Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

The Numerical Solutions of Partial Differential Equations using Two-Three-Dimensions Differential Transform Method

Rohit Nagargoje[1], Avinash Khambayat[2]

[1.] Research Scholar, Department of Mathematics, School of Science, Sandip University Nasik, Maharashtra

Email: rnagargoje92@gmail.com

[2.] Professor, Department of Mathematics, School of Science, Sandip University Nasik, Maharashtra

Email: avinash.khambayat@sandipuniversity.edu.in

Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract: Differential Transform Method is the one of the novel and unique methods which provides series solution. The approach mainly rests on the Differential transform method (DTM) which is one of the approximate methods to achieved exact solution of one dimensional up to multidimensional. The method can easily be applied on many problems in the field of Differential Equations and Partial differential equations (PDE). DTM is used to reduces the size of calculus work and gives the solution of higher order two-dimensional, three-dimensional, multi-dimensional differential equation.

Keywords: Differential transformed method, two-dimensional, three-dimensional, multi-dimensional partial differential equation.

1. Introduction

The analytical approaches of the Differential Transform Method have been succeeded to solve differential equations and integral equations in many fields. The basic concept of the Differential Transform Method is based on the Taylor series. The Differential Transform Method was first introduced by J.K. Zhou in a study about electrical circuits. The Differential Transform Method obtains an analytical solution in the form of a polynomial equations [1]. Furthermore, A variety of methods, exact approximate and purely numerical are available for the solution of differential equations. Most of these methods are computationally intensive because they are trial-and error in nature, or need complicated symbolic computations. Furthermore, the differential transformation technique is one of the numerical methods for ordinary differential equations. The concept of differential transformation was first proposed by Zhou [2]. It is different from the traditional high order Taylor's series method, which requires symbolic competition of the necessary derivatives of the data functions. The Taylor series method has computationally taken a long time for big orders; by using this method, it is possible to obtain highly accurate results (up to 3^{th} order) or exact solutions for differential equations [3]. With this technique the given partial differential equation and related initial conditions are transformed into a recurrence equation that finally leads to the solution of a system of algebraic equations as coefficients of a power series solution [4]. This method is useful for obtaining exact and approximate solutions of linear and nonlinear ordinary and partial differential equations [5-6]. In this method there is no need for linearization or perturbations, large computational work and round-off errors are avoided. It has been used to solve effectively, easily and accurately a large class of linear and nonlinear problems with approximations. It is possible to solve system of differential equations, differential algebraic equations, difference equations, differential difference equations, partial differential equations [7]. Moreover, the applying of DTM is effective for solving two-dimensional and three-dimensional PDEs with initial value problems [8-10]. It causes scientific findings in the for mentioned study fields. It is critical to investigate different techniques for integrating these PDEs. The DTM is a highly successful and efficient instrument for addressing both one-dimensional and multidimensional problems [11]. This method uses a sequential method to create analytical solutions in the form of polynomials, which is based on the Taylor series expansion [12]. The DTM has been used to solving both linear and nonlinear differential equations, including the KdV and MKdV equations given by Angalgil & Ayaz, 2009 in [13]. The two-point boundary value problem [14]. (Chenand & Liu, 1998), the linear parabolic-hyperbolic partial differential equations [15]. (Biazar et al., 2010), the two-dimensional nonlinear Gas dynamic, and the Klien-Gordon equations [16].

The differential transform approach is also taken into consideration for solving the three-dimensional linear Helmholtz problem in the following form:

$$l\frac{\partial z^2}{\partial p^2}+m\frac{\partial z^2}{\partial q^2}+n\frac{\partial z^2}{\partial r^2}+\lambda z=G(p,q,r)$$

With the initial conditions:

$$z(o,q,r)=g_1(q,r)=z_p(o,q,r)=g_2(q,r)$$

$$z(p,o,r)=g_3(p,r)=z_q(p,o,r)=g_4(p,r)$$

$$z(p,q,o)=g_5(p,q)=z_r(p,q,o)=g_6(p,q)$$

Where $g_1(q,r),g_2(q,r),g_3(p,r),g_4(p,r),g_5(p,q),g_6(p,q)$ and l,m,n,λ are given function and constant respective [17].

This equation has wide applications in various filed such as electrical and mechanical engineering and physics. These equations the reader is referred to (Zwilinger, 1992 Burdenand and Faires, 1993) [18]. Jafari and Zabini solved the above equations by homotope perturbation method and homotop analysis method respectively [19]. (Jafari.et.al.2010b and 2010c). In this paper we apply DTM for Helmholtz equation and Schrodinger equations, Partial Differential Equation [20-21].

2. Methods

In this research article we proposed, the Differential Transformation Method (DTM) has been successfully applied to find exact and approximate solution of the second order differential equations. The method was used in a direct way without using linearization, perturbation or restrictive assumptions. And solving Partial differential equations in two, three-dimensional linear and nonlinear, have been solved effectively using the Differential Transform Method. Therefore, it is not affected by computation round off errors and one is not faced with the necessities of large computer memory and time.

Definitions of Multi-dimensional DTM:

We define - dimensional differential transform and fundamental operation of the function $z(x_1, x_2, \ldots x_m)$ as,

$$Z(k_1, k_2, \ldots k_m) = \frac{1}{k_1! k_2! \ldots k_m!} \left[\frac{\partial^{k_1+k_2+\ldots+k_m} z(x_1, x_2, \ldots, x_m)}{\partial x_1^{k_1}, \partial x_2^{k2}, \ldots \partial x_m^{k_m}} \right]_{(0,0,\ldots.0)} \tag{1}$$

Where $z(x_1, x_2, \ldots x_m)$ is original function and $Z(k_1, k_2, \ldots k_m)$ is transformed function.

The differential inverse transform of $z(x_1, x_2, \ldots x_m)$ is defined as follows

$$z(x_1, x_2, \ldots x_m) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \ldots \sum_{k_m=0}^{\infty} Z(k_1, k_2, \ldots k_m)\, x_1^{k_1} x_2^{k_2} \ldots x_m^{k_m} \tag{2}$$

and from equation (1) and (2) we can conclude

$$z(x_1, x_2, \ldots x_m) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \ldots \sum_{k_m=0}^{\infty} \frac{1}{k_1! k_2! \ldots k_m!} \left[\frac{\partial^{k_1+k_2+\ldots+k_m} z(x_1, x_2, \ldots x_m)}{\partial x_1^{k_1}, \partial x_2^{k2}, \ldots \partial x_m^{k_m}} \right]_{(0,0,\ldots.0)} x_1^{k_1} x_2^{k_2} \ldots x_m^{k_m} \tag{3}$$

Theorem 1: If $z(p_1, p_2 \ldots, p_m) = \lambda f\ (p_1, p_2 \ldots, p_m)$ then,

$$Z(k_1, k_2 \ldots, k_m) = \lambda f\ (k_1, k_2, \ldots, k_m)$$

Theorem 2: If $z(p_1, p_2 \ldots, p_m) = \frac{\partial f(p_1, p_2, \ldots, p_m)}{\partial p_1}$ then,

$$Z(k_1, k_2 \ldots, k_m) = (k_i + 1) F\ (k_1, k_2, \ldots, (k_i + 1), \ldots, k_m)$$

Theorem 3: If $z(p_1, p_2 \ldots, p_m) = p_1{}^{h_1}, p_2{}^{h_2} \ldots p_m{}^{h_m}$ then,

$$Z(k_1, k_2 \ldots, k_m) = \delta(k_1 - h_1)\delta(k_2 - h_2) \ldots \delta(k_m - h_m)$$

where,

$$\delta(k_i - h_i) = \begin{cases} 1 & k_i = h_i \\ 0 & otherwise \end{cases}$$

Theorem 4: If $z(p_1, p_2 \ldots, p_m) = p_1{}^{h_1}, p_2{}^{h_2} \ldots \sin(a_{x_i} + b) \ldots p_m{}^{h_m}\ then\ Z(k_1, k_2 \ldots, k_m)$

$$= \delta(k_1 - k_2) \ldots \frac{a^{k_i}}{k_{i!}} \sin\left(\frac{k_i \pi}{2} + b\right) \ldots \delta(k_m - h_m)$$

Theorem 5: If $z(p_1, p_2 \ldots, p_m) = p_1{}^{h_1}, p_2{}^{h_2} \ldots \cos(a_{x_i} + b) \ldots p_m{}^{h_m}\ then\ Z(k_1, k_2 \ldots, k_m)$

$$= \delta(k_1 - k_2) \ldots \frac{a^{k_i}}{k_{i!}} \cos\left(\frac{k_i \pi}{2} + b\right) \ldots \delta(k_m - h_m)$$

3. Results

Solving two numerical using DTM:

Example [1] Consider the following Two-Dimensional Schrodinger equations:

$$\frac{\partial^2 z}{\partial p^2}+\frac{\partial^2 z}{\partial q^2}-4z=(24p^2-5p^4)\cos(\mathrm{q}) \tag{4}$$

With the initial condition $z(0,q)=0\,, z_p(0,q)=0$ (5)

The exact solution can be expressed as $z(p,t)=p^4\cos(\mathrm{q})$

Taking the differential transform of (4)

$$(k_1+2)\,(k_1+1)\,Z\,(k_1+2k_2)+(k_2+2)\,(k_2+1)\,Z\,(k_1,k_2+2)-4Z\,(k_1,k_2)$$
$$=24\,\delta(k_1-2)\frac{1}{k_2!}\cos\left(\frac{k_2\pi}{2}\right)-5\delta(k_1-4)\frac{1}{k_2!}\cos\left(\frac{k_2\pi}{2}\right) \tag{6}$$

From the initial condition given by equation (5)

$Z\big(0,k_{2,}\big)=0$

$Z(1,\ k_2\ )=0$

$k_2=0,1,2,\ldots$ (7)

Substituting equation (6) into equation (7)

By means of recursive method, the results are

$$Z(k_{1,}k_{2,})=\begin{cases}\frac{1}{k_2!}\cos\left(\frac{k_2\pi}{2}\right) & ,if\ k_1=4\\ 0 & ,\ o.w.\end{cases}$$

We obtained the series solution as

$$Z(p,q)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}Z(k_1,k_2)p^{k_1}q^{k_2}=p^4\cos(\mathrm{q})$$

Which is exact solution.

Example [2] Consider the following Three-Dimensional Helmholtz equations:

$$\frac{\partial^2 z}{\partial p^2}+\frac{\partial^2 z}{\partial q^2}-\frac{\partial^2 z}{\partial r^2}-8z=(24p^2-8p^4)\cos(\mathrm{q})\sin(r) \tag{8}$$

With the initial condition $z(0,q,r)=0\,, z_p(0,q,r)=0$ (9)

The exact solution can be expressed as $z(p,t)=p^4\cos(\mathrm{q})\sin(r)$

Taking the differential transform of (8)

$(k_1+2)\ (k_1+1)\ Z\ (k_1+2,\ k_2,\ k_3)\ +\ (k_2+2)\ (k_2+1)\ Z\ (k_1,\ k_2+2,\ k_3)-\ (k_3+2)\ (k_3+1)\ Z\ (k_1,\ k_2,\ k_3+2)-\ 8Z\ (k_1,\ k_2,\ k_3)$

$$= 24\delta(k_1 - 2)\frac{1}{k_2!}\cos\left(\frac{k_2\pi}{2}\right)\frac{1}{k_3!}\sin\left(\frac{k_3\pi}{2}\right) - 8\delta(k_1 - 4)\frac{1}{k_2!}\cos\left(\frac{k_2\pi}{2}\right)\frac{1}{k_3!}\sin\left(\frac{k_3\pi}{2}\right) \qquad (10)$$

From the initial condition given by equation (9)

$Z(0, k_{2,}\, k_3) = 0$

$Z(1, k_{2,}\, k_3) = 0$

$k_{2,}k_3 = 0,1,2, \dots$ (11)

Substituting equation (11) into equation (10)

By means of recursive method, the results are

$Z(k_{1,}k_{2,}k_{3,}) = 0 \qquad if\ k_1 \neq 8\ \&\ k_{2,}k_{3,} = 0,1,2, \dots$

$Z(8\ k_{2,}k_3 = \frac{1}{k_2!}\cos\,(\frac{k_2\pi}{2})\frac{1}{k_3!}\sin\,(\frac{k_3\pi}{2}) \qquad if\ k_{2,}k_{3,} = 0,1,2, \dots$

We obtained the series solution as

$$Z(p,q,r) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty} Z(k_1k_2,k_3)\, p^{k_1}q^{k_2}r^{k_3} = p^4\cos(\mathrm{q})\sin(r)$$

Which is exact solution.

Example [3] Consider PIDE,

$$xu_x = u_{tt} + xsint - \int_0^t sin(t-z)\, u(x,z)\, dz$$

With initial conditions $u(x,0) = 0, u_t(x,0) = x$ and boundary condition $u(1,t) = t$

Solution: Given,

$$xu_x = u_{tt} + xsint - \int_0^t sin(t-z)u(x,z)dz$$

$$u_{tt} = xu_x - xsint - sint\int_0^t coszu(x,z)dz + cost\int_0^t sinzu(x,z)dz \qquad (12)$$

With $u(x,0) = 0,\ \ u_t(x,0) = x$ (13)

And $u(1,t) = t$ (14)

Appling two dimensional DTM on both sides of equations (12),

$$\therefore U(k,h+2)=\frac{1}{(h+1)(h+2)}\left\{\begin{array}{l}\sum_{\alpha=0}^{k}\delta(\alpha-1)(k-\alpha+1)U(k-\alpha+1,h)-\frac{\delta(k-1)\,sin\left(\frac{h\pi}{2}\right)}{h!}\\ -\sum_{\beta=1}^{h}\sum_{s=0}^{\beta-1}\left(\frac{1}{\beta}\right)\left(\frac{sin\left(\frac{(h-\beta)\pi}{2}\right)}{(h-\beta)!}\right)\left(\frac{cos\left(\frac{s\pi}{2}\right)}{s!}\right)U(k,\beta-s-1)\\ +\sum_{\beta=0}^{h}\sum_{s=0}^{\beta-1}\left(\frac{1}{\beta}\right)\left(\frac{cos\left(\frac{(h-\beta)\pi}{2}\right)}{(h-\beta)!}\right)\left(\frac{sin\left(\frac{s\pi}{2}\right)}{s!}\right)U(k,\beta-s-1)\end{array}\right\}$$

$\because$ Using trigonometric identity, $sinA.cosB-cosA.sinB=\sin(A-B)$

We get,

$U(k,h+2)=$

$$\frac{1}{(h+1)(h+2)}\left\{\begin{array}{l}\sum_{\alpha=0}^{k}\delta(\alpha-1)(k-\alpha+1)U(k-\alpha+1,h)-\frac{\delta(k-1)\,sin\left(\frac{h\pi}{2}\right)}{h!}\\ -\sum_{\beta=1}^{h}\sum_{s=0}^{\beta-1}\left(\frac{1}{\beta}\right)\left(\frac{sin\left(\frac{(h-\beta-s)\pi}{2}\right)}{(h-\beta)!\;\;s!}\right)U(k,\beta-s-1)\end{array}\right\}$$

(15)

Now applying two-dimensional DTM on initial conditions (13) and boundary conditions (14) we get,

$U(k,0)=0\;;\;\forall k,$ (16)

$\therefore U(0,0)=U(1,0)=U(2,0)=U(3,0)=\cdots..=0$

$U(k,1)=\delta(k-1)=\begin{cases}1; & k=1\\ 0; & k\neq 1\end{cases}$ (17)

$\therefore U(0,1)=0,\;\; U(1,1)=1,\;\; U(2,1)=U(3,1)=U(4,1)=\cdots..0$

$U(1,h)=\delta(h-1)=\begin{cases}1; & h=1\\ 0; & h\neq 1\end{cases}$ (18)

$\therefore U(1,0)=0,\;\; U(1,1)=1,\;\; U(1,2)=U(1,3)=U(1,4)=\cdots..0$

Put $h=0,1,2,3,4,....$ in equation (15) and using equations (16) to (18),

If $h=0$,

$$U(k,2)=\frac{1}{2}\left\{\begin{array}{c}\sum_{\alpha=0}^{k}\delta(\alpha-1)(k-\alpha+1)\;U(k-\alpha+1,0)-\\ \frac{\delta(k-1)\,sin\,0}{h!}-0\end{array}\right\}$$

$$\therefore U(k,2)=\frac{1}{2}\left\{\sum_{\alpha=0}^{k}\delta(\alpha-1)(k-\alpha+1)\,U(k-\alpha+1{,}0)-0\;-0\right\}$$

As $U(k,0)=0;\ \forall k, \qquad \therefore U(k-\alpha+1,0)=0$

$\therefore U(k,2)=0, \qquad \forall k$

$$\Rightarrow U(0{,}2)=0\,,U(1{,}2)=0\,,U(2{,}2)=0\,,U(3{,}2)=0\,,U(4{,}2)=0\,,\ldots.. \tag{19}$$

If $h=1$,

$$U(k,3)=\frac{1}{6}\left\{\begin{gathered}\sum_{\alpha=0}^{k}\delta(\alpha-1)(k-\alpha+1)\,U(k-\alpha+1,1)-\\ \frac{\delta(k-1)\,sin\left(\frac{\pi}{2}\right)}{1!}-\sum_{\beta=1}^{1}\sum_{s=0}^{\beta-1}\left(\frac{1}{\beta}\right)\left(\frac{sin\left(\frac{(1-\beta-s)\,\pi}{2}\right)}{(1-\beta)!\;\;s!}\right)U(k,\beta-s-1)\end{gathered}\right\}$$

$$\text{If } k=0 \quad \therefore U(0{,}3)=\frac{1}{6}\{0-0-0\}=\ 0$$

$$\text{If } k=1 \quad \therefore U(1{,}3)=\frac{1}{6}\left\{\sum_{\alpha=0}^{1}\delta(\alpha-1)(2-\alpha)\,U(2-\alpha,1)-\delta(1-1)\right\}$$

$$=\frac{1}{6}\{0+(1)(1)\,U(1{,}1)-1\}=0 \qquad \because U(1{,}1)=1$$

$$\text{If } k=2 \quad \therefore U(2{,}3)=\frac{1}{6}\left\{\sum_{\alpha=0}^{2}\delta(\alpha-1)(3-\alpha)\,U(3-\alpha,1)-\delta(2-1)\right\}$$

$$=\frac{1}{6}\{0+(1)(2)\,U(2{,}1)+0-0\}=0 \qquad \because U(2{,}1)=0$$

$\therefore U(k,3)=0, \qquad \forall k$

$$\Rightarrow U(0{,}3)=0\,,U(1{,}3)=0\,,U(2{,}3)=0\,,U(3{,}3)=0\,,U(4{,}3)=0\,,\ldots.. \tag{20}$$

If $h=2$,

$U(k,4)=$

$$\frac{1}{12}\left\{\begin{gathered}\sum_{\alpha=0}^{k}\delta(\alpha-1)(k-\alpha+1)U(k-\alpha+1{,}2)-\\ \frac{\delta(k-1)\,sin(\pi)}{2!}-\sum_{\beta=1}^{2}\sum_{s=0}^{\beta-1}\left(\frac{1}{\beta}\right)\left(\frac{sin\left(\frac{(2-\beta-s)\pi}{2}\right)}{(2-\beta)!\;\;s!}\right)U(k,\beta-s-1)\end{gathered}\right\}$$

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

$$\therefore U(k,4) = \frac{1}{12}\left\{\begin{array}{l} \sum_{\alpha=0}^{k} \delta(\alpha-1)(k-\alpha+1)U(k-\alpha+1,2) + \frac{\delta(k-1)sin\pi}{2} \\ -\sum_{s=0}^{0} \frac{sin\left(\frac{(1-s)\pi}{2}\right)}{1!\,s!}U(k,-s) - \sum_{s=0}^{1} \frac{sin\left(\frac{-s\pi}{2}\right)}{0!\,s!}U(k,1-s) \end{array}\right\}$$

$$\therefore U(k,4) = \frac{1}{12}\{0+0-U(k,0)-0+U(k,0)\}$$

$$\therefore U(k,4) = 0, \quad \forall k \because U(k,0) = 0$$

$$\Longrightarrow U(0,4) = 0, U(1,4) = 0, U(2,4) = 0, U(3,4) = 0, U(4,4) = 0 \qquad (21)$$

Similarly, we get,

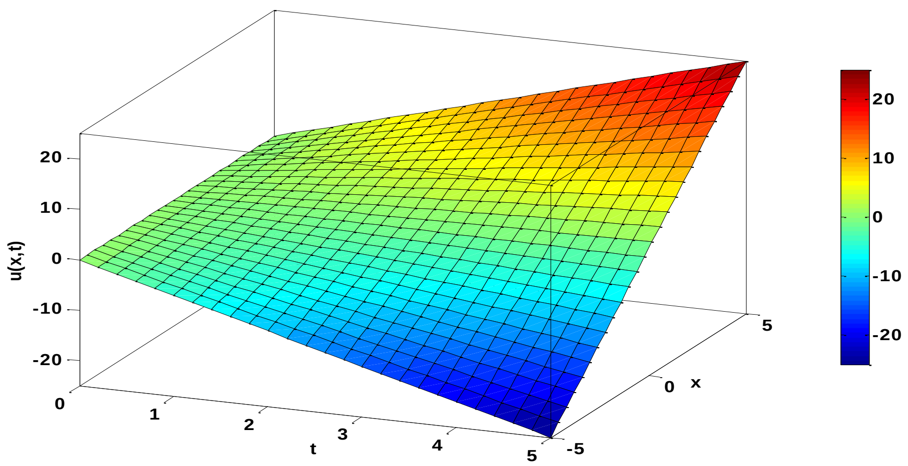
$$U(k,4) = U(k,5) = 0\ U(k,6) = U(k,7) = \cdots .. = 0, \quad \forall k$$

We know that,

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U(k,h)x^k t^h$$

$$= \{U(0,0) + U(1,0)x + U(2,0)x^2 + U(3,0)x^3 + \cdots .\}$$
$$+ \{U(0,1)t + U(1,1)xt + U(2,1)x^2 t + \cdots .\}$$
$$+ \{U(0,2)t^2 + U(1,2)xt^2 + U(2,2)x^2t^2 + \cdots\} + \cdots .$$

$$\therefore \boldsymbol{u(x,t) = xt}$$



(Fig. 1.3: ***Graph of*** $\boldsymbol{u(x,t) = xt}$ ***for*** $\boldsymbol{-20 \le u \le 20;\ -5 \le x \le 5\ ;\ 0 \le t \le 4}$**)**

4. Conclusion

In this paper, we proposed the Differential Transformation Method (DTM) has been successfully applied to find exact and approximate solution of the second order differential equations. The method was used in a direct way without using linearization, perturbation or restrictive assumptions. While, this equation solving Partial differential equations in two, three-dimensional linear and nonlinear, have been solved effectively using the Differential Transform Method. Therefore, it is not affected by computation round off errors and one is not faced with the necessities of large computer memory and time. Moreover, this method provides a closed-series form solutions with the convergence region A specific advantage of this method is provided to exact and accurate solutions up to the 3rd dimensional DTM over any purely numerical method. It may be concluded that DTM is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of differential equations.

References

[1] Zhou JK. Differential Transformation and Its Applications for Electrical Circuits. Huazhong University Press, Wuhan China (In Chinese) google schola. 1986; 2:413-20.

[2] Zou L, Wang Z, Zong Z. Generalized differential transform method to differential-difference equation. Physics Letters A. 2009 Nov 2;373(45):4142-51.

[3] Barton D. Taylor series methods for ordinary differential equations: an evaluation. Mathematical Software. 1971:369-90.

[4] Denef J, Lipshitz L. Power series solutions of algebraic differential equations. Mathematische annalen. 1984 Jun; 267:213-38.

[5] Kanth AR, Aruna K. Differential transform method for solving linear and non-linear systems of partial differential equations. Physics Letters A. 2008 Nov 17;372(46):6896-8.

[6] Patil N, Khambayat A. Differential transform method for system of linear differential equations. Research Journal of Mathematical and Statistical Sciences ISSN. 2014 Mar; 2320:6047.

[7] Sewell G. The numerical solution of ordinary and partial differential equations. John Wiley & Sons; 2005 Jul 25.

[8] Chen CO, Ho SH. Solving partial differential equations by two-dimensional differential transform method. Applied Mathematics and computation. 1999 Dec 1;106(2-3):171-9.

[9] Ayaz F. On the two-dimensional differential transform method. Applied Mathematics and computation. 2003 Nov 10;143(2-3):361-74.

[10] Richardson SM, Cornish AR. Solution of three-dimensional incompressible flow problems. Journal of fluid Mechanics. 1977 Sep;82(2):309-19.

[11] Gregor J. The multidimensional $ z $-transform and its use in solution of partial difference equations. Kybernetika. 1988;24(7):1-3.

[12] Eda Yülüklü, Ahmet and Yasir Khan, “A Taylor Series Based Method for Solving Nonlinear Sine-Gordon and Klein-Gordon Equations”, World Applied Sciences Journal, 12 (1): 21-27, 2011, ISSN 1818-4952.

[13] Kangalgil F, Ayaz F. Solitary wave solutions for the KdV and mKdV equations by differential transform method. Chaos, Solitons & Fractals. 2009 Jul 15;41(1):464-72.

[14] Jafari H, Sadeghi S, Biswas A. The differential transform method for solving multidimensional partial differential equations. Indian Journal of Science and Technology. 2012 Feb;5(2):2009-12.

[15] Li J, Chen YT. Computational partial differential equations using MATLAB®. Crc Press; 2019 Sep 26.

[16] Jafari H, Sadeghi S, Biswas A. The differential transform method for solving multidimensional partial differential equations. Indian Journal of Science and Technology. 2012 Feb;5(2):2009-12.

[17] Maponi P, Misici L, Zirilli F. An inverse problem for the three-dimensional vector Helmholtz equation for a perfectly conducting obstacle. Computers & Mathematics with Applications. 1991 Jan 1;22(4-5):137-46.

[18] Jafari H, Sadeghi S, Biswas A. The differential transform method for solving multidimensional partial differential equations. Indian Journal of Science and Technology. 2012 Feb;5(2):2009-12.

[19] Jafari H, Ghorbani M, Ghasempour S. A note on exact solutions for nonlinear integral equations by a modified homotopy perturbation method. New Trends in Mathematical Sciences. 2013;1(2):22-6.

[20] Alshammari S, Abuasad S. Exact solutions of the 3D fractional helmholtz equation by fractional differential transform method. Journal of Function Spaces. 2022;2022(1):7374751.

[21] Borhanifar A, Abazari R. Numerical study of nonlinear Schrödinger and coupled Schrödinger equations by differential transformation method. Optics Communications. 2010 May 15;283(10):2026-31.