commuting with $r\in S$, we arrive

$$[\sigma\varsigma,r]=0 \text{ for all } \sigma,\varsigma\in M,\ r\in S. \tag{3.6}$$

and so,

$$\sigma[\varsigma,r]+[\sigma,r]\varsigma=0 \text{ for all } \sigma,\varsigma\in M,\ r\in S. \tag{3.7}$$

Replacing ς by ςk in (3.7) and using (3.7), we see that

$$\sigma\varsigma[k,r]=0 \text{ for all } \sigma,\varsigma,k\in M,\ r\in S.$$

On replacing σ by $[k,r]$, we get
$[k,r]\varsigma[k,r]=0$ for all $\varsigma,k\in M$, $r\in S$.
That is,
$[k,r]\varsigma S[k,r]\varsigma=(0)$ for all $\varsigma,k\in M$, $r\in S$.
Since S is a semiprime ring, we have
$[k,r]\varsigma=0$ for all $\varsigma,k\in M$, $r\in S$.
Taking ς to be $t[k,r]$, $t\in S$, we see that
$[k,r]t[k,r]=0$.
By the semiprimeness of S, we get $M\subseteq Z(S)$. Thus, S contains a nonzero central ideal. The proof is complete. □

Theorem 3.2. For a fixed integer $n\geq 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits a nonzero symmetric n-derivation $D:\mathrm{S}^n\rightarrow S$ with trace $\delta:S\rightarrow S$ satisfying $\delta(\sigma)\delta(\varsigma)\pm\sigma\circ\varsigma\in\mathrm{Z(S)}$ for all $\sigma,\varsigma\in M$, then S has a nonzero central ideal.

Proof: It is given that

$$\delta(\sigma)\delta(\varsigma)\pm\sigma\circ\varsigma\in\mathrm{Z(S)} \text{ for all } \sigma,\varsigma\in M. \tag{3.8}$$

Replacing ς by $\varsigma+mk$ for $k\in M$ in (3.8) and $1\leq m\leq n-1$, we arrive at

$$\delta(\sigma)(\delta(\varsigma)+\delta(mk)+\sum_{t=1}^{n-1}{}^{n}C_t\ \mathrm{D}(\underbrace{\varsigma,\ldots,\varsigma}_{(n-t)-times},\underbrace{mk,\ldots,mk}_{t-times}))\pm\sigma\circ\varsigma\pm\sigma\circ mk\in Z(S) \text{ for all } \sigma,\varsigma,k\in M. \tag{3.9}$$

Considering the given condition, we establish

$$\delta(\sigma)\sum_{t=1}^{n-1}{}^{n}C_t\ \mathrm{D}(\underbrace{\varsigma,\ldots,\varsigma}_{(n-t)-times},\underbrace{mk,\ldots,mk}_{t-times})\in Z(S). \tag{3.10}$$

With the help of Lemma 2.1, we see that
$n\delta(\sigma)D(\varsigma,\ldots,\varsigma,k)\in Z(S)$ for all $\sigma,\varsigma,k\in M$.
Since S is $n!$-torsion free, we have

$$\delta(\sigma)D(\varsigma,\ldots,\varsigma,k)\in Z(S) \text{ for all } \sigma,\varsigma,k\in M. \tag{3.11}$$

Writing ς in place of k in (3.11), we get

$$\delta(\sigma)\delta(\varsigma)\in\mathrm{Z(S)} \text{ for all } \sigma,\varsigma\in M. \tag{3.12}$$

Using the hypothesis, we obtain that

$$\sigma\circ\varsigma\in Z(S) \text{ for all } \sigma,\varsigma\in M. \tag{3.13}$$

That is, $M\circ M\in Z(S)$. Hence, by using Lemma 2.3, S contains a nonzero central ideal. □

Theorem 3.3. For a fixed integer $n\geq 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits two nonzero symmetric n-derivations $D:\mathrm{S}^n\rightarrow S$ with trace $\delta: S \to S$ satisfying $\delta(\sigma)\delta(\varsigma) \pm [\sigma,\varsigma] \in \mathrm{Z(S)}$ for all $\sigma,\varsigma \in M$, then S has a nonzero central ideal.

Proof: It is assumed that

$$\delta(\sigma)\delta(\varsigma) \pm [\sigma,\varsigma] \in \mathrm{Z(S)} \text{ for all } \sigma,\varsigma \in M. \tag{3.14}$$

Replacing ς by $\varsigma + mk$ for $k \in M$ (3.14) and $1 \le m \le n-1$, we arrive at

$$\delta(\sigma)(\delta(\varsigma)+\delta(mk)+\sum_{t=1}^{n-1} {}^{n}C_t\ \mathrm{D}(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times},\underbrace{mk,...,mk}_{t-times})) \pm [\sigma,\varsigma] \pm [\sigma,mk] \in Z(S) \text{ for all } \sigma,\varsigma,k \in M. \tag{3.15}$$

Using the given condition, we get

$$\delta(\sigma)\sum_{t=1}^{n-1} {}^{n}C_t\ \mathrm{D}(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times},\underbrace{mk,...,mk}_{t-times}) \in Z(S). \tag{3.16}$$

With the help of Lemma 2.1, we obtain

$n\delta(\sigma)D(\varsigma,...,\varsigma,k) \in Z(S)$ for all $\sigma,\varsigma,k \in M$.

Since S is $n!$-torsionfree,we have

$$\delta(\sigma)D(\varsigma,...,\varsigma,k) \in Z(S) \text{ for all } \sigma,\varsigma,k \in M. \tag{3.17}$$

Writing ς in place of k, we get

$$\delta(\sigma)\delta(\varsigma) \in \mathrm{Z(S)} \text{ for all } \sigma,\varsigma \in M. \tag{3.18}$$

From the hypothesis, we attain

$$[\sigma,\varsigma] \in \mathrm{Z(S)} \text{ for all } \sigma,\varsigma \in M. \tag{3.19}$$

That is, $[M,M] \in Z(S)$. Hence, by using Lemma 2.2, S contains a nonzero central ideal.□

From the earlier results, we arrive at the following corollary:

Corollary3.4: For a fixed integer $n \ge 2$, let S be an $n!$-torsion free semiprime ring. Suppose that S admits two nonzero symmetric n-derivations $D: \mathrm{S}^n \to S$ with trace $\delta: S \to S$ satisfying any one of the conditions:

$\delta(\sigma)\delta(\varsigma) \pm \sigma\varsigma \in Z(S)$ for all $\sigma,\varsigma \in S$,

$\delta(\sigma)\delta(\varsigma) \pm \sigma \circ \varsigma \in Z(S)$ for all $\sigma,\varsigma \in S$,

$\delta(\sigma)\delta(\varsigma) \pm [\sigma,\varsigma] \in Z(S)$ for all $\sigma,\varsigma \in S$ then S is commutative.

Theorem 3.5. For a fixed integer $n \ge 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits two nonzero symmetric n-derivations $D: \mathrm{S}^n \to S$ with trace $\delta: S \to S$ satisfying $\delta[\sigma,\varsigma] \pm \sigma\varsigma \in \mathrm{Z(S)}$ for all $\sigma,\varsigma \in M$, then S has a nonzero central ideal.

Proof: It is given that

$$\delta[\sigma,\varsigma] \pm \sigma\varsigma \in \mathrm{Z(S)} \text{ for all } \sigma,\varsigma \in M. \tag{3.20}$$

Replacing ς by $\varsigma + mk$ for $k \in M$ in (3.20) and $1 \le m \le n-1$, we get

$$\delta([\sigma,\varsigma]+[\sigma,mk]) \pm \sigma\varsigma + \sigma mk \in Z(S) \text{ for all } \sigma,\varsigma,k \in M. \quad (3.21)$$

By reducing, we arrive at

$$\delta([\sigma,\varsigma]+[\sigma,mk]) + \sum_{t=1}^{n-1} {}^{n}C_t \, D(\underbrace{[\sigma,\varsigma],...,[\sigma,\varsigma]}_{(n-t)-times},\underbrace{[\sigma,mk],...,[\sigma,mk]}_{t-times}) \pm [\sigma,\varsigma] \pm [\sigma,mk] \in Z(S)$$

for all $\sigma,\varsigma,k \in M$. (3.22)

Considering the hypothesis, we get

$$\sum_{t=1}^{n-1} {}^{n}C_t \, D(\underbrace{[\sigma,\varsigma],...,[\sigma,\varsigma]}_{(n-t)-times},\underbrace{[\sigma,mk],...,[\sigma,mk]}_{t-times}) \in Z(S) \text{ for all } \sigma,\varsigma,k \in M. \quad (3.23)$$

Consequently, we obtain the following:

$mP_1(\sigma,\varsigma,k) + m^2 P_2(\sigma,\varsigma,k) + \cdots + m^{n-1} P_{n-1}(\sigma,\varsigma,k) = 0$ for all $\sigma,\varsigma,k \in M$. Where

$$P_t(\sigma,\varsigma,k) = {}^{n}C_t D(\underbrace{[\sigma,\varsigma],...,[\sigma,\varsigma]}_{(n-t)-times},\underbrace{[\sigma,mk],...,[\sigma,mk]}_{t-times}) - [{}^{n}C_t D(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times},\underbrace{k,...,k}_{t-times}),\sigma]$$

Signifies the sum of terms contain k appears t -times.
In consideration of Lemma2.1 and the restriction of torsion-freeness in S, we get

$$D([\sigma,\varsigma],...,[\sigma,\varsigma],[\sigma,mk]) \in Z(S) \text{ for all } \sigma,\varsigma,k \in M. \quad (3.24)$$

Replacing k by ς, we get

$$\delta([\sigma,\varsigma]) \in Z(S) \text{ for all } \sigma,\varsigma \in M. \quad (3.25)$$

Applying the hypothesis once again, we obtain
$\sigma\varsigma \in Z(S)$ for all $\sigma,\varsigma \in M$.
Consequently, the result can be derived using the same approach as (3.5) in Theorem 3.1.□

Following results can analogously be obtained.

Theorem 3.6. For a fixed integer $n \ge 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits two nonzero symmetric n-derivations $D: S^n \to S$ with trace $\delta: S \to S$ satisfying $\delta[\sigma,\varsigma] \pm \sigma \circ \varsigma \in Z(S)$ for all $\sigma,\varsigma \in M$, then S has a nonzero central ideal.

Theorem 3.7. For a fixed integer $n \ge 2$, let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of S. Suppose that S admits two nonzero symmetric n-derivations $D: S^n \to S$ with trace $\delta: S \to S$ satisfying $\delta[\sigma,\varsigma] \pm [\sigma,\varsigma] \in Z(S)$ for all $\sigma,\varsigma \in M$, then S has a nonzero central ideal.

Corollary3.8: For a fixed integer $n \geq 2,$ let S be an $n!$-torsion free semiprime ring. Assume that S admits two nonzero symmetric n-derivations $D: S^n \to S$ with trace $\delta: S \to S$ satisfying any one of the conditions:
$\delta[\sigma,\varsigma] \pm \sigma\varsigma \in Z(S)$ for all $\sigma,\varsigma \in S,$
$\delta[\sigma,\varsigma] \pm \sigma \circ \varsigma \in Z(S)$ for all $\sigma,\varsigma \in S,$
$\delta[\sigma,\varsigma] \pm [\sigma,\varsigma] \in Z(S)$ for all $\sigma,\varsigma \in S$ thus, S is commutative.

Theorem 3.9. For a fixed integer $n \geq 2,$ let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of $S.$ Suppose that S admits two nonzero symmetric n-derivations $D: S^n \to S$ with trace $\delta: S \to S$ satisfying $\delta(\sigma) \circ \delta(\varsigma) \pm \sigma\varsigma \in Z(S)$ for all $\sigma,\varsigma \in M,$ then S has a nonzero central ideal.

Proof: It is given that
$\delta(\sigma) \circ \delta(\varsigma) \pm \sigma\varsigma \in Z(S)$ for all $\sigma,\varsigma \in M.$ (3.26)
Replacing ς by $\varsigma + mk$ for $k \in M$ in (3.26) and $1 \leq m \leq n-1,$ we arrive at

$$\delta(\sigma) \circ \delta(\varsigma) + \delta(\sigma) \circ \delta(mk) + \delta(\sigma) \circ \sum_{t=1}^{n-1} {}^nC_t \; D(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times}, \underbrace{mk,...,mk}_{t-times}) \pm \sigma\varsigma \pm \sigma mk \in Z(S)$$

for all $\sigma,\varsigma,k \in M.$ (3.27)
On using the hypothesis, we arrive at

$$\delta(\sigma) \circ \sum_{t=1}^{n-1} {}^nC_t \; D(\underbrace{\varsigma,...,\varsigma}_{(n-t)-times}, \underbrace{mk,...,mk}_{t-times}) \in Z(S) \text{ for all } \sigma,\varsigma,k \in M. \quad (3.28)$$

Using Lemma2.1, we obtain that
$\delta(\sigma) \circ D(\varsigma,...,\varsigma,k) \in Z(S)$ for all $\sigma,\varsigma,k \in M.$ (3.29)
Writing ς in place of k, we get
$\delta(\sigma) \circ \delta(\varsigma) \in Z(S)$ for all $\sigma,\varsigma \in M.$ (3.30)
From the hypothesis, we see that
$\sigma\varsigma \in Z(S)$ for all $\sigma,\varsigma \in M.$
(3.31)
Consequently, the result can be derived using the same approach as (3.5) in Theorem 3.1. □

Following results can analogously be obtained.

Theorem 3.10. For a fixed integer $n \geq 2,$ let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of $S.$ Suppose that S admits two nonzero symmetric n-derivations $D: S^n \to S$ with trace $\delta: S \to S$ satisfying $\delta(\sigma) \circ \delta(\varsigma) \pm \sigma \circ \varsigma \in Z(S)$ for all $\sigma,\varsigma \in M,$ then S has a nonzero central ideal.

Theorem 3.11. For a fixed integer $n \geq 2,$ let S be an $n!$-torsion free semiprime ring and M be a nonzero ideal of $S.$ Suppose that S admits two nonzero symmetric n-derivations $D:\ S^n \to S$ with trace $\delta : S \to S$ satisfying $\delta(\sigma) \circ \delta(\varsigma) \pm [\sigma, \varsigma] \in Z(S)$ for all $\sigma, \varsigma \in M,$ then S has a nonzero central ideal.

Corollary3.12: For a fixed integer $n \geq 2,$ let S be an $n!$-torsion free semiprime ring. Assume that S admits two nonzero symmetric n-derivations $D:\ S^n \to S$ with trace $\delta : S \to S$ satisfying any one of the conditions:

$\delta(\sigma) \circ \delta(\varsigma) \pm \sigma\varsigma \in Z(S)$ for all $\sigma, \varsigma \in S$,

$\delta(\sigma) \circ \delta(\varsigma) \pm \sigma \circ \varsigma \in Z(S)$ for all $\sigma, \varsigma \in S$,

$\delta(\sigma) \circ \delta(\varsigma) \pm [\sigma, \varsigma] \in Z(S)$ for all $\sigma, \varsigma \in S$ thus, S is commutative.

Example:

Suppose the ring $S = \left\{ \begin{bmatrix} u & v \\ 0 & 0 \end{bmatrix} / u, v \in Z \right\}$. Consider $M = \left\{ \begin{bmatrix} u & 0 \\ 0 & 0 \end{bmatrix} / u, v \in Z \right\}$ be a nonzero ideal of $S.$ Denote $W_i = \begin{bmatrix} u_i & v_i \\ 0 & 0 \end{bmatrix} \in S,\ u_{i,} v_i \in Z,\ 1 \leq i \leq n$ and define as $D:\ S^n \to S \quad D(W_1, W_2 ... W_n) = \begin{bmatrix} w_1 w_2 ... w_n & 0 \\ 0 & 0 \end{bmatrix}$ with trace $\delta : S \to S$ define by $\delta\left(\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} b^n & 0 \\ 0 & 0 \end{bmatrix}$. It is evident that is a symmetric n-derivation, and all the conditions in the Theorems are satisfied. However, M is nonzero ideal and S does not contain any nonzero central ideal.

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