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Aggregate Operators of Neutrosophic Vague Hypersoft Set

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Abstract: A new set NVHSS is defined. This essay covers fundamental operators such as union, intersection, complement, subset, empty set, and same set etc., of NVHSS. Appropriate examples are provided along with the implementation and validity. Proposed operations will be crucial in future decision-making for more accuracy and precision in areas such as management issues, personal selection, and many more.

Keywords: SS, Neutrosophic SS, Hyper soft set, NV , Neutrosophic Vague soft set, VHSS, Neutrosophic Vague Hypersoft set.

1. Introduction

Atanassov's theory only addresses incomplete data that takes into account both membership and non-membership values; intuitionistic fuzzy set theory is unable to cope with inconsistent and imprecise data. The NS was created by Smarandache to address such inconsistent and imprecise data [3]. Molodtsov was the first person to describe the concept of SS [2] as a a brand-new numerical tool for handling problems with unclear circumstances. Maji [5] offered the concept of an NSS with the required functions. The concept of the potential NSS was created by Karaaslan. Saqlain. Samlai et al. By merging the plithogenic sets and hypersoft sets, Martin and Smarandache created the plithogenic hypersoft set in [4]. Saqlain et al. Neutrosophic vague set theory was studied by Alkhazaleh (2015). Das et al[1]. Muhammad saqlain ,Sana Moni Muhammad Naveed and Florentin smarandache[2020] established a AONHSS. Anjan Mukherjee . [7]Rana Muhammed Zulqarnain Xiao Long xin Muhammad saqlain ,Florentin smarandache[2020] presented GAONHSs .This article tries to establish a concept called as Neutrosophic Vague Hyper Soft Set (NVHSS).

2. Preliminaries

Definition 2.1:[3]

Let ɱ be the worldwide collection and Ȩ be the collection of characteristics with respectively to ɱ. Let Ᵽ (ɱ) be the power set of ɱ and Ă ⊆ Ȩ . A pair (Ⅎ, Ă) is known as SS over ɱ and its provided as

Ⅎ: Ă → Ᵽ(ɱ)

It is also depicted as: (Ⅎ, Ă) = [Ⅎ (ȩ) ∈ Ᵽ (ɱ): ȩ (Ⅎ, Ă) = {Ⅎ (ȩ) ∈ Ᵽ (ɱ): ȩ ∈ Ȩ , Ⅎ (ȩ) = ∅ *if* ȩ ≠ Ąȩ≠ Ă]

Definition 2.2: [6]

Let ɱ be worldwide collection and Ᵽ(ɱ) be a power set of ɱ. Consider as ƙ1, ƙ2, ƙ3,….. ƙn and for $n \geq 1$, There are a few unique characteristics such as ƙ1, ƙ2, ƙ3,….. ƙn and Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn are sets with the following constraints for corresponding values and characteristics, correspondingly Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn with $Ᵽ_k \cap Ᵽ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \dots n\}$ and Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn = S. Ⅎ is a mapping from Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn to Ᵽ(ɱ)

Definition 2.3:[1]

Let a NV set on the worldwide collection

Ă={<X, Ʈ (xu), Ị(xu),F(xu)>:x∈X}

Since Ʈ (xu) is TMF , Ị (xu) is IMF ,Ƒ(xu) is FMF where Ʈ (xu)=[$Ʈ^+$, $Ʈ^-$] , Ị(xu)=[$Ĭ^+$, $Ĭ^-$] , Ƒ (xu)= [$Ƒ^+$, $Ƒ^-$]

$Ʈ^+$=1-$Ƒ^-$, $Ƒ^+$ =1- $Ʈ^-$

$0 \leq$ Ʈ (xu) + Ĭ (xu) + Ƒ(xu) ≤ 3.

Definition 2.4: [7]

Let Ⅎ (ɱ) be the set of NV of ɱ and Ă ⊆ Ḝ . A pair ((Ⅎ, Ă) is called a NSS over ɱ and its mapping is provided as Ⅎ: A ⟶ Ᵽ (ɱ)

Definition 2.5: [5]

Let ɱ be worldwide collection and Ᵽ(ɱ) be a power set of ɱ. Consider as ƙ1, ƙ2, ƙ3,….. ƙn and for $n \geq 1$, There are a few unique characteristics such as ƙ1, ƙ2, ƙ3,….. ƙn and Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn are sets with the following constraints for corresponding values and characteristics, correspondingly Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn with $Ᵽ_k \cap Ᵽ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \dots n\}$ and Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn = S. Ⅎ is a mapping from Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn to Ᵽ(ɱ) and Ⅎ (Ᵽ1, Ᵽ2,Ᵽ3,……. Ᵽn) ={< x, Ť (xu), Ĭ (xu), Ƒ (xu) >, $x \in$ ɱ} Where Ť(xu)=[$Ť^+$, $Ť^-$] , Ĭ (xu)=[$Ĭ^+$,$Ĭ^-$] , Ƒ (xu)=[$Ƒ^+$,$Ƒ^-$]

$Ť^+$=1-$Ƒ^-$, $Ƒ^+$ =1- $Ť^-$

$0 \leq$ Ť(xu) + Ĭ (xu) + Ƒ (xu) ≤ 3.

Definition 2.6:[5]

Consider two NVSs , A_{NS} and B_{NV} the union is a NVHS, C_{NV} written as $C_{NV}=A_{NV} \cup B_{NV}$, following by

$Ʈ_{CNV}(xu)=[\max(Ʈ^-_{ANV}, Ʈ^-_{BNV}), \max(Ʈ^+_{ANV}, Ʈ^+_{BNV})]$

$Ĭ_{CNV}(xu)=[\min(Ĭ^-_{ANV}, Ĭ^-_{BNV}), \min(Ĭ^+_{ANV}, Ĭ^+_{BNV})]$

$Ƒ_{CNV}(xu)=[\min(Ƒ^-_{ANV}, Ƒ^-_{BNV}), \min(Ƒ^+_{ANV}, Ƒ^+_{BNV})]$

Definition 2.7:[5]

Consider two NVSs , A_{NS} and B_{NV} the intersection is a NVHS, C_{NV}, written as $C_{NV}=A_{NV} \cap B_{NV}$, following by

Ʈ$_{CNV}$(xu)=[min($Ʈ^{-}_{ANV}$,$Ʈ^{-}_{BNV}$),min($Ʈ^{+}_{ANV}$,$Ʈ^{+}_{BNV}$)]

$Ĭ_{CNV}$(xu)=[max(I^{-}_{ANV} ,$Ĭ^{-}_{BNV}$),max($Ĭ^{+}_{ANV}$,$Ĭ^{+}_{BNV}$)]

$Ƒ_{CNV}$(xu)=[max($Ƒ^{-}_{ANV}$,$Ƒ^{-}_{BNV}$),max($Ƒ^{+}_{ANV}$,$Ƒ^{+}_{BNV}$)]

3.1. Neutrosophic Vague Hypersoft Set (NVHSS)

Let Ⅎ($Ş^1$) and Ⅎ($Ş^2$) be two worldwide collection and Ᵽ(ɱ) be a power set ɱ of Consider as $ƙ_1$, $ƙ_2$, $ƙ_3$,….. $ƙ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ƙ_1$, $ƙ_2$, $ƙ_3$,….. $ƙ_n$ and $Ᵽ_1$, $Ᵽ_2$,$Ᵽ_3$,……. $Ᵽ_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly $Ᵽ_1$, $Ᵽ_2$,$Ᵽ_3$,……. $Ᵽ_n$ with $Ᵽ_k \cap Ᵽ_l = \emptyset$, The relationship between $k \neq l$ and k, l$\epsilon\{1,2,3 \dots n\}$ and $Ᵽ_1$, $Ᵽ_2$,$Ᵽ_3$,……. $Ᵽ_n$ = S. Ⅎ is a mapping from $Ᵽ_1$, $Ᵽ_2$,$Ᵽ_3$,……. $Ᵽ_n$ to Ᵽ(ɱ) and Ⅎ ($Ᵽ_1$, $Ᵽ_2$,$Ᵽ_3$,……. $Ᵽ_n$) ={< x, Ť (xu), Ĭ (xu), Ƒ (xu) >, $x \in$ ɱ} where Ť, Ĭ, Ƒ are membership values respectively such that Ť, Ĭ, Ƒ : ɱ ⟶ [0^-, 1^+] and Where

Ť(xu)=[$Ť^+$, $Ť^-$] , Ĭ (xu)=[$Ĭ^+$,$Ĭ^-$] , Ƒ (xu)=[$Ƒ^+$,$Ƒ^-$]

$Ť^+$=1-$Ƒ^-$, $Ƒ^+$ =1- $Ť^-$

0 ≤ Ť(xu) + Ĭ (xu) + Ƒ (xu) ≤ 3.

Example 3.2:

Let ɱ be the group of decision-makers to choose the optimal car type as

ɱ = { X^1, X^2, X^3}

Consider the collection of characteristics as

$¥^1$ = Car type, $¥^2$ = Seat, $¥^3$ = Variate.

And their respective characteristics are following by

$¥^1$ = Car type = {Convertible, Coupe, Sedon}

$¥^2$ = Seat = {7 S, 4 S ,5 S}

$¥^3$ = Variate = {Low end , Medium end, Top end}

Let the function be Ⅎ : $¥^1 \times ¥^2 \times ¥^3 \longrightarrow$ Ᵽ(ɱ)

Table 1: car type

$¥^1$(Car *type*)	X^1	X^2	X^3
Convertible	[0.2, 0.3],[0.2,0.4],[0.7,0.8]	[0.2, 0.4],[0.2,0.3],[0.6,0.8]	[0.4,0.5],[0.1,0.5,[0.5,0.6]
Coupe	[0.2,0.4],[0.3,0.4],[0.6,0.8]	[0.2,0.5],[0.1,0.2],[0.5,0.8]	[0.3,0.5],[0.2,0.4],[0.5,0.7]
Variate	[0.4,0.6],[0.3,0.5],[0.6, 0.4]	[0.4,0.5],[0.2,0.4],[0.5,0.6]	[0.3,0.6],[0.1,0.2],[0.4, 0.7]

Table 2: Seat

	X^1	X^2	X^3
¥² (Seat)			
7 S	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5 0.6]	[0.1,0.5],[0.2,0.3],[0.5,0.9]
5 S	[0.1,0.2],[0.2,0.4],[0.8,0.9]	[0.2,0.3],[0.1,0.3],[0.7,0.8]	[0.6,0.8],[0.1,0.3],[0.2,0.4]
4 S	[0.4,0.5],[0.2,0.4],[0.5,0.6]	[0.1,0.3],[0.1,0.3],[0.7,0.9]	[0.1,0.2],[0.1,0.2],[0.8,0.9]

Table 3: Variate

¥³ (Variate)	X^1	X^2	X^3
Low end	[0.2,0.7],[0.1,0.2],[0.3,0.8]	[0.3,0.6],[0.3,0.4],[0.4,0.7]	[0.3,0.4],[0.1,0.2],[0.6,0.7]
Medium	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]	[0.1,0.2],[0.3,0.4],[0.8,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0,8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]	[0.1,0.5],[0.2,0.3],[0.5,0.9]

Definition 3.3

Ɨ : ¥¹ × ¥² ×¥³ ⟶P(ɱ)

Let's assume Ɨ(Ş) = Ɨ(Convertible, 7 Seat, Top end) = {X^1, X^2}. Then NVHSS of above assumed relation is Ɨ(Ş) =Ɨ(Convertible, 7 Seat, Top end) = {< X^1, (Convertible {[0.2, 0.3],[0.2,0.4],[0.7,0.8]}, 7 Seat {[0.2,0.5],[0.2,0.3],[0.5,0.8] }, Top end {[0.2,0.5],[0.2,0.3],[0.5,0,8]}) >

< X^2,(Convertible {[0.2, 0.4],[0.2, 0.3],[0.6,0.8]},7 Seat {[0.4,0.5],[0.1,0.2],[0.5 0.6]}, Top end {[0.4,0.5],[0.1,0.2],[0.5,0.6]}) >}

Its tabular form is given as

Table 4: NVHSS

Ɨ(Ş¹)	X^1	X^2
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Definition 3.4:

Let Ɨ(Ş¹) and Ɨ(Ş²) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as $ƙ_1$, $ƙ_2$, $ƙ_3$,….. $ƙ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ƙ_1$, $ƙ_2$, $ƙ_3$,….. $ƙ_n$ and $Ᵽ_1$, $Ᵽ_2$,$Ᵽ_3$,…..... $Ᵽ_n$

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are sets with the following constraints for corresponding values and characteristics, correspondingly $Ƀ_1$, $Ƀ_2$,$Ƀ_3$,…..... $Ƀ_n$ with $Ƀ_k \cap Ƀ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \ldots n\}$ and $Ƀ_1$, $Ƀ_2$,$Ƀ_3$,…..... $Ƀ_n$ = S. then Ⅎ(Ş^1) is the NVHSSsubset of Ⅎ(Ş^2) if

T(Ⅎ(Ş^1))) ≤ T(Ⅎ(Ş^1))

I(Ⅎ(Ş^1))) ≤ I(Ⅎ(Ş^1))

F(Ⅎ(Ş^1))) ≥ F(Ⅎ(Ş^1))

Example 3.5

Consider the two NVHSS, Ⅎ(Ş^1) and NVHSS, Ⅎ(Ş^2) over the same universe ɱ = {X^1, X^2, X^3}. The NVHSS Ⅎ(Ş^1) = Ⅎ(Convertible$s$, 7 Seat, Top end) = {X^1, X^2} is the subset of NVHSS Ⅎ(Ş^2) = Ⅎ(Convertible$s$,7 Seat) = {X^1} if T(Ⅎ(Ş^1)) ≤ T(Ⅎ(Ş^2)) , I(Ⅎ(Ş^1)) ≤ I(Ⅎ(Ş^2)) , F(Ⅎ(Ş^1)) ≥ FℲ(Ş^2)) . It is provided in tabular form below.

Table 5: NVHSS Ⅎ(Ş^1)

Ⅎ(Ş^1)	X^1	X^2
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Table 6: NVHSS Ⅎ(Ş^2)

Ⅎ(Ş^2)= Ⅎ(Convertible, 7 Seat)	X^1
Convertible	[0.3,0.4],[0.2,0.5],[0.6,0.7]
7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]

This can also be written as

T(Ⅎ(Ş^1)) ⊆ T(Ⅎ(Ş^2)) = Ⅎ (Convertible, 7 Seat , Top end) ⊂ Ⅎ (Convertible, 7 Seat)

= {<X^1, (Convertible {[0.2,0.3],[0.2,0.4],[0.7, 0.8] },7 Seat {[0.4,0.5],[0.2,0.3],[0.5,0.6]}, Top end {[0.2,0.5],[0.2,0.3],[0.5,0.8] }) >,<X^2, (Convertible{[0.2,0.4],[0.2,0.3],[0.6,0.8] },7 Seat{[0.1,0.3],[0.2,0.3],[0.7,0.9] },Top end {[0.4,0.5],[0.1,0.2],[0.5,0.6] }) >,}

⊂ {< X^1, (Convertible{[0.3,0.4],[0.2,0.5],[0.6,0.7] },7 Seat {[0.1,0.3],[0.3,0.4],[0.7,0.9]})>}

This illustrates the value of membership of Convertible for X^1 in both sets is ([0.2,0.3],[0.2,0.4],[0.7, 0.8]) and ([0.3,0.4],[0.2,0.5],[0.6,0.7]) which meet the requirements of the definition of NVHSS subset .

Definition 3.6:

Let Ⅎ(Ş^{1}) and Ⅎ(Ş^{2}) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as ƙ$_1$, ƙ$_2$, ƙ$_3$,….. ƙ$_n$ and for $n \geq 1$, There are a few unique characteristics such as ƙ$_1$, ƙ$_2$, ƙ$_3$,….. ƙ$_n$ and Ᵽ$_1$, Ᵽ$_2$,Ᵽ$_3$,…..... Ᵽ$_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly Ᵽ$_1$, Ᵽ$_2$,Ᵽ$_3$,…..... Ᵽ$_n$ with Ᵽ$_k$∩Ᵽ$_l$ = ∅, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \dots n\}$ and Ᵽ$_1$, Ᵽ$_2$,Ᵽ$_3$,…..... Ᵽ$_n$ = S. then Ⅎ(Ş^{1}) is the Neutrosophic vague equal Hypersoft of Ⅎ(Ş^{2}) if

T(Ⅎ(Ş^{1})) = T(Ⅎ(Ş^{2}))

I(Ⅎ(Ş^{1})) = I(Ⅎ(Ş^{2}))

F(Ⅎ(Ş^{1})) = F(Ⅎ(Ş^{2}))

Example 3.7

Consider the two NVHSS Ⅎ(Ş^{1}) and NVHSS Ⅎ(Ş^{2}) over the same worldwide ɱ = {X^{1}, X^{2}, X^{3} }. The NVHSS F(S^{1}) = Ⅎ(Ş^{1}) =Ⅎ (Convertible, 7 Seat, Top end) = {X^{1}, X^{2}} is the equal to NVHSS Ⅎ(Ş^{2})= Ⅎ(Convertible, 7 Seat) = {X^{1}} if T(Ⅎ(Ş^{1})) = T(Ⅎ(Ş^{2}))) , I(Ⅎ(Ş^{1})) = I(Ⅎ(Ş^{2})) , F(Ⅎ(Ş^{1})) =F(Ⅎ(Ş^{2})).

Table 7: NVHSS Ⅎ(Ş^{1})

Ⅎ(Ş^{1})	X^{1}	X^{2}
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Table 8: NVHSS Ⅎ(Ş^{2})

Ⅎ(Ş^{2})	X^{1}
Convertible	[0.2,0.3],[0.1,0.4],[0.7,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]

This can also be written as

(Ⅎ(Ş^{1}) =Ⅎ(Ş^{2})= Ⅎ(Convertible, 7 Seat, Top end)

= (({< X^{1}, (Convertible {[0.2,0.3],[0.2,0.4],[0.7, 0.8] },7 Seat {[0.4,0.5],[0.2,0.3],[0.5,0.6] },Top end {[0.2,0.5],[0.2,0.3],[0.5,0.8] }) >,< X^{2}, (Convertible{[0.2,0.4],[0.2,0.3],[0.6,0.8] },7 Seat{[0.1,0.3],[0.2,0.3],[0.7,0.9] },Top end {[0.4,0.5],[0.1,0.2],[0.5,0.6]}) >}

={< X^{1}(Convertible{[0.3,0.4],[0.2,0.5],[0.6,0.7] },7 Seat {[0.1,0.3],[0.3,0.4],[0.7,0.9]}}) >}

This illustrates the value of membership of Convertible for X^{1} in both sets is([0.2,0.3],[0.2,0.4],[0.7, 0.8]) ([0.2,0.3],[0.2,0.4],[0.7, 0.8]) which satisfy the Definition of NVEHSS. The characteristics of NVHSS Ⅎ(Ş^{1}) and NVHSS Ⅎ(Ş^{2}).

Definition 3.8:

Let Ꟊ(Ş¹) and Ꟊ(Ş²) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ with $Ꞵ_k \cap Ꞵ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \ldots n\}$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ = S. then Ꟊ(Ş¹) is Empty NVHS set if

{ T(Ꟊ(Ş¹))= 0

I(Ꟊ(Ş¹))= 0

F(Ꟊ(Ş¹))= 0]

Example 3.9

Consider the NVHSS Ꟊ(Ş¹) over the universe ɱ = {X^1, X^2, X^3 } .

The NVHSS Ꟊ(Ş¹)= Ꟊ(Convertible, 7 Seat, Top end) = {X^1, X^2} is said to be null NVHSS if its NVHSS values are 0.

Table 9: NVHSS Ꟊ(Ş¹)

Ꟊ(Ş¹)	X^1	X^2
Convertible	[0,0],[0,0],[0,0]	[0,0],[0,0],[0,0]
7 Seat	[0,0],[0,0],[0,0]	[0,0],[0,0],[0,0]
Top end	[0,0],[0,0],[0,0]	[0,0],[0,0],[0,0]

This can also be written as

Ꟊ(Ş¹) = Ꟊ (Convertible, 7 Seat, Top end)

= {< X^1, (Convertible ({[0,0],[0, 0],[0,0]}), 7 Seat {[0,0],[0, 0],[0,0]}, Top end({[0,0],[0, 0],[0,0]}) >,

< X^4, (Convertible ({[0,0],[0, 0],[0,0]}), 7 Seat {[0,0],[0, 0],[0,0]}, Top end({[0,0],[0, 0],[0,0] }) >}

Definition 3.10:

Let Ꟊ(Ş¹) and Ꟊ(Ş²) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ with $Ꞵ_k \cap Ꞵ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \ldots n\}$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ = S. then $Ꟊ^c$(Ş¹) of Ꟊ(Ş¹)

if Ꟊ(Ş¹): (→ $Ý^1$ ×→ $Ý^2$ ×→ $Ý^3$ … → $Ý^n$) → Ᵽ(ɱ) such that

[T(Ꟊ(Ş¹))= T(Ꟊ(Ş¹))

I(Ꟊ(Ş¹))= I(Ꟊ(Ş¹))

F(Ⅎ(Ş¹))= F(Ⅎ(Ş¹))]

Example 3.11

Consider the NVHSS Ⅎ(Ş¹) over the universe ɱ = {X¹, X², X³ }. The compliment of NVHSS Ⅎ(Ş¹) = Ⅎ(Convertible, 7 Seat, Top end) = {X¹, X²} is given as T^C(Ⅎ(Ş¹)) = F(Ⅎ(Ş¹)) , I^C(Ⅎ(Ş¹)) = I(Ⅎ(Ş¹)), F^C(Ⅎ(Ş¹)) = T(Ⅎ(Ş¹)).

Table 10: NVHSS Ⅎ(Ş¹)

F^C(Ş¹)	X¹	X²
Not Convertible	[0.7,0.8],[0.2,0.4],[0.2,0.3]	[0.6,0.4],[0.2,0.3],[0.2,0.8]
Not 7 Seat	[0.5,0.6],[0.2,0.3],[0.4,0.5]	[0.7,0.9],[0.2,0.3],[0.1,0.3]
Not Top end	[0.5,0.8],[0.2,0.3],[0.2,0.5]	[0.5,0.6],[0.1,0.2],[0.4,0.5]

This can also be written as

Ⅎ(Ş¹) = Ⅎ (*not* Convertible, 7 Seat, Top end)

= {< X¹, (*not* Convertible {[0.7,0.8],[0.2,0.4],[0.2,0.3] }, *not* 7 Seat {[0.5,0.6],[0.2,0.3],[0.4,0.5] }, *not* Top end {[0.5,0.8],[0.2,0.3],[0.2,0.5] }) >,< X²(*not* Convertible {[0.6,0.4],[0.2,0.3],[0.2,0.8] }, *not* 7 Seat {[0.7,0.9],[0.2,0.3],[0.1,0.3] }, *not* Top end {[0.5,0.6],[0.1,0.2],[0.4,0.5]) >}

This illustrates the value of membership of Convertible for X¹ in Ⅎ (S¹) is {[0.7,0.8],[0.2,0.4],[0.2,0.3] }, and its compliment is([0.2,0.3],[0.2,0.4],[0.7,0.8]) which satisfy the Definition of CNVHSS. This shows that {[0.7,0.8],[0.2,0.4],[0.2,0.3] }is the compliment of ([0.2,0.3],[0.2,0.4],[0.7,0.8]) and The same applied to the remaining qualities of NVHSS Ⅎ (S¹).

Definition 3.12:

Let Ⅎ(Ş¹) and Ⅎ(Ş²) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as $ƙ_1$, $ƙ_2$, $ƙ_3$,….. $ƙ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ƙ_1$, $ƙ_2$, $ƙ_3$,….. $ƙ_n$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ with $Ꞵ_k \cap Ꞵ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \ldots n\}$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ = S. then Ⅎ(Ş¹) ∪ Ⅎ(Ş²) is given as

$$T\left(Ⅎ(Ş^1) \cup Ⅎ(Ş^2)\right) = \begin{cases} T\left(Ⅎ(Ş^1)\right) \; if \; x \in Ş^1 \\ T\left(Ⅎ(Ş^2)\right) \; if \; x \in Ş^2 \\ \max\left(T\left(Ⅎ(Ş^1)\right), T\left(Ⅎ(Ş^2)\right)\right) \; if \; x \in Ş^1 \cap Ş^2 \end{cases}$$

$$I\left(Ⅎ(Ş^1) \cup Ⅎ(Ş^2)\right) = \begin{cases} I\left(Ⅎ(Ş^1)\right) \; if \; x \in Ş^1 \\ I\left(Ⅎ(Ş^2)\right) \; if \; x \in Ş^2 \\ \min\left(I\left(Ⅎ(Ş^1)\right), I\left(Ⅎ(Ş^2)\right)\right) \; if \; x \in Ş^1 \cap Ş^2 \end{cases}$$

$$F\big(Ꟶ(Ş^1) \cup Ꟶ(Ş^2)\big) = \begin{cases} F\big(Ꟶ(Ş^1)\big)\ if\ x \in Ş^1 \\ F\big(Ꟶ(Ş^2)\big)\ if\ x \in Ş^2 \\ \min\Big(F\big(Ꟶ(Ş^1)\big), F\big(Ꟶ(Ş^2)\big)\Big)\ if\ x \in Ş^1 \cap Ş^2 \end{cases}$$

Example3.13

Consider the two NVHSS Ꟶ($Ş^1$) and NVHSS Ꟶ($Ş^2$) over the same universe ɱ = {X^1, X^2, X^3}. Tabular representation of NVHSS Ꟶ($Ş^1$) = Ꟶ(Convertible ,7 Seat ,Top end) = {X^1, X^2} and NVHSS Ꟶ($Ş^2$) = Ꟶ(Convertible,7 Seat) = {X^1} is given below

Table 11: NVHSS Ꟶ($Ş^1$)

Ꟶ($Ş^1$)	X^1	X^2
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Table 12: NVHSS Ꟶ(S^2)

Ꟶ($Ş^2$)	X^1
Convertible	[0.3,0.4],[0.2,0.5],[0.6,0.7]
7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]

Then the union of above NVHSS is given as

Table 13: Union of NVHSS Ꟶ($Ş^1$) and NVHSS Ꟶ($Ş^1$)

Ꟶ($Ş^1$) ∪ Ꟶ($Ş^2$)	X^1	X^2
Convertible	[0.3,0.4],[0.2,0.4],[0.6,0.7]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0,0],[0, 0]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

This can also be written as

Ꟶ($Ş^1$) ∪ Ꟶ($Ş^2$)= Ꟶ(Convertible,7 Seat , Top end) ∪ Ꟶ (Convertible,7 Seat)

= {< X^1, (Convertible {[0.3,0.4],[0.2,0.4],[0.6,0.7]}, 7 Seat{[0.4,0.5],[0.2,0.3],[0.5,0.6]}, Top end{[0.2,0.5],[0,0],[0, 0]}) >,

< X^2,(Convertible {[0.2,0.4],[0.2,0.3],[0.6,0.8]}, 7 Seat{[0.1,0.3],[0.2,0.3],[0.7,0.9]}, Top end{[0.4,0.5],[0.1,0.2],[0.5,0.6]}) >}

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Definition 3.14:

Let Ꟶ(Ş¹) and Ꟶ(Ş²) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as ƙ$_1$, ƙ$_2$, ƙ$_3$,….. ƙ$_n$ and for $n \geq 1$, There are a few unique characteristics such as ƙ$_1$, ƙ$_2$, ƙ$_3$,….. ƙ$_n$ and Ᵽ$_1$, Ᵽ$_2$,Ᵽ$_3$,…..., Ᵽ$_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly Ᵽ$_1$, Ᵽ$_2$,Ᵽ$_3$,…..., Ᵽ$_n$ with Ᵽ$_k$∩Ᵽ$_l$ = ∅, The relationship between k ≠ l and k, l∈{1,2,3 … n} and Ᵽ$_1$, Ᵽ$_2$,Ᵽ$_3$,….... Ᵽ$_n$ = S. then Ꟶ(Ş¹) ∩ Ꟶ(Ş²)

$$T(Ꟶ(S^1) \cap Ꟶ(S^2)) = \begin{cases} T(Ꟶ(Ş^1)) \; if \; x \in Ş^1 \\ T(Ꟶ(Ş^2)) \; if \; x \in Ş^2 \\ \min\left(T(Ꟶ(Ş^1)), T(Ꟶ(Ş^2))\right) \; if \; x \in Ş^1 \cap Ş^2 \end{cases}$$

$$I(Ꟶ(S^1) \cap Ꟶ(S^2)) = \begin{cases} I(Ꟶ(S^1)) \; if \; x \in Ş^1 \\ I(Ꟶ(Ş^2)) \; if \; x \in Ş^2 \\ \max\left(I(Ꟶ(Ş^1)), I(Ꟶ(Ş^2))\right) \; if \; x \in Ş^1 \cap Ş^2 \end{cases}$$

$$F(Ꟶ(Ş^1) \cap Ꟶ(Ş^2)) = \begin{cases} F(Ꟶ(Ş^1)) \; if \; x \in Ş^1 \\ F(Ꟶ(Ş^2)) \; if \; x \in Ş^2 \\ \max\left(F(Ꟶ(Ş^1)), F(Ꟶ(Ş^2))\right) \; if \; x \in Ş^1 \cap Ş^2 \end{cases}$$

Example 3.15

Consider the two NVHSS Ꟶ(Ş¹) and NVHSS Ꟶ(Ş²) over the same universe ɱ = {X¹, X², X³}. Tabular representation of NVHSS Ꟶ(Ş¹) = Ꟶ(Convertible ,7 Seat ,Top end) = {X¹, X²} and NVHSS Ꟶ(Ş²) = Ꟶ(Convertible,7 Seat) = {X¹} is given below

Table 14: NVHSS Ꟶ(Ş¹)

Ꟶ(Ş¹)	X¹	X²
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Table 15: NVHSS Ꟶ(Ş²)

Ꟶ(Ş²)	X¹
Convertible	[0.3,0.4],[0.2,0.5],[0.6,0.7]
7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]

Then the intersection of above NVHSS is given as

Table 16: NVHSS Ꟊ(Ș^1) and NVHSS Ꟊ(Ș^2)

Ꟊ(Ș^1) ∩ Ꟊ(Ș^2)	X^1
Convertible	[0.2,0.3],[0.2,0.5],[0.7,0.8]
	[0.1,0.3],[0.3,0.4],[0.7,0.9]
7 Seat	
Top end	[0 , 0],[0.2, 0.3],[0.5,0.8]

This can also be written as

Ꟊ(Ș^1) ∩ Ꟊ(Ș^2) = Ꟊ(Convertible,7 Seat , Top end) ∩ Ꟊ (Convertible,7 Seat)

= {< X^1, (Convertible{[0.2,0.3],[0.2,0.5],[0.7,0.8]}, 7 Seat{[0.1,0.3],[0.3,0.4],[0.7,0.9]}, Top end{[0,0],[0.2,0.3],[0.5,0.8]}) >}

Definition 3.16:

Let Ꟊ(Ș^1) and Ꟊ(Ș^2) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ with $Ꞵ_k \cap Ꞵ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \dots n\}$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ = S. then Ꟊ(Ș^1) ∧ Ꟊ(Ș^2) = Ꟊ (Ș^1 ×Ș^2)

T(Ș^1 ×Ș^2) = min (T(Ꟊ(Ș^1)), T(Ꟊ(Ș^2)))

I(Ș^1 ×Ș^2) = max (I(Ꟊ(Ș^1)), T(Ꟊ(Ș^2)))

F(Ș^1 ×Ș^2) = max (F(Ꟊ(Ș^1)), T(Ꟊ(Ș^2)))

Example:3.17

Consider the two NVHSS Ꟊ(Ș^1) and NVHSS Ꟊ(Ș^2) over the same universe ɱ = {X^1, X^2, X^3}. Tabular representation of NVHSS Ꟊ(Ș^1) = Ꟊ(Convertible ,7 Seat ,Top end) = {X^1, X^2} and NVHSS Ꟊ(Ș^2) = Ꟊ(Convertible,7 Seat) = {X^1} is given below

Table 17: NVHSS Ꟊ(Ș^1)

Ꟊ(Ș^1)	X^1	X^2
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]
7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Table 18: NVHSS Ⅎ(Ș^2)

Ⅎ(Ș^2)	X^1
Convertible	[0.3,0.4],[0.2,0.5],[0.6,0.7]
7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]

Then the AND Operation of above NVHSS is given as

Table 19: AND of NVHSS Ⅎ(Ș^1) and NVHSS Ⅎ(Ș^2)

Ⅎ(Ș^1) ∧ Ⅎ(Ș^2)	X^1	X^2
Convert × Convert	[0.2,0.3],[0.2,0.5],[0.7,0.8]	[0,0],[0.2,0.3],[0.6,0.8]
Convert × 7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]	[0,0],[0.2,0.3],[0.6,0.8]
7 Seat × Convert	[0.3,0.4],[0.2,0.5],[0.6,0.7]	[0,0],[0.2,0.3],[0.7,0.9]
7 Seat × 7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]	[0,0],[0.2,0.3],[0.7,0.9]
Top end × Convert	[0.2,0.4],[0.2,0.5],[0.6,0.8]	[0,0],[0.1,0.2],[0.5,0.6]
Top end × 7 Seat	[0.1,0.3],[0.3,0.4],[0.7,0.9]	[0,0],[0.1,0.2],[0.5,0.6]

Definition 3.18:

Let Ⅎ(Ș^1) and Ⅎ(Ș^2) be two Neutrosophic Vague Hypersoft set over ɱ. Consider as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and for $n \geq 1$, There are a few unique characteristics such as $ꝁ_1$, $ꝁ_2$, $ꝁ_3$,….. $ꝁ_n$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ are sets with the following constraints for corresponding values and characteristics, correspondingly $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ with $Ꞵ_k \cap Ꞵ_l = \emptyset$, The relationship between $k \neq l$ and k, $l \epsilon \{1,2,3 \ldots n\}$ and $Ꞵ_1$, $Ꞵ_2$,$Ꞵ_3$,…..... $Ꞵ_n$ = S. then Ⅎ(Ș^1) ∨ Ⅎ(Ș^2) = Ⅎ (Ș^1 ×Ș^2) is given as

T(Ș^1 ×Ș^2) = max (T(Ⅎ(Ș^1)), T(Ⅎ(Ș^2)))

I(Ș^1 ×Ș^2) = min (I(Ⅎ(Ș^1)), T(Ⅎ(Ș^2)))

F(Ș^1 ×Ș^2) = min (F(Ⅎ(Ș^1)), T(Ⅎ(Ș^2)))

Example 3.18

Consider the two NVHSS Ⅎ(Ș^1) and NVHSS Ⅎ(Ș^2) over the same universe ɱ = {X^1, X^2, X^3}. Tabular representation of NVHSS Ⅎ(Ș^1) = Ⅎ(Convertible ,7 Seat ,Top end) = {X^1, X^2} and NVHSS Ⅎ(Ș^2) = Ⅎ(Convertible,7 Seat) = {X^1} is given below

Table 20: NVHSS Ⅎ(Ș^1)

Ⅎ(Ș^1)	X^1	X^2
Convertible	[0.2,0.3],[0.2,0.4],[0.7, 0.8]	[0.2,0.4],[0.2,0.3],[0.6,0.8]

7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0.2,0.3],[0.7,0.9]
Top end	[0.2,0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0.1,0.2],[0.5,0.6]

Table 21: NVHSS $Ⅎ(Ş^{2})$

$Ⅎ(Ş^{2})$	X^{1}
Samsung	[0.3,0.4],[0.2,0.5],[0.6,0.7]
6 GB	[0.1,0.3],[0.3,0.4],[0.7,0.9]

Then the OR Operation of above NVHSS is given as

Table 22: OR of NVHSS $Ⅎ(Ş^{1})$ and NVHSS $Ⅎ(Ş^{2})$

$Ⅎ(Ş^{1}) \vee Ⅎ(Ş^{2})$	X^{1}	X^{2}
Convert× Convert	[0.3,0.4],[0.2,0.4],[0.6,0.7]	[0.2,0.4],[0,0],[0,0]
Convert × 7 Seat	[0.2,0.3],[0.2,0.4],[0.7,0.8]	[0.2,0.4],[0,0],[0,0]
7 Seat × Convert	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0,0],[0,0]
7 Seat × 7 Seat	[0.4,0.5],[0.2,0.3],[0.5,0.6]	[0.1,0.3],[0,0],[0,0]
Top end × Convert	[0.3,0.5],[0.2,0.3],[0.5,0.7]	[0.4,0.5],[0,0],[0,0]
Top end × 7 Seat	[0.2, 0.5],[0.2,0.3],[0.5,0.8]	[0.4,0.5],[0,0],[0,0]

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