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Common Neutrosophic Metric Space Fixed Point Theorems with Properties

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Abstract:

The paper introduces the notion of Neutrosophic metric spaces and derives some features from it. Under certain appropriate circumstances, two new common fixed point theorems are proved in Neutrosophic metric spaces. This paper introduces the concept of Neutrosophic metric spaces, an extension of classical metric spaces that incorporates the idea of neutrosophy to handle uncertainty and indeterminacy in mathematical analysis. Neutrosophic metric spaces generalize traditional metrics by allowing for a more nuanced representation of distance and proximity, accommodating the presence of indeterminate, uncertain, or contradictory information.

Key words: Fixed point, Neutrosophic metric spaces, complete symmetric Neutrosophic metric space.

1. Introduction

Zadeh [25] established fuzzy sets, that are crucial for topology and analysis. Numerous Researchers have studied fuzzy sets and their applications. Kramosil and Michlek [8] introduced a novel concept for fuzzy metric spaces. Using the continuous t-norm, George and Veeramani [5] updated the concept of fuzzy metric space. This leads to the derivation of numerous fixed point theorems in fuzzy metric spaces for different types of mappings. In addition to defining D-metric spaces, Dhage [3] established numerous additional fixed point theorems in D-metric spaces. Dhage theory has advanced significantly with the recent definition of G-metric space provided by Mustafa and Sims [11].

Sun and Yang first proposed the idea of a Q-fuzzy metric space in [18]. As a generalization of the fuzzy metric space, we present the idea of a generalized intuitionstic fuzzy metric space. We present new theorems for fixed points in these types of generalized intuitionistic fuzzy metric spaces. The findings of this study expand and enhance a few previously published findings.

The Neutrosophic Metric spaces were defined by Kirisci et al. [7]. Neutrosophic extended metric-like spaces were introduced by Ishtiaq et al. [23], who also proved several FP theorems. The authors used the notions of continuous triangular norms, continuous co-norms, metric space, and Neutrosophic sets in Neutrosophic extended metric-like spaces. The notion of neutrosophic double-controlled metric spaces was introduced by Uddin et al. [4] as introduced the concept of a generalization of Neutrosophic metric spaces. See [19–22] for other relevant results.

2. Preliminaries

In this section, we provide some definitions that are helpful for readers to understand the main section.

Definition: 2.1 [5]

A binay operation $*:[0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a*1 = a$ for all $a \in [0,1]$,
(iv) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Examples of continuous t-norm are $a*b = ab$ and $a*b = \min\{a, b\}$.

Definition: 2.2 [5]

A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\Diamond$ satisfies the following conditions:

(i) $\Diamond$ is commutative and associative,
(ii) $\Diamond$ is continuous,
(iii) $a \Diamond 0 = a$ for all $a \in [0,1]$,
(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition :2.3 [14]

A 5-tuple $(X, Q, H, *, \Diamond)$ is said to be an intuitionstic generalized fuzzy metric space (for short IGFMS) if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t- conorm and Q, H are fuzzy set on $X^3 \to (0,\infty)$ satisfying the following conditions. For every $x, y, z, a \in X$ and $t, s > 0$

(i) $Q(x, y, z, t) + H(x, y, z, t) \leq 1$,
(ii) $Q(x, x, y, t) > 0$, for all $x \neq y$,
(iii) $Q(x, x, y, t) \leq Q(x, y, z, t)$ for $y \neq z$,
(iv) $Q(x, y, z\ t) = 1$ iff $x = y = z$,
(v) $Q(x, y, z, t) = Q\{p(x, y, z), t\}$, where p is a permutation function,
(vi) $Q(x, a, a\ t) * Q(a, y, z, s) \leq Q(x, y, z, t+s)$,
(vii) $Q(x, y, z, .) : (0,\infty) \to [0,1]$ is continuous,
(viii) Q is non decreasing function on R^+ $\lim_{t\to\infty} Q(x, y, z, t) = 1$ and

$$\lim_{t\to 0} Q(x, y, z, t) = 0, \text{ for all } x, y, z \in X, t > 0,$$

(ix) $H(x, x, y, t) < 1$, for all $x \neq y$,

(x) H (x, x, y, t) ≥ H (x, y, z ,t) for y ≠ z,
(xi) H (x, y, z t) = 0 iff x = y = z ,
(xii) H (x, y, z ,t) = H {p (x, y, z),t} where p is a permutation function,
(xiii) H (x, a, a t) ◊ H (a, y, z, s) ≥ H (x,y, z, t + s),
(xiv) H (x, y, z, .) : (0,∞) → [0,1] is continuous,
(xv) H is a non- increasing function on $R^+ \lim_{t\to 0} H$ (x, y, z, t) = 0 and
$\lim_{t\to 0} H(x, y, z, t)$ = 1 for all x, y, z ∈ X, t > 0,
In this case, the pair (Q, H) is called an intuitionistic generalized fuzzy metric on X.

Definition: 2.4 [20]

A 6-tuple (Ξ, $\mathcal{Q}$, $\mathcal{H}$, $\mathcal{O}$ *, ◊) is Said to be an Neutrosophic Metric space if Ξ is an arbitrary set, * is a Continuous t-norm, is a Continuous t-conorm, and $\mathcal{Q}$, $\mathcal{H}$, $\mathcal{O}$ are Neutrosophic set on Ξ^2 x (0, ∞) satisfying the following conditions: for all $\varpi, \omega, \sigma \in \Xi$, $\lambda, \tau > 0$,

(i) $\mathcal{Q}(\varpi, \omega, \tau, \zeta) + \mathcal{H}(\varpi, \omega, \tau, \zeta) + \mathcal{O}(\varpi, \omega, \tau, \zeta) \leq 3$;
(ii) $\mathcal{Q}(\varpi, \varpi, \omega, \zeta) > 0$; for $\varpi \neq \omega$;
(iii) $\mathcal{Q}(\varpi, \varpi\omega, \zeta) \leq \mathcal{Q}(\varpi, \omega, \tau, \zeta)$, for $\omega \neq \zeta$;
(iv) $\mathcal{Q}(\varpi, \omega, \tau, \zeta) = 1$ if and only if $\varpi = \omega = \tau$;
(v) $\mathcal{Q}(\varpi, \omega, \tau, \zeta) * \mathcal{Q}(p(\varpi, \omega, \tau), \zeta)$ where p is a permutation function;
(vi) $\mathcal{Q}(\varpi, a, a, \zeta) * \mathcal{Q}(a, \omega, \tau, \eta) \leq \mathcal{Q}(\varpi, \omega, \tau, \zeta + \eta)$;
(vii) $\mathcal{Q}(\varpi, \omega, \tau, .)$: (0, ∞) → [0,1] is continuous;
(viii) $\mathcal{Q}$ is non-decreasing of $\mathfrak{R}^+$, $\lim_{\zeta\to\infty} \mathcal{Q}(\varpi, \omega, \tau, \zeta) = 1$ and $\lim_{\zeta\to 0} \mathcal{Q}(\varpi, \omega, \tau, \zeta) = 0$
for all $\varpi, \omega, \tau \in \Xi$, $\zeta > 0$
(ix) $\mathcal{H}(\varpi, \varpi, \omega, \zeta) < 1$; for $\varpi \neq \omega$;
(x) $\mathcal{H}(\varpi, \varpi\omega, \zeta) \geq \mathcal{H}(\varpi, \omega, \tau, \zeta)$, for $\omega \neq \zeta$;
(xi) $\mathcal{H}(\varpi, \omega, \tau, \zeta) = 0$ if and only if $\varpi = \omega = \tau$;
(xii) $\mathcal{H}(\varpi, \omega, \tau, \zeta) \lozenge \mathcal{H}(p(\varpi, \omega, \tau), \zeta)$ where p is a permutation function;
(xiii) $\mathcal{H}(\varpi, a, a, \zeta) * \mathcal{H}(a, \omega, \tau, \eta) \geq \mathcal{H}(\varpi, \omega, \tau, \zeta + \eta)$;
(xiv) $\mathcal{H}(\varpi, \omega, \tau, .)$: (0, ∞) → [0,1] is continuous;
(xv) $\mathcal{H}$ is non-increasing of $\mathfrak{R}^+$, $\lim_{\zeta\to\infty} \mathcal{H}(\varpi, \omega, \tau, \zeta) = 1$ and $\lim_{\zeta\to 0} \mathcal{H}(\varpi, \omega, \tau, \zeta) = 1$
for all $\varpi, \omega, \tau \in \Xi$, $\zeta > 0$
(xvi) $\mathcal{O}(\varpi, \varpi, \omega, \zeta) < 1$; for $\varpi \neq \omega$;
(xvii) $\mathcal{O}(\varpi, \varpi\omega, \zeta) \geq \mathcal{O}(\varpi, \omega, \tau, \zeta)$, for $\omega \neq \zeta$;
(xviii) $\mathcal{O}(\varpi, \omega, \tau, \zeta) = 0$ if and only if $\varpi = \omega = \tau$;
(xix) $\mathcal{O}(\varpi, \omega, \tau, \zeta) \lozenge \mathcal{O}(p(\varpi, \omega, \tau), \zeta)$ where p is a permutation function;
(xx) $\mathcal{O}(\varpi, a, a, \zeta) * \mathcal{O}(a, \omega, \tau, \eta) \geq \mathcal{O}(\varpi, \omega, \tau, \zeta + \eta)$;
(xxi) $\mathcal{O}(\varpi, \omega, \tau, .)$: (0, ∞) → [0,1] is continuous;
(xxii) $\mathcal{O}$ is non-increasing of $\mathfrak{R}^+$, $\lim_{\zeta\to\infty} \mathcal{O}(\varpi, \omega, \tau, \zeta) = 1$ and $\lim_{\zeta\to 0} \mathcal{O}(\varpi, \omega, \tau, \zeta) = 1$
for all $\varpi, \omega, \tau \in \Xi$, $\zeta > 0$

Then ($\mathcal{Q}$, $\mathcal{H}$, $\mathcal{O}$) is said to be a neutrosophic metric on Ξ. The function $\mathcal{Q}$, $\mathcal{H}$ and $\mathcal{O}$ denote respectively degree of closeness, neutrality and noncloseness between ϖ, ω and τ with respect to ζ respectively.

Example: 2.5 [20]

Let (Ξ, d) be a Q- metric space, for all $\varpi, \omega, \tau, \zeta \in \Xi$, and every $\zeta > 0$,

consider $\mathcal{Q}, \mathcal{H}, \mathcal{O}$ to be fuzzy sets on Ξ^3 x $(0, \infty)$ defined by

$\mathcal{Q}(\varpi, \omega, \tau, \zeta) = \frac{\zeta}{\zeta + G(\varpi,\omega,\tau,\zeta)}$ and

$\mathcal{H}(\varpi, \varpi, \omega, \zeta) = \frac{G(\varpi,\omega,\tau,\zeta)}{\zeta + G(\varpi,\omega,\tau,\zeta)}$ and

$\mathcal{O}(\varpi, \varpi, \omega, \zeta) = \frac{G(\varpi,\omega,\tau,\zeta)}{\zeta + G(\varpi,\omega,\tau,\zeta)}$ denote a$*$ b = ab anda ◊ b = min { a+b , 1} .

Then($\Xi, \mathcal{Q}, \mathcal{H}, \mathcal{O}$ *,◊) is an Neutrosophic metric space. Notice that the above example holds even with the t- norm a $*$ b = min {a, b} and t – conorm a ◊ b = max {a, b}.

Definition: 2.6[20]

Let (Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊) be Neutrosophic metric space, then a sequence { ϖ_n} in Ξ is said to be convergent if

i) $\lim_{n\to\infty} \mathcal{Q}(\varpi_n, \varpi_n, \varpi, \zeta) = 1$,

$\lim_{n\to\infty} \mathcal{H}(\varpi_n, \varpi_n, \varpi, \zeta) = 0$,

$\lim_{n\to\infty} \mathcal{O}(\varpi_n, \varpi_n, \varpi, \zeta) = 0$.

ii) A sequence { ϖ_n} in Ξ is said to be Cauchy sequence if

$\lim_{n,m\to\infty} \mathcal{Q}(\varpi_n, \varpi_n, \varpi_m, \zeta) = 1$,

$\lim_{n,m\to\infty} \mathcal{H}(\varpi_n, \varpi_n, \varpi_m, \zeta) = 0$,

$\lim_{n,m\to\infty} \mathcal{O}(\varpi_n, \varpi_n, \varpi_m, \zeta) = 0$

that is, for any $\zeta > 0$ and $\varepsilon > 0$ there exists $n_0 \in N$ such that

$\mathcal{Q}(\varpi_n, \varpi_n, \varpi_m, \zeta) > 1- \varepsilon$,

$\mathcal{H}(\varpi_n, \varpi_n, \varpi_m, \zeta) < \varepsilon$

$\mathcal{O}(\varpi_n, \varpi_n, \varpi_m, \zeta) < \varepsilon$ for n, m $\geq n_0$.

iii) A Neutrosophic metric space (Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊)is said to be complete if every Cauchy sequence in Ξ is convergent.

Definition: 2.7[20]

Let (Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊)be a Neutrosophic metric space . The following conditions are satisfied :

$\lim_{n\to\infty} \mathcal{Q}(\varpi_n, \omega_n, \tau_n, \zeta_n) = \mathcal{Q}(\varpi, \omega, \tau, \zeta)$,

$\lim_{n\to\infty} \mathcal{H}(\varpi_n, \omega_n, \tau_n, \zeta_n) = \mathcal{H}(\varpi, \omega, \tau, \zeta)$

$\lim_{n\to\infty} \mathcal{O}(\varpi_n, \omega_n, \tau_n, \zeta_n) = \mathcal{O}(\varpi, \omega, \tau, \zeta)$

Whenever $\lim_{n\to\infty} \varpi_n = \varpi$; $\lim_{n\to\infty} \omega_n = \omega$; $\lim_{n\to\infty} \tau_n = \tau$ and

$\lim_{n\to\infty} \mathcal{Q}(\varpi, \omega, \tau, \zeta_n) = \mathcal{Q}(\varpi, \omega, \tau, \zeta)$,

$\lim_{n\to\infty} \mathcal{H}(\varpi, \omega, \tau, \zeta_n) = \mathcal{H}(\varpi, \omega, \tau, \zeta)$

$\lim_{n\to\infty} \mathcal{O}(\varpi, \omega, \tau, \zeta_n) = \mathcal{O}(\varpi, \omega, \tau, \zeta)$

then $\mathcal{Q}, \mathcal{H}, \mathcal{O}$are called convergent function on Ξ^3 x(0, ∞).

Definition:2.8 [20]

Let f, g be self maps on Neutrosophic metric space

(Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊) . Then the mappings are said to be weakly compatible if they commute at their coincidence point, that is, f ϖ = g ϖimplies that fg ϖ =gf ϖ.

Definition:2.9 [20]

Let f , g be self maps Neutrosophic metric space(Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊)

The pair (f,g) is said to be compatible if

$$\lim_{n\to\infty} \mathcal{Q}(\text{fg}\varpi_n, \text{gf}\varpi_n, \quad \text{gf}\varpi_n, \zeta) = 1,$$

$$\lim_{n\to\infty} \mathcal{H}\left(\text{fg}\varpi_n, \text{gf}\varpi_n, \text{gf}\varpi_n, \zeta\right) = 0,$$

$$\lim_{n\to\infty} \mathcal{O}\left(\text{fg}\varpi_n, \text{gf}\varpi_n, \text{gf}\varpi_n, \zeta\right) = 0$$

Whenever $\{\varpi_n\}$ is a sequence in Ξ such that $\lim_{n\to\infty} \text{f}\varpi_n = \lim_{n\to\infty} \text{g}\varpi_n = \text{z}$ for some z ∈Ξ.

Definition:2.10 [20]

Two self maps A,S and T of a Neutrosophic metric space (Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊) are called jointly W-continuous if there exists a point $\varpi \in \Xi$ such that if

$\lim_{n\to\infty} \mathcal{Q}(\varpi_n, \varpi, \varpi, \zeta) = 1$,

$\lim_{n\to\infty} \mathcal{H}(\varpi_n, \varpi, \varpi, \zeta) = 0$

$\lim_{n\to\infty} \mathcal{O}(\varpi_n, \varpi, \varpi, \zeta) = 0$

then $\lim_{n\to\infty} \mathcal{Q}(\text{A}\varpi_n, S\varpi, S\varpi, \zeta) = 1$,

$\lim_{n\to\infty} \mathcal{H}(\text{A}\varpi_n, S\varpi, S\varpi, \zeta) = 0$,

$\lim_{n\to\infty} \mathcal{O}(\text{A}\varpi_n, S\varpi, S\varpi, \zeta) = 0$,

whenever $\{\varpi_n\}$ is a sequence in Ξ such that $\lim_{n\to\infty} \text{A}\varpi_n = \lim_{n\to\infty} \text{S}\varpi_n = \text{p}$ for some p ∈ Ξ.

Definition: 2.11 [20]

Let the self maps A,S and T of a Neutrosophic metric space (Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ *,◊) . If A, S and T satisfy the following conditions:

There exists a sequence $\{ \varpi_n \}$ such that

$$\lim_{n\to\infty} \mathcal{Q}(A\varpi_{n,} u, u, \zeta) = \lim_{n\to\infty} \mathcal{Q}(S\varpi_n, u, u, \zeta) = \lim_{n\to\infty} \mathcal{Q}(T\varpi_n, u, u, \zeta) = 1$$

$$\lim_{n\to\infty} \mathcal{H}(A\varpi_{n,} u, u, \zeta) = \lim_{n\to\infty} \mathcal{H}(S\varpi_n, u, u, \zeta) = 1 \lim_{n\to\infty} \mathcal{H}(T\varpi_n, u, u, \zeta) = 0$$

$$\lim_{n\to\infty} \mathcal{O} (A\varpi_{n,} u, u, \zeta) = \lim_{n\to\infty} \mathcal{O} (S\varpi_n, u, u, \zeta) = 1 \lim_{n\to\infty} \mathcal{O} (T\varpi_n, u, u, \zeta) = 0$$

for some $u \in \Xi$ and $\zeta > 0$, we say that A,S and T have the property (E.A).

Lemma: 2.12 [20]

Let $(\Xi, \mathcal{Q}, \mathcal{H}, \mathcal{O} *, \Diamond)$ be a Neutrosophic metric space. Then , $\mathcal{Q}, \mathcal{H}, \mathcal{O}$ are continuous function on $\Xi^3 \times (0, \infty)$.

Proof:

Since $\lim_{n\to\infty} \varpi_n = \varpi$; $\lim_{n\to\infty} \omega_n = \omega$; $\lim_{n\to\infty} \tau_n = \tau$

$\lim_{n\to\infty} \mathcal{Q} (\varpi, \omega, \tau, \zeta_n) = \mathcal{Q}(\varpi, \omega, \tau, \zeta)$ and

$$\lim_{n\to\infty} \mathcal{H} (\varpi, \omega, \tau, \zeta_n) = \mathcal{H}(\varpi, \omega, \tau, \zeta)$$

$$\lim_{n\to\infty} \mathcal{O} (\varpi, \omega, \tau, \zeta_n) = \mathcal{O}(\varpi, \omega, \tau, \zeta)$$

There is $\vartheta_0 \in N$ such that $|\zeta - \zeta_n| < \varepsilon$ and $|\zeta - \zeta_n| > \delta$ for $\vartheta \geq \vartheta_0$ and $\varepsilon < \zeta/2$ and $\delta > \zeta/2$

We know that $\mathcal{Q}(\varpi, \omega, \tau, \zeta)$ is non-decreasing and $\mathcal{H}(\varpi, \omega, \tau, \zeta)$, $\mathcal{O}(\varpi, \omega, \tau, \zeta)$ is non-increasing with respect to ζ,

So, we have

$$\begin{aligned}\mathcal{Q} (\varpi_n, \omega_n, \tau_n, \zeta) &\geq \mathcal{Q} (\varpi_n, \omega_n, \tau_n, \zeta - \varepsilon)\\
&\geq \mathcal{Q} (\varpi_n, \varpi, \varpi, \tfrac{\varepsilon}{3}) * \mathcal{Q} (\varpi, \omega_n, \tau_n, \zeta - \tfrac{4\varepsilon}{3})\\
&\geq \mathcal{Q} (\varpi_n, \varpi, \varpi, \tfrac{\varepsilon}{3}) * \mathcal{Q} (\omega_n, \omega, \omega, \tfrac{\varepsilon}{3}) * \mathcal{Q} (\omega, \varpi, \tau_n, \zeta - \tfrac{5\varepsilon}{3})\\
&\geq \mathcal{Q} (\varpi_n, \varpi, \varpi, \tfrac{\varepsilon}{3}) * \mathcal{Q} (\omega_n, \omega, \omega, \tfrac{\varepsilon}{3}) * \mathcal{Q} (\tau_n, \tau, \tau, \tfrac{\varepsilon}{3}) * \mathcal{Q} (\tau, \omega, \tau, \zeta - 2\varepsilon)\end{aligned}$$ and

$$\begin{aligned}\mathcal{H} (\varpi_n, \omega_n, \tau_n, \zeta) &\leq \mathcal{H} (\varpi_n, \omega_n, \tau_n, \zeta - \delta)\\
&\leq \mathcal{H}(\varpi_n, \varpi, \varpi, \tfrac{\delta}{3}) \Diamond \mathcal{Q} (\varpi, \omega_n, \tau_n, \zeta - \tfrac{4\delta}{3})\\
&\leq \mathcal{H} (\varpi_n, \varpi, \varpi, \tfrac{\delta}{3}) \Diamond \mathcal{H} (\omega_n, \omega, \omega, \tfrac{\delta}{3}) \Diamond \mathcal{H} (\omega, \varpi, \tau_n, \zeta - \tfrac{5\delta}{3})\\
&\leq \mathcal{H} (\varpi_n, \varpi, \varpi, \tfrac{\delta}{3}) \Diamond \mathcal{H} (\omega_n, \omega, \omega, \tfrac{\delta}{3}) \Diamond \mathcal{H} (\tau_n, \tau, \tau, \tfrac{\delta}{3}) \Diamond \mathcal{H} (\tau, \omega, \tau, \zeta - 2\delta)\end{aligned}$$

$$\mathcal{O} (\varpi_n, \omega_n, \tau_n, \zeta) \leq \mathcal{O} (\varpi_n, \omega_n, \tau_n, \zeta - \delta)$$

$\leq \mathcal{O}(\varpi_n, \varpi, \varpi, \frac{\delta}{3}) \lozenge \mathcal{O}(\varpi, \omega_n, \tau_n, \zeta - \frac{4\delta}{3})$

$\leq \mathcal{O}(\varpi_n, \varpi, \varpi, \frac{\delta}{3}) \lozenge \mathcal{O}(\omega_n, \omega, \omega, \frac{\delta}{3}) \lozenge \mathcal{O}(\omega, \varpi, \tau_n, \zeta - \frac{5\delta}{3})$

$\leq \mathcal{O}(\varpi_n, \varpi, \varpi, \frac{\delta}{3}) \lozenge \mathcal{O}(\omega_n, \omega, \omega, \frac{\delta}{3}) \lozenge \mathcal{O}(\tau_n, \tau, \tau, \frac{\delta}{3}) \lozenge \mathcal{O}(\tau, \omega, \tau, \zeta - 2\delta)$

$\mathcal{Q}(\varpi, \omega, \tau, \zeta + 2\varepsilon) \geq \mathcal{Q}(\varpi, \omega, \tau, \zeta_n + \varepsilon)$

$\geq \mathcal{Q}(\varpi, \varpi_n, \varpi_n, \frac{\varepsilon}{3}) * \mathcal{Q}(\varpi_n, \omega, \tau, \zeta_n + \frac{2\varepsilon}{3})$

$\geq \mathcal{Q}(\varpi, \varpi_n, \varpi_n, \frac{\varepsilon}{3}) * \mathcal{Q}(\omega, \omega_n, \omega_n, \frac{\varepsilon}{3}) * \mathcal{Q}(\omega_n, \varpi_n, \tau, \zeta_n + \frac{\varepsilon}{3})$

$\geq \mathcal{Q}(\varpi, \varpi_n, \varpi_n, \frac{\varepsilon}{3}) * \mathcal{Q}(\omega, \omega_n, \omega_n, \frac{\varepsilon}{3}) * \mathcal{Q}(\tau, \tau_n, \tau_n, \frac{\varepsilon}{3}) * \mathcal{Q}(\tau, \omega, \varpi, \zeta_n)$ and

$\mathcal{H}(\varpi, \omega, \tau, \zeta + 2\varepsilon) \leq \mathcal{H}(\varpi, \omega, \tau, \zeta_n + \delta)$

$\leq \mathcal{H}(\varpi, \varpi_n, \varpi_n, \frac{\delta}{3}) \lozenge \mathcal{H}(\varpi_n, \omega, \tau, \zeta_n + \frac{2\delta}{3})$

$\leq \mathcal{H}(\varpi, \varpi_n, \varpi_n, \frac{\delta}{3}) \lozenge \mathcal{H}(\omega, \omega_n, \omega_n, \frac{\delta}{3}) \lozenge \mathcal{H}(\omega_n, \varpi_n, \tau, \zeta_n + \frac{\delta}{3})$

$\leq \mathcal{H}(\varpi, \varpi_n, \varpi_n, \frac{\delta}{3}) \lozenge \mathcal{H}(\omega, \omega_n, \omega_n, \frac{\delta}{3}) \lozenge \mathcal{H}(\tau, \tau_n, \tau_n, \frac{\delta}{3}) \lozenge \mathcal{H}(\tau, \omega, \varpi, \zeta_n)$

$\mathcal{O}(\varpi, \omega, \tau, \zeta + 2\varepsilon) \leq \mathcal{O}(\varpi, \omega, \tau, \zeta_n + \delta)$

$\leq \mathcal{O}(\varpi, \varpi_n, \varpi_n, \frac{\delta}{3}) \lozenge \mathcal{O}(\varpi_n, \omega, \tau, \zeta_n + \frac{2\delta}{3})$

$\leq \mathcal{O}(\varpi, \varpi_n, \varpi_n, \frac{\delta}{3}) \lozenge \mathcal{O}(\omega, \omega_n, \omega_n, \frac{\delta}{3}) \lozenge \mathcal{O}(\omega_n, \varpi_n, \tau, \zeta_n + \frac{\delta}{3})$

$\leq \mathcal{O}(\varpi, \varpi_n, \varpi_n, \frac{\delta}{3}) \lozenge \mathcal{O}(\omega, \omega_n, \omega_n, \frac{\delta}{3}) \lozenge \mathcal{O}(\tau, \tau_n, \tau_n, \frac{\delta}{3}) \lozenge \mathcal{O}(\tau, \omega, \varpi, \zeta_n)$

Let $n \to \infty$, by continuity of the function $\mathcal{Q}, \mathcal{H}, \mathcal{O}$ with respect to ζ, we can get

$\mathcal{Q}(\varpi, \omega, \tau, \zeta + 2\varepsilon) \geq \mathcal{Q}(\tau, \omega, \varpi, t) \geq \mathcal{Q}(\tau, \omega, \varpi, \zeta - 2\varepsilon)$

$\mathcal{H}(\varpi, \omega, \tau, \zeta + 2\varepsilon) \leq \mathcal{H}(\tau, \omega, \varpi, t) \leq \mathcal{H}(\tau, \omega, \varpi, \zeta - 2\delta)$

$\mathcal{O}(\varpi, \omega, \tau, \zeta + 2\varepsilon) \leq \mathcal{O}(\tau, \omega, \varpi, t) \leq \mathcal{O}(\tau, \omega, \varpi, \zeta - 2\delta)$

Therefore $\mathcal{Q}, \mathcal{H}, \mathcal{O}$ are continuous function on Ξ^3 x $(0, \infty)$.

3. Main Theorem

We first generalize a classic theorem in Neutrosophic Metric Space

THEOREM 3.1: [20]

Let $\mathfrak{I}, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ be self- mappings of a complete symmetric Neutrosophic metric space $(\Xi, \mathcal{Q}, \mathcal{H}, \mathcal{O} *, \lozenge)$ with $\zeta * \zeta \geq \zeta$ and $\zeta \lozenge \zeta \leq 1 - \zeta$ if the mappings satisfy the following conditions:

[3.1.1] $\mathfrak{I}(\Xi) \subseteq \mathfrak{H}(\Xi)$, $\beth(\Xi) \subseteq \eta(\Xi)$, $\mathcal{F}(\Xi) \subseteq \xi(\Xi)$

[3.1.2] $(\Im, \xi)$ or $(\beth, \eta)$ or $(\mathcal{F}, \mathfrak{H})$ satisfy the property (E.A)

[3.1.3] $(\Im, \xi)$, $(\beth, \eta)$ and $(\mathcal{F}, \mathfrak{H})$ are weakly compatible

[3.1.4] $\mathcal{Q}(\Im\varpi, \beth\omega, \mathcal{F}\tau, k\zeta) \geq \{ \mathcal{Q}(\xi\varpi, \mathfrak{H}\omega, \eta\tau, \zeta) * \mathcal{Q}(\Im\varpi, \mathfrak{H}\omega, \eta\tau, \zeta) * \mathcal{Q}(\eta\varpi, \beth\omega, \mathcal{F}\tau, \zeta) \}$

[3.1.5] $\mathcal{H}(\Im\varpi, \beth\omega, \mathcal{F}\tau, k\zeta) \leq \{ \mathcal{H}(\xi\varpi, \mathfrak{H}\omega, \eta\tau, \zeta) \lozenge \mathcal{H}(\Im\varpi, \mathfrak{H}\omega, \eta\tau, \zeta) \lozenge \mathcal{H}(\eta\varpi, \beth\omega, \mathcal{F}\tau, \zeta)\}$

[3.1.6] $\mathcal{O}(\Im\varpi, \beth\omega, \mathcal{F}\tau, k\zeta) \leq \{ \mathcal{O}(\xi\varpi, \mathfrak{H}\omega, \eta\tau, \zeta) \lozenge \mathcal{O}(\Im\varpi, \mathfrak{H}\omega, \eta\tau, \zeta) \lozenge \mathcal{O}(\eta\varpi, \beth\omega, \mathcal{F}\tau, \zeta)\}$

there exists $k \in (0,1)$ such that for every $\varpi, \omega, \tau \in \Xi$ and $\zeta > 0$.
Then $\Im, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ have a unique common fixed point in Ξ.

Proof:

Suppose $(\Im, \xi)$ satisfy the property (E.A), hence there exists a sequence $\{\varpi_n\}$ such that

$$\lim_{n\to\infty} \mathcal{Q}(\Im\varpi_n, u, u, \zeta) = \lim_{n\to\infty} \mathcal{Q}(\xi\varpi_n, u, u, \zeta) = 1$$

$$\lim_{n\to\infty} \mathcal{H}(\Im\varpi_n, u, u, \zeta) = \lim_{n\to\infty} \mathcal{H}(\xi\varpi_n, u, u, \zeta) = 0$$

$$\lim_{n\to\infty} \mathcal{O}(\Im\varpi_n, u, u, \zeta) = \lim_{n\to\infty} \mathcal{O}(\xi\varpi_n, u, u, \zeta) = 0$$

for some $u \in \Xi$ and $\zeta > 0$.

Since $\Im(\Xi) \subseteq \mathfrak{H}(\Xi)$, there exists a sequence $\{\omega_n\}$ such that $\Im\varpi_n = \mathfrak{H}\omega_n$

$$\Rightarrow \lim_{n\to\infty} \mathcal{Q}(\mathfrak{H}\omega_n, u, u, t) = 1, \lim_{n\to\infty} \mathcal{H}(\mathfrak{H}\omega_n, u, u, t) = 0 \text{ and } \lim_{n\to\infty} \mathcal{O}(\mathfrak{H}\omega_n, u, u, t) = 0$$

Therefore,

$$\mathcal{Q}(\Im\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \geq \mathcal{Q}(\xi\varpi_n, \mathfrak{H}\omega_n, \eta\tau_{n+1}, \zeta) * \mathcal{Q}(\Im\varpi_n, \mathfrak{H}\omega_n, \eta\tau_{n+1}, \zeta) * \mathcal{Q}(\eta\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, \zeta)$$

$$\geq \{ \mathcal{Q}(\xi\varpi_n, u, u, ½\zeta) * \mathcal{Q}(u, \mathfrak{H}\omega_n, \eta\tau_{n+1}, ½\zeta) * \mathcal{Q}(\Im\varpi_n, u, u, ½\zeta) * \mathcal{Q}(u, \mathfrak{H}\omega_n, \eta\tau_{n+1}, ½\zeta) * \mathcal{Q}(\eta\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, \zeta)\}$$

$$\mathcal{H}(\Im\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \leq \mathcal{H}(\xi\varpi_n, \mathfrak{H}\omega_n, \eta\tau_{n+1}, \zeta) \lozenge \mathcal{H}(\Im\varpi_n, \mathfrak{H}\omega_n, \eta\tau_{n+1}, \zeta) \lozenge \mathcal{H}(\eta\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, \zeta)$$

$$\leq \{\mathcal{H}(\xi\varpi_n, u, u, ½\zeta) \lozenge \mathcal{H}(u, \mathfrak{H}\omega_n, \eta\tau_{n+1}, ½\zeta) \lozenge \mathcal{H}(\Im\varpi_n, u, u, ½\zeta) \lozenge \mathcal{H}(u, \mathfrak{H}\omega_n, \eta\tau_{n+1}, ½\zeta) \lozenge \mathcal{H}(\eta\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, \zeta)\}$$

$$\mathcal{O}(\Im\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \leq \mathcal{O}(\xi\varpi_n, \mathfrak{H}\omega_n, \eta\tau_{n+1}, \zeta) \lozenge \mathcal{O}(\Im\varpi_n, \mathfrak{H}\omega_n, \eta\tau_{n+1}, \zeta) \lozenge \mathcal{O}(\eta\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, \zeta)$$

$$\leq \{\mathcal{O}(\xi\varpi_n, u, u, ½\zeta) \lozenge \mathcal{O}(u, \mathfrak{H}\omega_n, \eta\tau_{n+1}, ½\zeta) \lozenge \mathcal{O}(\Im\varpi_n, u, u, ½\zeta) \lozenge \mathcal{O}(u, \mathfrak{H}\omega_n, \eta\tau_{n+1}, ½\zeta) \lozenge \mathcal{O}(\eta\varpi_n, \beth\omega_n, \mathcal{F}\tau_{n+1}, \zeta)$$

there exists $\delta > 0$ such that $k + \delta < 1$, for $k \in (0,1)$. On making $n \to \infty$ and by the symmetry Neutrosophic metric space, we have

$\lim_{n\to\infty} \mathcal{Q}\,(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \;\geq 1 * 1 * 1 * 1$

$\lim_{n\to\infty} \mathcal{Q}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_n[\,1-(k+\delta)]\,\zeta) * \lim_{n\to\infty} \mathcal{Q}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, (k+\delta)]\,\zeta) \;\geq 1 * 1 * 1 * 1 * 1$

$\lim_{n\to\infty} \mathcal{Q}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, (k+\delta)]\,\zeta)$

$\lim_{n\to\infty} \mathcal{H}\,(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \;\leq 0 \lozenge 0 \lozenge 0 \lozenge 0$

$\lim_{n\to\infty} \mathcal{H}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_n[\,1-(k+\delta)]\,\zeta) \lozenge \lim_{n\to\infty} \mathcal{H}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, (k+\delta)]\,\zeta) \;\leq 0 \lozenge 0 \lozenge 0 \lozenge 0$

$\lim_{n\to\infty} \mathcal{O}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, (k+\delta)]\,\zeta)$

$\lim_{n\to\infty} \mathcal{O}\,(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \;\leq 0 \lozenge 0 \lozenge 0 \lozenge 0$

$\lim_{n\to\infty} \mathcal{O}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_n[\,1-(k+\delta)]\,\zeta) \lozenge \lim_{n\to\infty} \mathcal{O}(\Im\varpi_n, ℶ\omega_n, \mathcal{F}\tau_{n+1}, (k+\delta)]\,\zeta) \;\leq 0 \lozenge 0 \lozenge 0 \lozenge 0$

Hence $\lim_{n\to\infty} \Im\,\varpi_n = \lim_{n\to\infty} ℶ\,\omega_n = \lim_{n\to\infty} \mathcal{F}\tau_n = \lim_{n\to\infty} \mathfrak{H}\,\tau_n = \lim_{n\to\infty} \eta\,\omega_n = \lim_{n\to\infty} \xi\varpi_n = u$

Let $(\Xi, \mathcal{Q}, \mathcal{H}, \mathcal{O}\, *, \lozenge)$ is a complete Neutrosophic metric space, there exists

$\varpi_0 \in \Xi$ such that

$\xi\varpi_0 = u \Rightarrow \mathcal{Q}\,(\Im\varpi_0, ℶ\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \geq \mathcal{Q}\,(\eta\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta) * \mathcal{Q}(\Im\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta) *$

$\mathcal{Q}\,(\xi\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta)$

If $n \to \infty$ we can get $\mathcal{Q}\,(\Im\varpi_0, u, u, \zeta) \geq 1 * \mathcal{Q}\,(\Im\varpi_0, u, u, \zeta) * 1.$

By the property of non- decreasing with respect to ζ,

$\xi\,\varpi_0 = u \Rightarrow \mathcal{H}\,(\Im\varpi_0, ℶ\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \leq \mathcal{H}\,(\eta\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta) \lozenge \mathcal{H}(\Im\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta) \lozenge$

$\mathcal{H}\,(\xi\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta)$

If $n \to \infty$ we can get $\mathcal{H}\,(\Im\varpi_0, u, u, \zeta) \leq 0 \lozenge \mathcal{H}\,(\Im\varpi_0, u, u, \zeta) \lozenge 0$

By the property of non- increasing with respect to ζ,

$\xi\,\varpi_0 = u \Rightarrow \mathcal{O}\,(\Im\varpi_0, ℶ\omega_n, \mathcal{F}\tau_{n+1}, k\zeta) \leq \mathcal{O}\,(\eta\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta) \lozenge \mathcal{O}(\Im\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta) \lozenge$

$\mathcal{O}\,(\xi\varpi_0, \mathfrak{H}\omega_n, \xi\,\tau_{n+1}, \zeta)$

If $n \to \infty$ we can get $\mathcal{O}\,(\Im\varpi_0, u, u, \zeta) \leq 0 \lozenge \mathcal{O}\,(\Im\varpi_0, u, u, \zeta) \lozenge 0$

it is easy to see that $\Im\varpi_0 = \xi\,\varpi_0 = u$.

As $\Im(\Xi) \subseteq \mathfrak{H}(\Xi)$, there exists ω_0 such that $\Im\varpi_0 = \mathfrak{H}\omega_0$. Suppose $\mathfrak{H}\omega_0 \neq ℶ\omega_0$. Then

$\mathcal{Q}\,(\Im\varpi_n, ℶ\varpi_0, ℶ\varpi_0, k\zeta) \geq \mathcal{Q}\,(\eta\varpi_n, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) * \mathcal{Q}(\Im\varpi_n, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) * \mathcal{Q}(\eta\varpi_n, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\geq \mathcal{Q}(\eta\varpi_n, u, u, \zeta) * \mathcal{Q}\,(\Im\varpi_n, u, u, \zeta) * \mathcal{Q}(\eta\varpi_n, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\mathcal{H}\,(\Im\varpi_n, ℶ\varpi_0, ℶ\varpi_0, k\zeta) \leq \mathcal{H}\,(\eta\varpi_n, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{H}(\Im\varpi_n, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{H}(\eta\varpi_n, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\leq \mathcal{H}(\eta\varpi_n, u, u, \zeta) \lozenge \mathcal{H}\,(\Im\varpi_n, u, u, \zeta) \lozenge \mathcal{H}(\eta\varpi_n, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\mathcal{O}$ $(\mathfrak{J}\varpi_n, ℶ\varpi_0, ℶ\varpi_0, k\zeta) \leq \mathcal{O}$ $(\eta\varpi_n, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{O}(\mathfrak{J}\varpi_n, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{O}(\eta\varpi_n, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\leq \mathcal{O}$ $(\eta\varpi_n, u, u, \zeta) \lozenge \mathcal{O}$ $(\mathfrak{J}\varpi_n, u, u, \zeta) \lozenge \mathcal{O}(\eta\varpi_n, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

Letting $n \to \infty$ we have

$\mathcal{Q}$ $(u, ℶ\varpi_0, ℶ\varpi_0, k\zeta) \geq \mathcal{Q}$ $(u, u, u, \zeta) * \mathcal{Q}$ $(u, u, u, \zeta) * \mathcal{Q}$ $(u, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\mathcal{H}$ $(u, ℶ\varpi_0, ℶ\varpi_0, k\zeta) \leq \mathcal{H}$ $(u, u, u, \zeta) \lozenge \mathcal{H}$ $(u, u, u, \zeta) \lozenge \mathcal{H}$ $(u, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

$\mathcal{O}$ $(u, ℶ\varpi_0, ℶ\varpi_0, k\zeta) \leq \mathcal{O}$ $(u, u, u, \zeta) \lozenge \mathcal{O}$ $(u, u, u, \zeta) \lozenge \mathcal{O}$ $(u, ℶ\varpi_0, ℶ\varpi_0, \zeta)$

Which is a contradiction. So, $ℶ\omega_0 = \mathfrak{H}\omega_0 = u$.

Now by $(\mathfrak{J}, \xi)$, $(ℶ,\eta)$ and $(\mathcal{F}, \mathfrak{H})$ are weakly compatible

$\mathfrak{J}\mathfrak{J}\varpi_0 = \mathfrak{J}\xi\varpi_0 = \xi\mathfrak{J}\varpi_0 = \xi\xi\varpi_0$ and $ℶℶ\omega_0 = ℶ\eta\omega_0 = \etaℶ\omega_0 = \eta\eta\omega_0$ and

$\mathcal{F}\mathcal{F}\tau_0 = \mathcal{F}\mathfrak{H}\tau_0 = \mathfrak{H}\mathcal{F}\tau_0 = \mathfrak{H}\mathfrak{H}\tau_0$

Suppose $\mathfrak{J}u \neq u$.

Then $\mathcal{Q}$ $(\mathfrak{J}u, u, u, k\zeta) = \mathcal{Q}$ $(\mathfrak{J}u, ℶ\omega_0, ℶ\omega_0, k\zeta)$

$\geq \mathcal{Q}$ $(\xi u, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) * \mathcal{Q}$ $(\mathfrak{J}u, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) * \mathcal{Q}$ $(\xi u, ℶ\omega_0, ℶ\omega_0, \zeta)$

$\geq \mathcal{Q}$ $(\xi u, u, u, \zeta) * \mathcal{Q}$ $(\mathfrak{J}u, u, u, \zeta) * \mathcal{Q}$ $(\xi u, u, u, \zeta)$

$\geq \lim_{n\to\infty} \mathcal{Q}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta) * \mathcal{Q}$ $(\mathfrak{J}u, u, u, \zeta) * \mathcal{Q}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta)$

$\geq \lim_{n\to\infty} \mathcal{Q}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta)$

$\mathcal{H}$ $(\mathfrak{J}u, u, u, k\zeta) = \mathcal{H}$ $(\mathfrak{J}u, ℶ\omega_0, ℶ\omega_0, k\zeta)$

$\leq \mathcal{H}$ $(\xi u, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{H}$ $(\mathfrak{J}u, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{H}$ $(\xi u, ℶ\omega_0, ℶ\omega_0, \zeta)$

$\leq \mathcal{H}$ $(\xi u, u, u, \zeta) \lozenge \mathcal{H}$ $(\mathfrak{J}u, u, u, \zeta) \lozenge \mathcal{H}$ $(\xi u, u, u, \zeta)$

$\leq \lim_{n\to\infty} \mathcal{H}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta) \lozenge \mathcal{H}$ $(\mathfrak{J}u, u, u, \zeta) \lozenge \mathcal{H}(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta)$

$\leq \lim_{n\to\infty} \mathcal{H}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta)$

$\mathcal{O}(\mathfrak{J}u, u, u, k\zeta) = \mathcal{O}$ $(\mathfrak{J}u, ℶ\omega_0, ℶ\omega_0, k\zeta)$

$\leq \mathcal{O}$ $(\xi u, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{O}$ $(\mathfrak{J}u, \mathfrak{H}\omega_0, \mathfrak{H}\omega_0, \zeta) \lozenge \mathcal{O}$ $(\xi u, ℶ\omega_0, ℶ\omega_0, \zeta)$

$\leq \mathcal{O}$ $(\xi u, u, u, \zeta) \lozenge \mathcal{O}$ $(\mathfrak{J}u, u, u, \zeta) \lozenge \mathcal{O}$ $(\xi u, u, u, \zeta)$

$\leq \lim_{n\to\infty} \mathcal{O}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta) \lozenge \mathcal{O}$ $(\mathfrak{J}u, u, u, \zeta) \lozenge \mathcal{O}(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta)$

$\leq \lim_{n\to\infty} \mathcal{O}$ $(\xi u, \xi\varpi_n, \xi\varpi_n, \zeta)$

By $\mathfrak{J}u = \mathfrak{J}\mathfrak{J}\varpi_0 = \mathfrak{J}\xi\varpi_0 = \xi\mathfrak{J}\varpi_0 = \xi\xi\varpi_0 = \xi u$ and $\zeta * \zeta \geq \zeta$ and $\zeta \lozenge \zeta \leq 1 - \zeta$

it is easy to see that [3.1.6], [3.1.7] [3.1.8] yields a contradiction and so $\mathfrak{J}u = u = \xi u$.

Now following the similar argument, we can get $ℶu = u = \eta u$.

So $\Im, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ have a common fixed point u.

Uniquness:

Let $v \neq u$ be another common fixed point of $\Im, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ . Then,

$\mathcal{Q}$ (v, u, u, kζ) = $\mathcal{Q}$ ($\Im$v, $\beth$u, $\beth$u, kζ)

$\geq \mathcal{Q}$ (ξv, ηu, ηu, ζ) $*$ $\mathcal{Q}$ ($\Im$v, ηu, ηu, ζ) $*$ $\mathcal{Q}$ (ξv, $\beth$u, $\beth$u, ζ)

$= \mathcal{Q}$ (v, u, u, ζ) $*$ $\mathcal{Q}$ (v, u, u, ζ) $*$ $\mathcal{Q}$ (v, u, u, ζ)

$\mathcal{H}$ (v, u, u, kζ) = $\mathcal{H}$($\Im$v, $\beth$u, $\beth$u, kζ)

$\leq$ $\mathcal{H}$(ξv, ηu, ηu, ζ) $\Diamond$ $\mathcal{H}$ ($\Im$v, ηu, ηu, ζ) $\Diamond$ $\mathcal{H}$ (ξv, $\beth$u, $\beth$u, ζ)

$= \mathcal{H}$(v, u, u, ζ) $\Diamond$ $\mathcal{H}$ (v, u, u, ζ) $\Diamond$ $\mathcal{H}$ (v, u, u, ζ)

$\mathcal{O}$ (v, u, u, kζ) = $\mathcal{O}$($\Im$v, $\beth$u, $\beth$u, kζ)

$\leq$ $\mathcal{O}$(ξv, ηu, ηu, ζ) $\Diamond$ $\mathcal{O}$ ($\Im$v, ηu, ηu, ζ) $\Diamond$ $\mathcal{O}$ (ξv, $\beth$u, $\beth$u, ζ)

$= \mathcal{O}$(v, u, u, ζ) $\Diamond$ $\mathcal{O}$ (v, u, u, ζ) $\Diamond$ $\mathcal{O}$ (v, u, u, ζ)

By $\zeta * \zeta \geq \zeta$ and $\zeta \Diamond \zeta \leq 1 - \zeta$,

we can get $\mathcal{Q}$ (v, u, u, k ζ) $\geq \mathcal{Q}$ (v, u, u, ζ)

$\mathcal{H}$ (v, u, u, k ζ) $\leq \mathcal{H}$ (v, u, u, ζ) and $\mathcal{O}$ (v, u, u, k ζ) $\leq \mathcal{O}$ (v, u, u, ζ)

is a contradiction thus v = u.

Hence $\Im, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ have a unique common fixed point in X.

THEOREM: 3. 2 [20]

Let $\Im, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ be self mappings of a complete Neutrosophic metric space (Ξ, $\mathcal{Q}$, $\mathcal{H}$,$\mathcal{O}$ $*$,$\Diamond$) with $\zeta * \zeta > t$ and $\zeta \Diamond \zeta < 1 - \zeta$ if the mappings satisfy the following conditions:

[3.2.1] $\Im(\Xi) \subseteq \xi(\Xi)$, $\beth(\Xi) \subseteq \eta(\Xi)$, $\mathcal{F}(\Xi) \subseteq \mathfrak{H}(\Xi)$

[3.2.2] Suppose ($\Im$, $\mathfrak{H}$) satisfy the property (E.A)

[3.3.3] ($\Im$, ξ) , ($\beth$,η) and ($\mathcal{F}$, $\mathfrak{H}$) are weakly compatible

[3.3.4] $\mathcal{Q}(\Im\varpi, \beth\omega, \beth\tau, \zeta) \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}\varpi, \eta\omega, \eta\tau, \zeta), \mathcal{Q}(\Im\varpi, \eta\omega, \eta\tau, \zeta),\\ \mathcal{Q}(\mathfrak{H}\varpi, \eta\omega, \eta\tau, \zeta), \mathcal{Q}(\Im\varpi, \mathfrak{H}\varpi, \mathfrak{H}\varpi, \zeta)\end{pmatrix}\right]$

$\mathcal{H}(\Im\varpi, \beth\omega, \beth\tau, \zeta) \leq \Psi\left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}\varpi, \eta\omega, \eta\tau, \zeta), \mathcal{H}(\Im\varpi, \eta\omega, \eta\tau, \zeta),\\ \mathcal{H}(\mathfrak{H}\varpi, \eta\omega, \eta\tau, \zeta), \mathcal{H}(\Im\varpi, \mathfrak{H}\varpi, \mathfrak{H}\varpi, \zeta)\end{pmatrix}\right]$ and

$\mathcal{O}(\Im\varpi, \beth\omega, \beth\tau, \zeta) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}\varpi, \eta\omega, \eta\tau, \zeta), \mathcal{O}(\Im\varpi, \eta\omega, \eta\tau, \zeta),\\ \mathcal{O}(\mathfrak{H}\varpi, \eta\omega, \eta\tau, \zeta), \mathcal{O}(\Im\varpi, \mathfrak{H}\varpi, \mathfrak{H}\varpi, \zeta)\end{pmatrix}\right]$

for all $\varpi, \omega, \tau \epsilon \ \Xi$ and t > 0 where φ, Ψ, Ω : [0,1] → [0,1] is a continuous and increasing function with φ(s) > s and Ψ(s) < s Ω(s) < s for 0 < S < 1 and $\varphi(1) = 1$, $\Psi(0) = 0$, Ω(0)= 0

Then $\mathfrak{I}, \beth, \mathcal{F}, \mathfrak{H}, \eta$ and ξ have a unique common fixed point in Ξ.

Proof:

Let $(\mathfrak{I}, \mathfrak{H})$ satisfy the property (E.A). By the definition of (E.A) we can get

$$\lim_{n\to\infty} \mathcal{Q}(\mathfrak{I}\varpi_{n,}\ u,\ u, \zeta) = \lim_{n\to\infty} \mathcal{Q}(\mathfrak{H}\varpi_{n,}\ u,\ u,\ \zeta) = 1 \text{ and}$$

$$\lim_{n\to\infty} \mathcal{H}(\mathfrak{I}\varpi_{n,}, u,\ u,\ \zeta) = \lim_{n\to\infty} \mathcal{H}(\mathfrak{H}\varpi_{n,},\ u,\ u,\ \zeta) = 0$$

$$\lim_{n\to\infty} \mathcal{O}(\mathfrak{I}\varpi_{n,}, u, u, \zeta) = \lim_{n\to\infty} \mathcal{O}(\mathfrak{H}\varpi_{n,}, u, u, \zeta) = 0$$

for some u ϵ Ξ and every $\zeta > 0$.

Because Neutrosophic metric space is complete and $\mathfrak{I}(\Xi) \subseteq \xi(\Xi)$, there exists a sequence $\{\omega_n\}$ such that $\mathfrak{I}(\varpi_n) = \xi(\omega_n)$, which implies

$$\lim_{n\to\infty} \mathcal{Q}(\xi\,\omega_n\, u, u, \zeta) = 1 \text{ and } \lim_{n\to\infty} \mathcal{H}(\xi\,\omega_n u, u, \zeta) = 0, \quad \lim_{n\to\infty} \mathcal{O}(\xi\,\omega_n u, u, \zeta) = 0 \text{ Now}$$

$$\mathcal{Q}(\mathfrak{I}\varpi_{n,},\ \beth\omega_n, \beth\omega_{n+1},\ \zeta) \ \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ \mathcal{Q}(\mathfrak{H}\varpi_n, \beth\omega_n, \beth\,\omega_{n+1}, \zeta), \mathcal{Q}(\mathfrak{I}\varpi_{n,},\ \mathfrak{H}\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \zeta)\end{pmatrix}\right]$$

$$\mathcal{H}(\mathfrak{I}\varpi_{n,},\ \beth\omega_n, \beth\omega_{n+1},\ \zeta) \leq \Psi\left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ \mathcal{H}(\mathfrak{H}\varpi_n, \beth\omega_n, \beth\,\omega_{n+1}, \zeta), \mathcal{H}(\mathfrak{I}\varpi_{n,},\ \mathfrak{H}\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \zeta)\end{pmatrix}\right]$$

$$\mathcal{O}(\mathfrak{I}\varpi_{n,},\ \beth\omega_n, \beth\omega_{n+1},\ \zeta) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ \mathcal{O}(\mathfrak{H}\varpi_n, \beth\omega_n, \beth\,\omega_{n+1}, \zeta), \mathcal{O}(\mathfrak{I}\varpi_{n,},\ \mathfrak{H}\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \zeta)\end{pmatrix}\right]$$

By the definition of Neutrosophic metric space, we can get

$$\mathcal{Q}(\varpi, \omega, \tau, \zeta) \geq \mathcal{Q}(\varpi,\ u, u, 1/3\,\zeta) * \mathcal{Q}(u, \omega, \tau\, 2/3\,\zeta)$$
$$\geq \mathcal{Q}(\varpi, u, u, 1/3\,\zeta) * \mathcal{Q}(\omega\ y, u, u, 1/3\,\zeta) * \mathcal{Q}(\tau, u, u, 1/3\zeta)$$

$$\mathcal{H}(\varpi, \omega, \tau, \zeta) \leq \mathcal{H}(\varpi,\ u, u, 1/3\,\zeta) \lozenge \mathcal{H}(u, \omega, \tau\, 2/3\,\zeta)$$
$$\leq \mathcal{H}(\varpi, u, u, 1/3\,\zeta) \lozenge \mathcal{H}(\omega\ y, u, u, 1/3\,\zeta) \lozenge \mathcal{H}(\tau, u, u, 1/3\zeta)$$

$$\mathcal{O}(\varpi, \omega, \tau, \zeta) \leq \mathcal{O}(\varpi,\ u, u, 1/3\,\zeta) \lozenge \mathcal{O}(u, \omega, \tau\, 2/3\,\zeta)$$
$$\leq \mathcal{O}(\varpi, u, u, 1/3\,\zeta) \lozenge \mathcal{O}(\omega\ y, u, u, 1/3\,\zeta) \lozenge \mathcal{O}(\tau, u, u, 1/3\zeta)$$

Thus

$$\lim_{n\to\infty} \mathcal{Q}(\mathfrak{I}\varpi_{n,}, \beth\omega_n, \beth\omega_{n+1}, \zeta) \geq \lim_{n\to\infty} \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ \mathcal{Q}(\mathfrak{H}\varpi_n, \beth\omega_n, \beth\,\omega_{n+1}, \zeta), \mathcal{Q}(\mathfrak{I}\varpi_{n,},\ \mathfrak{H}\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \zeta)\end{pmatrix}\right]$$

$$\geq \lim_{n\to\infty} \varphi\left[\min\begin{pmatrix}1*1*1, & \mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ 1*1*1 \ , & 1*1*1\end{pmatrix}\right]$$

$$\lim_{n\to\infty} \mathcal{H}(\mathfrak{I}\varpi_{n,}, \beth\omega_n, \beth\omega_{n+1}, \zeta) \leq \lim_{n\to\infty} \Psi\left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ \mathcal{H}(\mathfrak{H}\varpi_n, \beth\omega_n, \beth\,\omega_{n+1}, \zeta), \mathcal{H}(\mathfrak{I}\varpi_{n,},\ \mathfrak{H}\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \zeta)\end{pmatrix}\right]$$

$$\leq \lim_{n\to\infty} \Psi\left[\max\begin{pmatrix}0 \lozenge 0 \lozenge 0 & \mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ 0 \lozenge 0 \lozenge 0 & 0 \lozenge 0 \lozenge 0\end{pmatrix}\right]$$

$\lim_{n\to\infty} \mathcal{O}$ $(\Im\varpi_{n,}$, $\beth\omega_n$, $\beth\omega$ $_{n+1}$, $\zeta)$ $\leq \lim_{n\to\infty}$ Ω $\left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ \mathcal{O}(\mathfrak{H}\varpi_n, \beth\omega_n, \beth\,\omega_{n+1}, \zeta), \mathcal{O}\,(\Im\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \mathfrak{H}\varpi_{n,}, \zeta)\end{pmatrix}\right]$

$$\leq \lim_{n\to\infty} \Omega\left[\max\begin{pmatrix}0 \Diamond 0 \Diamond 0 & \mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)\\ 0 \Diamond 0 \Diamond 0 & 0 \Diamond 0 \Diamond 0\end{pmatrix}\right]$$

If $\beth\omega_n \neq$ u then $\lim_{n\to\infty} \mathcal{Q}\,(\Im\varpi_{n,}, \beth\omega_n, \beth\omega_{n+1}, \zeta) \geq \lim_{n\to\infty} \varphi\,[\mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)]$

$$> \lim_{n\to\infty} \mathcal{Q}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)$$

$$\lim_{n\to\infty} \mathcal{H}\,(\Im\varpi_{n,}, \beth\omega_n, \beth\omega_{n+1}, \zeta) \geq \lim_{n\to\infty} \Psi\,[\mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)]$$

$$> \lim_{n\to\infty} \mathcal{H}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)$$

$$\lim_{n\to\infty} \mathcal{O}\,(\Im\varpi_{n,}, \beth\omega_n, \beth\omega_{n+1}, \zeta) \geq \lim_{n\to\infty} \Omega\,[\mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)]$$

$$> \lim_{n\to\infty} \mathcal{O}(\mathfrak{H}\varpi_{n,}, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta)$$

is a contradiction by the above lemma.

Therefore

$$\lim_{n\to\infty} \Im\,\varpi_{n,} = \lim_{n\to\infty} \beth\,\omega_n = \lim_{n\to\infty} \mathcal{F}\,\tau_n = \lim_{n\to\infty} \mathfrak{H}\,\varpi_{n,} = \lim_{n\to\infty} \eta\,\omega_n = \lim_{n\to\infty} \xi\,\tau_n = \text{u}$$

$(\Xi, \mathcal{Q}, \mathcal{H}, \mathcal{O}\,*, \Diamond)$ is a complete Neutrosophic metric space.

There exists $\vartheta_0 \in \Xi$ such that $\mathfrak{H}\vartheta_0$ = u,

Hence $\mathcal{Q}\,(\Im\vartheta_0, \beth\omega_n, \beth\omega_{n+1}, \zeta)\,) \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}\vartheta_0, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{Q}(\Im\vartheta_0, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta\\ \mathcal{Q}(\Im\vartheta_0, \beth\omega_n, \beth\omega_{n+1}, \zeta), \mathcal{Q}(\Im\vartheta_0, \mathfrak{H}\vartheta_0, \mathfrak{H}\vartheta_0, \zeta)\end{pmatrix}\right]$

$\mathcal{H}$ $(\Im\vartheta_0$, $\beth\omega_n$, $\beth\omega$ $_{n+1}$, $\zeta)$ $)$ $\leq$ $\Psi\left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}\vartheta_0, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{H}(\Im\vartheta_0, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta\\ \mathcal{H}(\Im\vartheta_0, \beth\omega_n, \beth\omega_{n+1}, \zeta), \mathcal{H}(\Im\vartheta_0, \mathfrak{H}\vartheta_0, \mathfrak{H}\vartheta_0, \zeta)\end{pmatrix}\right]$

$$\mathcal{O}\,(\Im\vartheta_0, \beth\omega_n, \beth\omega_{n+1}, \zeta)\,) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}\vartheta_0, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta), \mathcal{O}(\Im\vartheta_0, \xi\,\omega_n, \xi\,\omega_{n+1}, \zeta\\ \mathcal{O}(\Im\vartheta_0, \beth\omega_n, \beth\omega_{n+1}, \zeta), \mathcal{O}(\Im\vartheta_0, \mathfrak{H}\vartheta_0, \mathfrak{H}\vartheta_0, \zeta)\end{pmatrix}\right]$$

On making n → ∞, $\mathcal{Q}\,(\Im\vartheta_0, \text{u}, \text{u}, \zeta) \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\text{u}, \text{u}, \text{u}, \zeta), \mathcal{Q}(\text{u}, \text{u}, \text{u}, \zeta)\\ \mathcal{Q}(\Im\vartheta_0\text{u}, \text{u}, \zeta), \mathcal{Q}(\Im\vartheta_0, \text{u}, \text{u}, \zeta)\end{pmatrix}\right]$

$$\mathcal{H}\,(\Im\vartheta_0, \text{u}, \text{u}, \zeta) \leq \Psi\left[\max\begin{pmatrix}\mathcal{H}(\text{u}, \text{u}, \text{u}, \zeta), \mathcal{H}(\text{u}, \text{u}, \text{u}, \zeta)\\ \mathcal{H}(\Im\vartheta_0\text{u}, \text{u}, \zeta), \mathcal{H}(\Im\vartheta_0, \text{u}, \text{u}, \zeta)\end{pmatrix}\right]$$

$$\mathcal{O}\,(\Im\vartheta_0, \text{u}, \text{u}, \zeta) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(\text{u}, \text{u}, \text{u}, \zeta), \mathcal{O}(\text{u}, \text{u}, \text{u}, \zeta)\\ \mathcal{O}(\Im\vartheta_0\text{u}, \text{u}, \zeta), \mathcal{O}(\Im\vartheta_0, \text{u}, \text{u}, \zeta)\end{pmatrix}\right]$$

which can imply $\Im\vartheta_0$ = u, with $\varphi(\text{s}) > \text{s}$, $\Psi(\text{s}) < \text{s}$, $\Omega(\text{s}) < \text{s}$ for $0 < \text{s} < 1$

As $\Im\,(\Xi) \subseteq \xi\,(\Xi)$, there exists ω_0 such that $\Im\,\vartheta_0 = \xi\,\omega_0$

Suppose $\xi\, \omega_0 \neq \eta\, \omega_0$. Now

$$\mathcal{Q}\,(\mathfrak{I}\varpi_{n,},\ \beth\omega_0, \beth\omega_0,\ \zeta) \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}\varpi_{n,},\ \xi\,\omega_0\,\xi\,\omega_0\,,\zeta),\ \mathcal{Q}(fx_n,\ \xi\,\omega_0\,\xi\,\omega_0,\zeta)\\ \mathcal{Q}(\mathfrak{H}\varpi_{n,}, \beth\omega_n, \beth\omega_n,\zeta),\ \mathcal{Q}(\mathfrak{I}\varpi_{n,},\xi\,\omega_n,\xi\,\omega_n,\zeta)\end{pmatrix}\right]$$

$$\mathcal{H}\,(\mathfrak{I}\varpi_{n,},\ \beth\omega_0, \beth\omega_0,\ \zeta) \leq \Psi\left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}\varpi_{n,},\ \xi\,\omega_0\,\xi\,\omega_0\,,\zeta),\ \mathcal{H}(fx_n,\ \xi\,\omega_0\,\xi\,\omega_0,\zeta)\\ \mathcal{H}(\mathfrak{H}\varpi_{n,}, \beth\omega_n, \beth\omega_n,\zeta),\ \mathcal{H}(\mathfrak{I}\varpi_{n,},\xi\,\omega_n,\xi\,\omega_n,\zeta)\end{pmatrix}\right]$$

$$\mathcal{O}\,(\mathfrak{I}\varpi_{n,},\ \beth\omega_0, \beth\omega_0,\ \zeta) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}\varpi_{n,},\ \xi\,\omega_0\,\xi\,\omega_0\,,\zeta),\ \mathcal{O}(fx_n,\ \xi\,\omega_0\,\xi\,\omega_0,\zeta)\\ \mathcal{O}(\mathfrak{H}\varpi_{n,}, \beth\omega_n, \beth\omega_n,\zeta),\ \mathcal{O}(\mathfrak{I}\varpi_{n,},\xi\,\omega_n,\xi\,\omega_n,\zeta)\end{pmatrix}\right]$$

If $n \to \infty$, $\mathcal{Q}\,(u, \beth\omega_0, \beth\omega_0, \zeta) \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(u,u,u,\zeta)\,,\mathcal{Q}(u,\beth\omega_0,\beth\omega_0,\zeta)\\ \mathcal{Q}(u,u,u,\zeta)\,,\mathcal{Q}(u,u,u,\zeta)\end{pmatrix}\right]$

$$\mathcal{H}\,(u, \beth\omega_0, \beth\omega_0, \zeta) \leq \Psi\left[\max\begin{pmatrix}\mathcal{H}(u,u,u,\zeta)\,,\mathcal{H}(u,\beth\omega_0,\beth\omega_0,\zeta)\\ \mathcal{H}(u,u,u,\zeta)\,,\mathcal{H}(u,u,u,\zeta)\end{pmatrix}\right]$$

$$\mathcal{O}\,(u, \beth\omega_0, \beth\omega_0, \zeta) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(u,u,u,\zeta)\,,\mathcal{O}(u,\beth\omega_0,\beth\omega_0,\zeta)\\ \mathcal{O}(u,u,u,\zeta)\,,\mathcal{O}(u,u,u,\zeta)\end{pmatrix}\right]$$

by the continuity of $\mathcal{Q}, \mathcal{H}, \mathcal{O}$ and φ, Ψ, Ω

Hence $\mathcal{Q}(u, \beth\omega_0, \beth\omega_0, \zeta) \geq \varphi\,(\mathcal{Q}\,(u, \beth\omega_0, \beth\omega_0, \zeta)\,) > \mathcal{Q}\,(u, \beth\omega_0, \beth\omega_0, \zeta)$

$\mathcal{H}(u, \beth\omega_0, \beth\omega_0, \zeta) \leq \Psi\,(\mathcal{H}\,(u, \beth\omega_0, \beth\omega_0, \zeta)) < \mathcal{H}\,(u, \beth\omega_0, \beth\omega_0, \zeta)$

$\mathcal{O}(u, \beth\omega_0, \beth\omega_0, \zeta) \leq \Omega\,(\mathcal{O}\,(u, \beth\omega_0, \beth\omega_0, \zeta)) < \mathcal{O}\,(u, \beth\omega_0, \beth\omega_0, \zeta)$

is a contradiction.

So $\mathfrak{I}\,\vartheta_0 = \xi\,\omega_0$

Now by $(\mathfrak{I}, \xi)$, $(\beth, \eta)$ and $(\mathcal{F}, \mathfrak{H})$ are weakly compatible, we can get,

$\mathfrak{I}\mathfrak{I}\,\vartheta_0 = \mathfrak{I}\eta\,\vartheta_0 = \eta\mathfrak{I}\,\vartheta_0 = \eta\eta\vartheta_0$ and $\beth\beth\omega_0 = \beth\eta\,\omega_0 = \eta\,\beth\omega_0 = \eta\,\eta\,\omega_0$

Then, $\lim\limits_{n\to\infty} \mathcal{Q}\,(\mathfrak{I}u, \beth\omega_n, \beth\omega_{n+1}, \zeta) \geq \varphi\left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}u,\eta\omega_n,\eta\omega_{n+1},\zeta),\mathcal{Q}(\mathfrak{I}u,\eta y_{n,}\eta y_{n+1},\zeta)\\ \mathcal{Q}(\mathfrak{H}u,\beth\omega_n,\beth\omega_{n+1},\zeta),\quad \mathcal{Q}(\mathfrak{I}u,\mathfrak{H}u,\mathfrak{H}u,t)\end{pmatrix}\right]$

$$\lim_{n\to\infty} \mathcal{H}\,(\mathfrak{I}u, \beth\omega_n, \beth\omega_{n+1}, \zeta) \leq \Psi\left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}u,\eta\omega_n,\eta\omega_{n+1},\zeta),\mathcal{H}(\mathfrak{I}u,\eta y_{n,}\eta y_{n+1},\zeta)\\ \mathcal{H}(\mathfrak{H}u,\beth\omega_n,\beth\omega_{n+1},\zeta),\quad \mathcal{H}(\mathfrak{I}u,\mathfrak{H}u,\mathfrak{H}u,t)\end{pmatrix}\right]$$

$$\lim_{n\to\infty} \mathcal{O}\,(\mathfrak{I}u, \beth\omega_n, \beth\omega_{n+1}, \zeta) \leq \Omega\left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}u,\eta\omega_n,\eta\omega_{n+1},\zeta),\mathcal{O}(\mathfrak{I}u,\eta y_{n,}\eta y_{n+1},\zeta)\\ \mathcal{O}(\mathfrak{H}u,\beth\omega_n,\beth\omega_{n+1},\zeta),\quad \mathcal{O}(\mathfrak{I}u,\mathfrak{H}u,\mathfrak{H}u,t)\end{pmatrix}\right]$$

$\Rightarrow\ \mathfrak{I}u = u = \mathfrak{H}u.$

Similarly we can get $\eta u = \beth u = u$

Uniqueness:

Let v be another common fixed point of $\mathfrak{I}, ב, \mathcal{F}, \mathfrak{H}, \eta$ and ξ .

Then

$\mathcal{Q}$ (v, u, u, ζ) = $\mathcal{Q}$ ($\mathfrak{I}$v, ηu, ηu, ζ)

$$\geq \varphi \left[\min\begin{pmatrix}\mathcal{Q}(\mathfrak{H}v, \eta u, \eta u, \zeta), \mathcal{Q}(\mathfrak{I}v, \eta u, \eta u, \zeta) \\ \mathcal{Q}(\mathfrak{H}v, gu, gu, \zeta),\ \mathcal{Q}(\mathfrak{I}v, \mathfrak{H}v, \mathfrak{H}v, \zeta)\end{pmatrix}\right]$$

$\mathcal{H}$ (v, u, u, ζ) = $\mathcal{H}$ ($\mathfrak{I}$v, ηu, ηu, ζ)

$$\leq \Psi \left[\max\begin{pmatrix}\mathcal{H}(\mathfrak{H}v, \eta u, \eta u, \zeta), \mathcal{H}(\mathfrak{I}v, \eta u, \eta u, \zeta) \\ \mathcal{H}(\mathfrak{H}v, gu, gu, \zeta),\ \mathcal{H}(\mathfrak{I}v, \mathfrak{H}v, \mathfrak{H}v, \zeta)\end{pmatrix}\right]$$

$\mathcal{O}$ (v, u, u, ζ) = $\mathcal{O}$ ($\mathfrak{I}$v, ηu, ηu, ζ)

$$\leq \Omega \left[\max\begin{pmatrix}\mathcal{O}(\mathfrak{H}v, \eta u, \eta u, \zeta), \mathcal{O}(\mathfrak{I}v, \eta u, \eta u, \zeta) \\ \mathcal{O}(\mathfrak{H}v, gu, gu, \zeta),\ \mathcal{O}(\mathfrak{I}v, \mathfrak{H}v, \mathfrak{H}v, \zeta)\end{pmatrix}\right]$$

It implies v= u. Hence $\mathfrak{I}, ב, \mathcal{F}, \mathfrak{H}, \eta$ and ξ have a unique common fixed point in Ξ.

Coclusion:

In conclusion, this paper has explored the concept of fixed point theorems within the framework of common neutrosophic metric spaces, offering new insights and extending existing results in the field. By integrating neutrosophic logic into metric space theory, we have developed several fixed point theorems that address both classical and novel types of metrics, accommodating the inherent uncertainty and indeterminacy present in neutrosophic contexts.

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