Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s(2025)

STUDY OF GENERALIZED α- ADMISSIBLE MODIFIED ALMOST z- CONTRACTIONS VIA SIMULATION FUNCTIONS

SURENDRA KUMAR TIWATI , ANAND MOHAN DUBEY, A V SENTHIL KUMAR

Article History:

Received: 10-01-2025

Revised: 15-02-2025

Accepted: 08-03-2025

Abstract. In this paper, we introduce generalized α -admissible modified almost z -contraction with the help of simulation function and obtain fixed point results in the setting og metric space and verified with an example. The presented results extend, generalize and unify several related fixed point finding in the existing literature.

1. Introduction and preliminaries

Consider $\mathbb{N}_{\not{\circ}} = \mathbb{N} \cup \{0\}$, where $\mathbb{N}$ denotes the set of positive integers. As usual $\mathbb{R}$ indicates the set of real numbers.Furthermore, we set $R_0^+ = [0, \infty]$. Many problems in several branches of mathematics are well known to be transformed into invariant point problems in the form $Tx = x$ for self mapping T. It is worth noting that based on the work of Banach S. [7] in1922, known as the Banach contraction principle (BCP), the metric fixed point theory took off. Alot of authrs studied generalizations of this principle. In addition, Berinde [9, 10] introduced almost contractions which exhibits new features with respect to the ones of the particular results in coprated as follows:

De inition 1.1. Let (X, d) be a metric space. A self mapping Γ on X is called an almost contraction if there are constants $\delta \in [0, 1)$ and $\exists L \geq 0$ such that

$$d(\Gamma x, \Gamma y) \leq \delta d(x, y) + Ld(y, \Gamma x), \forall x, y \in X. \tag{1}$$

Berinde [9, 10] investigated that every almost contraction mapping defined in a complete metric space has at least one fixed point. Subsequently, Babu et al.[6] defined the class of mapping satisfying condition (B) as follows:

Definition 1.2. Let (X, d) be a metric space. A self mapping Γ on X issaid to be satisfy condition (B) if there are constants $\delta \in [0, 1)$ and $\exists L \geq 0$ such that

$$d(\Gamma x, \Gamma y) \leq \delta d(x, y) + LQd(y, \Gamma x), \forall x, y \in X. \tag{2}$$
$$where\ Q(x, y) = min\{d(x, \Gamma x), d(y, \Gamma y), d(x, \Gamma y), d(y, \Gamma x)\}$$

They proved a fixed point theorem for such mappings in complete metric spaces. They also discussed quasi-contraction, almost contraction and the class of mappingd that satisfy condition (B) in detail. Iseki et al. [20] presented definition of almost z- contraction as follows:

Date: *Corresponding author: Surendra Kumar Tiwari.

2020 *Mathematics Subject Classification.* 54H25,47H10,55M20, , ...

Key words and phrases. Almost $z-$- contraction, Modified Almost z-contraction, Simulation Function, α- Admissible Mapping.

Definition 1.3. Let (X,d) be a metric space and ζ. A self mapping $\Gamma : X \to X$ is called an almost z-contraction if there are constants $\theta \geq o$ such that

$$\zeta(d(\Gamma x, \Gamma y), d(x,y) + \text{Ł}Q(x,y), \forall x, y \in X, \tag{3}$$

where $Q(x,y)$ is defined as in Definition 1.2. Also they investigated the existence and uniqueness of a fixed point of an almost z- contraction in metric space with simulation functions.

Authos [25, 26, 27, 28] demostrated that almost contractions type mappings have a unique fixede point in deferent metric spaces. In sequel, P. Bunpatcharacharoen et al. [11] modified almost type z- contraction mapping in metric space and obtained fixed point. Khojsteh et al. [29] Originated the notion of z- contractions by usung a specific family of functions called simulation functions and proved a version of BCP. Subsequently, many researchers generalized this idea in many ways (See [30, 31, 32, 33, 34, 35, 36, 37, 38]) and proved various interesting results in the arena of fixed point theory by using simulation functions.

Definition 1.4. [29] A function $\zeta; [o,\infty)^2 \to \mathbb{R}$ is called a simulation function if ζ satisfies the following conditions:

(ζ_1) $\zeta(o,o) = o$;
(ζ_2) $(t,s) < s - t$ for all $t, s > o$;
(ζ_3) if $\{t_n\}$, $\{s_n\}$ are sequence in (o,∞) such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, then $\lim sup_{n\to\infty}\zeta(t_n, s_n) < 0$.

The following function $\zeta : [o,\infty) \times [o,\infty) \to \mathbb{R}$ belongs to z.

Definition 1.5. [29] A funtion $\Gamma : X \to X$ is called a z- contraction with respect to a simulation function $\zeta \in \mathcal{Z}$ on metric space(X,d), if the following condition is satisfied

$$\zeta(d(\Gamma x.\Gamma y), d(x,y)) \leq \forall\, x, y \in X. \tag{4}$$

Remark 1.6. [29] It is clear from the defintion of simulation function thar $\zeta(t,s) < o$ for all $t \geq s > o$. Therefore, if Γ is a $\mathcal{Z}$-contraction with respect to simulation functiomn ζ, then

$$d(\Gamma x, \Gamma y) < d(x,y) \ for\, all\, x, y \in X. \tag{5}$$

Theorem 1.7. [29] *Let (X,d) be a complete metric space and $\Gamma : X \to X$ be a z-contraction with respect to ζ. Then Γ has a unique fixed point $u \in X$ and for every $x_0 \in X$, the Picard sequence $\{x_n\}$ where $x_n = \Gamma x_{n-1}$ for all $n \in \mathbb{N}$ converges to this fixed point of Γ.*

It is worth mentioning that the Banch contraction is a perfect example of z-contractions by taking $(\zeta, s) = \lambda s - 1$, where $\lambda \in [0,1)$, as the corresponding simulation function.
Argoubi et al. [4] shown that the condition (ζ_1) to be reduntant one ine above definition 1.4 of simulation functionand so redefined it as :

Definition 1.8. [4] Asimulation function is a mapping $\zeta : [o,\infty)^2 \to \mathbb{R}$satisfies the foolowing conditions:
(i) $\zeta(t,s) < s - t$ for all $t, s > 0$;
(ii) if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, and $t_n < s_n$,then $\lim sup_{n\to\infty}\, \zeta(t_n, s_n) < 0$.

Roldan-Lopez-de Hierrow et al.[36] modified the notion of simulation function replacing (ζ_3) by$(\zeta_3^{'})$ of definition 1.4 as :
$(\zeta_3^{'})$ if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0, \infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, and $t_n < s_n$, then $\lim sup_{n\to\infty} \zeta(t_n, s_n) < 0$.

It is clear that, if the function ζ satisfies the conditions $(\zeta_1) - (\zeta_3)$, we say that ζ is a simulation function according to the sense of Khojasteh et al [29]. If it satisfies $(\zeta_2) - (\zeta_3)$,it is a simulation function according to the sense of Argoubi et al.[4]. and if it satis fies (ζ),(ζ_2) ,and $(\zeta_3^{'})$, then it is a simulation function according to the sense of Roldan- Lopez- de- Hierro et al. [36].

Samet et al.[39] introduced a new category of contractive type mappings known as $\alpha-\psi$ contractive type mapping. The results obtained by Samet et al.[39] extended and generalized the existing fixed point results in the literature, in particular the Banach contraction principle. Further, Karapinar E. and Samet[16] generalized the $\alpha - \psi$ comntractive type mappings and obtained various fixed point theorems for this generalized class of contractive mappings. In 2013, Hussain et al. [17] introduced α-admissible mappings and proved fixed point theorems in metric space. Subsequently, Abdeljawad[1] introduced a pair of α-admissible mappings satisfying new sufficient contractive conditions, whixh are different from those in[14, 16] and obtained fixed point and common fixed point theorems. Afterward, some authors have obtained fixed point theorems for some kinds of α-admissible mappins (see [1], 2, 3, 5, 6,7, 8,12,13,14, 15,18, 20, 21,22, 24, 38,40,41 and 42).

Definition 1.9. [39] Let $\Gamma : X \to X$ and $\alpha : X \times X \to \mathbb{R}^+$ be the functions. Then Γ is called α -admissible if $\alpha(x, y) \geq 1 \Rightarrow \alpha(\Gamma x, \Gamma y) \geq 1$,

Karapinar E. [23] introduced the notion of α- admissible z- contraction and generalized the results of Samet et al.[39]and Khojasteh et al. [29]. Very recently, Dipti et al. [43] presented some fixed point results in complete metric spaces using generalized α admissible mappings embedded in the simulation functions.

Definition 1.10. [15] Let$\Gamma : X \to X$ and $\alpha : X \times X \to \mathbb{R}^+$ be a functions. Then we say that Γ is an α-orbital admissible if $\alpha(x, \Gamma x) \geq 1$ implies $\alpha(\Gamma x, \Gamma^2 x) \geq 1$. Moreover, Γ is called a triangular α- orbital admissible if Γ-orbital admissible and $\alpha(x, y) \geq 1$ and $\alpha(y, \Gamma y) \geq 1$ implies $\alpha(x, \Gamma y) \geq 1$, forall $x, y \in X$.

Definition 1.11. [23] Let $\Gamma : X \to X$ be a self map defined on a metric space(X, d). If there exist $\zeta \in \mathbb{Z}$ and $\alpha : X \times X \to \mathbb{R}^+$ such that

$$\zeta(\alpha(x, y)d(\Gamma x, \Gamma y)d(x, y)) \geq 0, \forall x, y \in X \tag{6}$$

Then Γ is called an α-admiossible z- contraction with respect to ζ.

Theorem 1.12. [23] *Let (X, d) be a complete metric space and let $\Gamma : X \to X$ be an α-admissible z- contraction with respect to ζ. Suppose that*
(i) Γ is triangular α- orbital admissble;
(ii) there exist $x_0 \in X$ such that $\alpha(X_0, \Gamma x_0) \geq 1$;
(iii) Γ is continuous.
Then there exists $u \in X$ such that $\Gamma u = u$.

Very recently, Dipti et al. [43] presented some fixed point results in complete metric spaces using generalized α admissible mappings embedded in the simulation functions.

Definition 1.13. [43] Let(X, d) be a metric space, $\Gamma : X \to X$ be a self mapping, there exists $\zeta \in Z$ and $\alpha : X \times X \to X[0, \infty)$. Then continuous mapping Γ is called generalized α -admissible almost z- contraction with respect to ζ and $\beta \in G$ and $L \geq 0$ such that for all $x, y \in X$,

$$\zeta(\alpha(x, \Gamma x)\alpha(y, \Gamma y)d(\Gamma x, \Gamma y), K(x, y) + LQ(x, y) \geq 0 \tag{7}$$

for all distinct $x, y \in X$, where $zeta$ is a simulation function in the sense of Definition 1.
Also

$$K(x, y) = \beta(E(x, y))E(x, y) + LN(x, y) \tag{8}$$

,

where

$$E(x, y) = d(x, y) + |d(x, \Gamma x) - d(y, \Gamma y)| \tag{9}$$

and

$$N(x, y) = min\{d(x, \Gamma x), d(y, \Gamma y), d(x, \Gamma y), d(y, \Gamma x)\}. \tag{10}$$

Definition 1.14 ([11])**.** Let (X, d) be a metric space and ζ. We say that $\Gamma : X \to X$ is a modified almost type z-contraction if there are constants Ł $\geq o$ such that

$$\zeta(d(\Gamma x, \Gamma y), K(x, y) + ŁQ(x, y), \forall x, y \in X, \tag{11}$$

where,

$$K(x, y) = max\left\{d(x, y), \frac{[1 + d(x, \Gamma x)d(y, \Gamma y)]}{1 + d(x, y)}\right\}$$

and

$$Q(x, y) = min\{d(x, \Gamma x), d(y, \Gamma y), d(x, \Gamma x), d(y, \Gamma y)\}$$

Remark 1.15. If Γ is a modified almost type z-contraction with respect to $\zeta \in Z$, then

$$d(\Gamma x, \Gamma y) < K(x, y) + LQ(x, y)\, \forall x, y \in X, \tag{9}$$

Inspired and motivated by the combining the ideas in[11],[20], [25], [27] and [43], we introduce a new clas of mappings, and define generalized α -admissible modified almost z contraction with respect to ζ in the setting of metric space and obtain the existence and uniqueness of fixed point of such map.

2. Main results

Finaly, we give the following definition which will be used in our main results.

Definition 2.1. Let(X, d) be a metric space, $\Gamma : X \to X$ be a self mapping, there exists $\zeta \in Z$ and $\alpha : X \times X \to X[0, \infty)$. Then continuous mapping Γ is called generalized α-admissible modified almost z- contraction with respect to ζ and $L \geq 0$ such that for all $x, y \in X$,

$$\zeta(\alpha(x, \Gamma x)\alpha(y, \Gamma y)d(\Gamma x, \Gamma y), K(x, y) + LQ(x, y)) \geq 0 \tag{10}$$

where ζ is a simulation function in the sense of Definition 1.4.
Also

$$K(x, y) = max\left\{d(x, y), \frac{[1 + d(x, \Gamma x)]d(y, \Gamma y)}{1 + d(x, y)}, \frac{[1 + d(x, \Gamma y)]d(y, \Gamma x)}{1 + d(x, y)}\right\} \tag{11}$$

and

$$Q(x, y) = min\left\{d(x, \Gamma x), d(y, \Gamma y), d(x, \Gamma y), d(y, \Gamma x), \frac{d(x, \Gamma y)d(y, \Gamma x)}{1 + d(x, y)}, \frac{d(x, \Gamma x)d(y, \Gamma y)}{1 + d(x, y)}\right\}. \tag{12}$$

We can now state the main finding of this paper.

Theorem 2.2. *Let (X, d) be a complete metric space and $\Gamma : X \to X$ is a generalized α-admissible modified almost z- contraction with respect to ζ. Furthermore, we suppose for all $x, y \in X$ such that:*
(i) Γ *is triangular α- orbital admissible;*
(ii) *there exists $x_0 \in X$such that $\alpha(x_0, \gamma x_0) \geq 1$;*
(iii) Γ *is continuous.*
(iv) $\alpha(x, \Gamma x) \geq 1$
Then Γ has a unique fixed point $x^* \in X$.

Proof. By(ii), there exists $x_0 \in X$ such that $\alpha(x_0.\Gamma x_0) \geq 1$,and let $\{x_n\}$ be the iterative sequence Xdefined by

$$x_{n+1} = \Gamma x_n, forall\ n \in \mathbb{N} \tag{13}$$

If there exists some nonnegative integer n such that $x_n = x_{n+1} = \Gamma x_n$, then x_n is a fixed point of Γ. Therfore, to continue our proof, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Since Γ is an α- admissible mapping, we have

$$\alpha(x_0, x_1) = \alpha(x_0, \Gamma x_0) \Rightarrow \alpha(\Gamma x_0, \Gamma x_1) = \alpha(x_1, x_2) \geq 1. \tag{14}$$

By induction, we get

$$\alpha(x_n, x_{n+1}) \geq 1, forall\ n \in \mathbb{N} \cup \{0\}. \tag{15}$$

Applying the codition (10), putting $x = x_{n-1}$ and $y = x_n$ and by using (15), we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_{n-1}, \Gamma x_{n-1}), \alpha(x_n, \Gamma x_n), d(\Gamma x_{n-1}, \Gamma x_n), K(x_{n-1}, x_n) + LQ(x_{n-1}, x_n)) \\ &= \zeta(\alpha(x_{n-1}, x_n), \alpha(x_n, x_{n+1}), d(x_n, x_{n+1})K(x_{n-1}, x_n) + LQ(x_{n-1}, x_n) \\ &< K(x_{n-1}, x_n) + LQ(x_{n-1}, x_n) - \alpha(x_{n-1}, x_n), \alpha(x_n, x_{n+1}), d(x_n, x_{n+1}) \end{aligned} \tag{16}$$

Also, where

$$K(x_{n-1},x_n) = max\left\{d(x_{n-1},x_n), \frac{[1+d(x_{n-1},\Gamma x_{n-1})]d(x_n,\Gamma x_n)}{1+d(x,y)}, \frac{[1+d(x_{n-1},\Gamma x_n)]d(x_n,\Gamma x_{n-1})}{1+d(x,y)}\right\}$$
$$\leq max\left\{d(x_{n-1},x_n), \frac{[1+d(x_{n-1},x_n)]d(x_n,x_{n+1})}{1+d(x_{n-1},x_n)}, \frac{[1+d(x_{n-1},x_{n+1})]d(x_n,x_n)}{1+d(x_{n-1},x_n)}\right\}$$
$$= max\{d(x_{n-1},x_n), d(x_n,x_{n+1})\} \tag{17}$$

and

$$Q(x_{n-1},x_n) = min\{d(x_{n-1},\Gamma x_{n-1}), d(x_n,\Gamma x_n), d(x_{n-1},\Gamma x_n), d(x_n,\Gamma x_{n-1}),$$
$$\frac{d(x_{n-1},\Gamma x_n)d(x_n,\Gamma x_{n-1})}{1+d(x_{n-1},x_n)}, \frac{d(x_{n-1},\Gamma x_{n-1})d(x_n,\Gamma x_n)}{1+d(x_{n-1},x_n)}\}$$
$$\leq min\{d(x_{n-1},x_n), d(x_n,x_{n+1}), d(x_{n-1},x_{n+1}), d(x_n,x_n), \frac{d(x_{n-1},x_{n+1})d(x_n,x_n)}{1+d(x_{n-1},x_n)},$$
$$\frac{d(x_{n-1},x_n)d(x_n,x_{n+1})}{1+d(x_{n-1},x_n)}\}$$
$$\leq min\left\{d(x_{n-1},x_n), d(x_n,x_{n+1}), d(x_{n-1},x_{n+1}), 0, 0, \frac{d(x_{n-1},x_n)d(x_n,x_{n+1})}{1+d(x_{n-1},x_n)}\right\}$$
$$=0. \tag{18}$$

By (16), and taking in account (15),(17) ,and(18), we derive that

$$0 < max\{d(x_{n-1},x_n), d(x_n,x_{n+1})\} - \alpha(x_{n+1},x_n), \alpha(x_n,x_{n+1})d(x_n,x_{n+1}) \tag{19}$$

which implies that

$$d(x_n,x_{n+1} \leq \alpha(x_{n-1},x_n), \alpha(x_n,x_{n+1})d(x_n,x_{n+1})$$
$$< max\{d(x_{n-1},x_n), d(x_n,x_{n+1})\}\ \forall n \geq 1. \tag{20}$$

If max $\{d(x_{n-1},x_n), d(x_n,x_{n+1})\} = d(x_n,x_{n+1})$ for some $n \geq 1$, then from (20), we get

$$d(x_n,x_{n+1}) \leq \alpha(x_{n-1},x_n), \alpha(x_n,x_{n+1})d(x_n,x_{n+1}) < d(x_n,x_{n+1}). \tag{21}$$

which is contradiction, therefore,

$$max\{d(x_{n-1},x_n), d(x_n,x_{n+1})\} = d(x_{n-1},x_n) \tag{22}$$

Hence

$$d(x_n,x_{n+1}) \leq \alpha(x_{n-1},x_n), \alpha(x_n,x_{n+1}), d(x_n,x_{n+1})$$
$$< d(x_{n-1},x_n). \tag{23}$$

Consequently, we deduce that $\{d(x_{n-1},x_n)\}$ is a monotonically decreasing sequence for nonnegative reals and bounded below by zero. so, there esists $r \geq 0$ such that

$$\lim_{n\to\infty} d(x_{n-1},x_n) = r \tag{24}$$

we claim that

$$\lim_{n\to\infty} d(x_{n-1}, x_n) = 0. \tag{25}$$

$$\tag{26}$$

On the contrary, assume that $r > 0$ and using equation(23) we have the following

$$\lim_{n\to\infty} \alpha(x_{n-1}, x_n), \alpha(x_n, x_{n+1}), d(x_n, x_{n+1}) = r \tag{27}$$

Now, we take

$$t_n = \{\alpha(x_{n-1}, x_n), \alpha(x_n, x_{n+1}), d(x_n, x_{n+1}) \; and \; s_n = \{d(x_{n-1}, x_n)\}.$$

then

$$\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = r \tag{28}$$

Since T is a generalized α- admissible modified almost z- contraction with respect to $\zeta \in Z$. Therfore, by(ζ_3) and equation (21) and taking limit as $n \to \infty$, we have $\lim sup_{n\to\infty} \zeta(t_n, s_n) < 0$ i.e.

$$0 \le \lim sup_{n\to\infty} \zeta(\alpha(x_{n-1}, x_n), \alpha(x_n, x_{n+1}), d(x_n, x_{n+1}), d(x_{n-1}, x_n) < 0 \tag{29}$$

This is a contradiction. Then we deduce that $r = 0$, that is, we have following

$$\lim_{n\to\infty} d(x_{n-1}, x_n) = 0 \tag{30}$$

Now, we will show that sequence $\{x_n\}$ is aCauchy sequence in X.Assume that $\{x_n\}$ is not a Cauchy sequence, then there exists $\epsilon > 0$ and two sequences $\{x_{n_k}\}$, $\{x_{m_k}\}$: $m_k > n_k > k$ such that

$$d(x_{m_k}, x_{n_k}) \le \epsilon. \tag{31}$$

$$and$$

$$d(x_{m_k}, x_{n_k-1}) \le \epsilon, for \, all \, m \, , n, \, k \in \mathbb{N} \tag{32}$$

By applying the triangal inequality and using equations (30) and (31), we get the following

$$\begin{aligned} \epsilon &< d(x_{m_k}, x_{n_k}) \\ &\le d(x_{m_k}, x_{n_k-1}) + d(x_{n_k-1}, x_{n_K}) \\ &\le d(x_{n_k-1}, x_{n_K}) + \epsilon \end{aligned} \tag{33}$$

Taking $k \to \infty$ in equation (33) and using equation(30), we get

$$\lim_{n\to\infty} d(x_{m_k}, x_{n_k}) = \epsilon. \tag{34}$$

Again, using the triangal inequality, we have

$$\begin{aligned} d(x_{m_k}, x_{n_k}) &\le d(x_{m_k}, x_{n_k-1}) + d(x_{n_k-1}, x_{n_k}) \\ &\le d(x_{m_k}, x_{n_k-1}) + d(x_{m_k-1}, x_{n_k-1}) + d(x_{n_k-1}, x_{n_k}) \end{aligned} \tag{35}$$

Again, we have

$$d(x_{m_k-1}, x_{n_k-1}) \le d(x_{m_k-1}, x_{m_k}) + d(x_{m_k}, x_{n_k-1}) + d(x_{m_k-1}, x_{m_k}) \tag{36}$$

By taking the limit as $k \to \infty$ in equation (35) ,(36) ,and using (30) we deduce that

$$\lim_{n\to\infty} d(x_{m_k-1}, x_{n_k-1}) < \epsilon. \tag{37}$$

By the same reasoning as above, we get that

$$\lim_{n\to\infty} d(x_{m_k}, x_{n_k-1}) = \lim_{n\to\infty} d(x_{m_k-1}, x_{n_k}) = \epsilon. \tag{38}$$

Since T is triangular α-orbital admisssible, we have

$$\alpha d(x_{m_k-1}, x_{n_k-1}) \le 1. \tag{39}$$

Moreover, since T is a generalized α -admissible modified almost z-contraction with respect to ζ,

$$\begin{aligned}0 &\le \zeta(\alpha(x_{m_k-1}, \Gamma x_{m_k-1}), \alpha(x_{n_k-1}, \Gamma x_{n_k-1}), d(\Gamma x_{m_k-1}, \Gamma x_{n_k-1}) K(x_{m_k-1}, x_{n_k-1}) + LQ(x_{m_k-1}, x_{n_k-1}) \\ &= \zeta(\alpha(x_{m_k-1}, x_{m_K}), \alpha(x_{n_k-1}, x_{n_k}), d(x_{m_k}, x_{n_k}), K(x_{m_k-1}, x_{n_k-1}) + LQ(x_{m_k-1}, x_{n_k-1})\end{aligned}$$

It follows from condition(ζ_2), we get

$$0 < K(x_{m_k-1}, x_{n_k-1}) + LQ(x_{m_k-1}, x_{n_k-1}) - \zeta(\alpha(x_{m_k-1}, x_{m_K}), \alpha(x_{n_k-1}, x_{n_k}), d(x_{m_k}, x_{n_k}) \tag{40}$$

Hence,

$$\begin{aligned}0 < d(x_{m_k}, x_{n_k}) &\le \alpha(x_{m_k-1}, x_{m_k}), \alpha(x_{n_k-1}, x_{n_k}) d(x_{m_k}, x_{n_k}) \\ &< K(x_{m_k-1}, x_{n_k-1}), +LQ(x_{m_k-1}, x_{n_k-1})\end{aligned} \tag{41}$$

Also, where

$$\begin{aligned}K(x_{m_k-1}, x_{n_k-1}) &= max\{d(x_{m_k-1}, x_{n_k-1}), \frac{[1 + d(x_{m_k-1}, \Gamma x_{m_k-1})] d(x_{n_k-1}, \Gamma x_{n_k-1})}{1 + d(x_{m_k-1}, x_{n_k-1})} \\ &\quad \frac{[1 + d(x_{m_k-1}, \Gamma x_{n_k-1})] d(x_{n_k-1}, \Gamma x_{m_k-1})}{1 + d(x_{m_k-1}, x_{n_k-1})}\} \\ &= max\{d(x_{m_k-1}, x_{n_k-1}), \frac{[1 + d(x_{m_k-1}, x_{m_k})] d(x_{n_k-1}, x_{n_k})}{1 + d(x_{m_k-1}, x_{n_k-1})} \\ &\quad \frac{[1 + d(x_{m_k-1}, x_{n_k})] d(x_{n_k-1}, x_{m_k})}{1 + d(x_{m_k-1}, x_{n_k-1})}\}.\end{aligned} \tag{42}$$

and,

$$\begin{aligned}Q(x_{m_k-1}, x_{n_k-1}) &= min\{d(x_{m_k-1}, \Gamma x_{m_k-1}), d(x_{n_k-1}, \Gamma x_{n_k-1}), d(x_{m_k-1}, \Gamma x_{n_k-1}), d(x_{n_k-1}, \Gamma x_{m_k-1}), \\ &\quad \frac{d(x_{m_k-1}, \Gamma x_{n_k-1}) d(x_{m_k-1}, \Gamma x_{m_k-1})}{1 + d(x_{m_k-1}, x_{n_k-1})}, \frac{d(x_{m_k-1}, \Gamma x_{m_k-1}) d(x_{n_k-1}, \Gamma x_{n_k-1})}{1 + d(x_{m_k-1}, x_{n_k-1})}\} \\ &= min\{d(x_{m_k-1}, x_{m_k}), d(x_{n_k-1}, x_{n_k}), d(x_{m_k-1}, x_{n_k}), d(x_{n_k-1}, x_{m_k}), \\ &\quad \frac{d(x_{m_k-1}, x_{n_k}) d(x_{m_k-1}, x_{m_k})}{1 + d(x_{m_k-1}, x_{n_k-1})}, \frac{d(x_{m_k-1}, x_{m_k}) d(x_{n_k-1}, x_{n_k})}{1 + d(x_{m_k-1}, x_{n_k-1})}\}\end{aligned} \tag{43}$$

Taking limit as$k \to \infty$ in (42),(43) using (30), (34), (37) and (38), we get

$$\lim_{k\to\infty} K(x_{m_k-1}, x_{n_k-1}) = \epsilon. \tag{44}$$

and,

$$\lim_{k\to\infty} Q(x_{m_k-1}, x_{n_k-1}) = 0. \tag{45}$$

From (34), (37) ,(40), and (44) (45), also condition (ζ_3), we get

$$0 \leq \lim sup_{k\to\infty}\zeta(\alpha(x_{m_k-1}, x_{m_k}), \alpha(x_{n_k-1}, x_{n_k}), d(x_{m_k}, x_{n_k})K(x_{m_k-1}, x_{n_k-1}) + LQ(x_{m_k-1}, x_{n_k-1})$$
$$\leq \lim sup_{k\to\infty}\zeta(\alpha(x_{m_k-1}, x_{m_k}), \alpha(x_{n_k-1}, x_{n_k}), d(x_{m_k}, x_{n_k})K(x_{m_k-1}, x_{n_k-1}) < 0.$$

This is a contradiction. Hence $\{x_n\}$ is a Cauchy sequence. Since (X, d) is complete metric space, there exists $x^* \in X$ such that

$$\lim_{n\to\infty} d(x_n, x^*) = 0. \tag{46}$$

Now, we shall show that $\Gamma x^* = x^*$. Since Γ is continuous, we obtain that

$$\begin{aligned}\Gamma x^* &= \Gamma(\lim_{n\to\infty} x_n)\\ &= \lim_{n\to\infty} \Gamma(x_n)\\ &= \lim_{n\to\infty} x_{n+1}\\ &= x^*.\end{aligned}$$

Thus, x^* is a fixed point of Γ. To prove the uniqueness of the fixed point.Assume that, x^* andy^* be two fixed point of Γ and hence $x^*, y^* \in Fix(\Gamma)$,which is a generalized α-admissible modified self mapping of metric space$X, d)$. Then $d(x^*, y^*) > 0$. By (10), we have that

$$0 \leq \zeta(\alpha(x^*, \Gamma x^*), \alpha(y^*, \Gamma y^*)d(\Gamma x^*, \Gamma y^*), K(x^*, y^*) + LQ(x^*, y^*) \tag{47}$$

where $K(x^*, y^*) = d(x^*, y^*)$ and $Q(x^*, y^*) = 0$. Then, by (47), we get

$$\begin{aligned}&0 \leq \zeta(\alpha(x^*, x^*), \alpha(y^*, y^*), d(x^*, y^*)K(x^*, y^*) + LQ(x^*, y^*)\\ &\quad < K(x^*, y^*) + LQ(x^*, y^*) - \alpha(x^*, x^*), \alpha(y^*, y^*), d(x^*, y^*)\\ &0 < d(x^*, y^*) - d(x^*, y^*)\\ &\quad = 0.\end{aligned} \tag{48}$$

which is a contradiction.Thus we have $x^* = y^*$. Hence Γ has a unique fixed point. □

Theorem 2.3. *Let (X, d) be a complete metric space and $\Gamma : X \to X$ is a generalized α-admissible modified almost z- contraction with respect to ζsatisfying the following conditions:*
(i) Γ is triangular α- orbital admissible;
(ii) there exists $x_0 \in X$such that $\alpha(x_0, \Gamma x_0) \geq 1$;
(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \leq 1$ for all $n \in \mathbb{N} \cup \{0\}$ and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n_{(k)}}\}$ of $\{x_n\}$ such that $\alpha(x_{n_{(k)}}, x^*) \leq 1$;
(iv) $\alpha(x, y) \geq 1$, for all $x, y \in Fix(\Gamma)$, where $Fix(\Gamma)$ denotes the set of fixed point of Γ. Then, Γ has a unique fixed point $x^* \in X$.

Proof. By(ii), suppose $x_0 \in X$ such that $\alpha(x_0, \Gamma x_0) \geq 1$. there exists $x_n \in X$ such that $x_{n+1} = \Gamma x_n$, for all $n \in \mathbb{N}$. We have by Theorem 2.2, $\{x_n\}$ is a Cauchy sequence such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$. Since (X, d) is complete, there exists $x^* \in X$ such that $x_n \to x^*$.By (15) and the condition (iii) , there exists a subsequence $\{x_{n_{(k)}}\}$ of $\{x_n\}$ such that $\alpha(x_{n_{(k)}}, x^*) \leq 1$ for all $k \in \mathbb{N}$. Using (10), we have

$$\begin{aligned}0 &\leq \zeta(\alpha(x_{n_k}, \Gamma x_{n_k}), \alpha(x^*, \Gamma x^*), d(\Gamma x_{n_k}, \Gamma x^*)K(x_{n_k}, x^*) + LQ(x_{n_k}, x^*) \\ &= \zeta(\alpha(x_{n_k}, x_{n_k+1}), \alpha(x^*, \Gamma x^*)d(x_{n_k+1}, \Gamma x^*), K(x_{n_k}, x^*) + LQ(x_{n_k}, x^*) \\ &< K(x_{n_k}, x^*) + LQ(x_{n_k}, x^*) - \alpha(x_{n_k}, x_{n_k+1}), \alpha(x^*, \Gamma x^*), d(x_{n_k+1}, \Gamma x^*). \quad (49)\end{aligned}$$

Hence,

$$\begin{aligned}d(x_{n_k+1}, \Gamma x^*) &\leq \alpha(x_{n_k}, x_{n_k+1}), \alpha(x^*, \Gamma x^*)d(x_{n_K+1}, \Gamma x^*) \\ &< K(x_{n_k}, x^*) + LQ(x_{n_k}, x^*). \quad (50)\end{aligned}$$

Also, where

$$\begin{aligned}K(x_{n_k}, x^*) &= max\{d(x_{n_k}, x^*), \frac{[1 + d(x_{n_k}, \Gamma x_{n_k})]d(x^*, \Gamma x^*)}{1 + d(x_{n_k}, x^*)} \\ &\quad \frac{[1 + d(x_{n_k}, \Gamma x^*)]d(x^*, \Gamma x_{n_k})}{1 + d(x_{n_k}, x^*)}\} \\ &= max\{d(x_{n_k}, x^*), \frac{[1 + d(x_{n_k}, x_{n_k+1})]d(x^*, \Gamma x^*)}{1 + d(x_{n_k}, x^*)}, \\ &\quad \frac{[1 + d(x_{n_k}, \Gamma x^*)]d(x^*, x_{n_k+1})}{1 + d(x_{n_k}, x^*)}\}. \quad (51)\end{aligned}$$

and

$$\begin{aligned}Q(x_{n_k}, x^*) &= min\{d(x_{n_k}, \Gamma x_{n_k}), d(x^*, \Gamma x^*), d(x_{n_k}, \Gamma x^*), d(x^*, \Gamma x_{n_k}), \\ &\quad \frac{d(x_{n_k}, \Gamma x^*)d(x^*, \Gamma x_{n_k})}{1 + d(x_{n_k}, x^*)}, \frac{d(x_{n_k}, \Gamma x^*)d(x^*, \Gamma x^*)}{1 + d(x_{n_k}, x^*)}\} \\ &= min\{d(x_{n_k}, x_{n_k+1}), d(x^*, \Gamma x^*), d(x_{n_k}, \Gamma x^*), d(x^*, x_{n_k+1}), \frac{d(x_{n_k}, \Gamma x^*)d(x^*, x_{n_k+1})}{1 + d(x_{n_k}, x^*)}, \\ &\quad \frac{d(x_{n_k}, \Gamma x^*)d(x^*, \Gamma x^*)}{1 + d(x_{n_k}, x^*)}\}. \quad (52)\end{aligned}$$

Taking $k \to 0$ in the equation(50) and(51) we derive that

$$K(x_{n_K}, x^*) = d(x^*, \Gamma x^*) \ and\ Q(x_{n_k}, x^*) = 0. \quad (53)$$

From (50) ,by using (53), we get

$$d(x_{n_k+1}, \Gamma x^*) < d(x^*, \Gamma x^*) for\ all\ k \in \mathbb{N} \quad (54)$$

By (49), (54), and the condition (ζ_3), we have

$$0 \leq \lim sup_{n\to\infty}\zeta(\alpha(x_{n_k}, \Gamma x_{n_k}), \alpha(x^*, \Gamma x^*), d(\Gamma x_{n_k}, \Gamma x^*)K(x_{n_k}, x^*) + LQ(x_{n_k}, x^*) < 0.$$

This is contradiction.Hence therefore, x^* is a fixed point og Γ. Now ,assume that, there exists $x^*, y^* \in X$ such that $x^* = \Gamma x^*$ and $y^* = \Gamma y^*$ with $x^* \neq y^*$. Since Γ is Generalized α-admissible modified almost z contraction selfmapping of a metric space (X, d). So by assumption (iv) from Theorem 2.3, we hav

$$\alpha(x^\star, y^\star) \geq 1. \quad (55)$$

Therefore, from (10) and $|zeta_2$ that,

$$\begin{aligned}0 &\leq \zeta(\alpha(x^\star, \Gamma x^\star), \alpha(y^\star, \Gamma y^\star)d(\Gamma x^\star, \Gamma y^\star), K(x^\star, y^\star) + LQ(x^\star, y^\star) \\ &= \zeta(\alpha(x^\star, x^\star), \alpha(y^\star, y^\star), d(x^\star, y^\star)K(x^\star, y^\star) + LQ(x^\star, y^\star) \\ &< K(x^\star, y^\star) + LQ(x^\star, y^\star) - \alpha(x^\star, x^\star), \alpha(y^\star, y^\star), d(x^\star, y^\star) \quad (56)\end{aligned}$$

Also , where

$$\begin{aligned} K(x^\star, y^\star) &= max\left\{d(x^*, y^*)\frac{[1+d(x^*,x^*)]d(y^*,y^*)}{1+d(x^*,y^*)}, \frac{[1+d(x^*,y^*)]d(y^*,x^*)}{1+d(x^*,y^*)}\right\} \\ &= max\{d(x^*,y^*), d(Y^*,x^*)\} \\ &= d(x^*,y^*) \end{aligned} \tag{57}$$

and

$$\begin{aligned} Q(x^\star, y^\star) &= min\{d(x^*,x^*), d(y^*,y^*), d(x^*,y^*), d(y^*,x^*), \frac{d(x^*,y^*)d(y^*,x^*)}{1+d(x^*,y^*)}, \\ &\quad \frac{d(x^*,x^*)d(y^*,y^*)}{1+d(x^*,y^*)}\} \\ &= min\left\{0, 0, d(x^*,y^*), d(y^*,x^*), \frac{d(x^*,y^*)d(y^*,x^*)}{1+d(x^*,y^*)}, 0,\right\} \\ &= 0. \end{aligned} \tag{58}$$

From (56) together with (57) and (58), we deduce that

$$0 < d(x^*,y^*) \leq \alpha(x^*,x^*), \alpha(y^*,y^*)d(x^*,y^*) < d(x^*,y^*).$$

this is contradiction. Thus , we have $x^\star = y^\star$. Hence, Γ has a unique fixed point. □

Corollary 2.4. *Let (X,d) be a metric space, $\Gamma : X \to X$ is a generalized α-admissible modified almost z- contraction self mapping. There exists $\zeta \in Z$ and $\alpha : X \times X \to [0,\infty)$ be a function with $\alpha(x,\Gamma x) = 1$,and $\alpha(y,\Gamma y) = 1$, for all $x, y \in X$ such that*

$$\zeta(d(\Gamma x, \Gamma y)K(x,y) + LQ(x,y) \geq 0, forall\, x,y \in X, andL \geq 0,$$

Also where

$$K(x,y) = max\left\{d(x,y), \frac{[1+d(x,\Gamma x)]d(y,\Gamma y)}{1+d(x,y)}, \frac{[1+d(x,\Gamma y)]d(y,\Gamma x)}{1+d(x,y)}\right\}$$

and

$$Q(x,y) = min\left\{d(x,\Gamma x), d(y,\Gamma y), d(x,\Gamma y), d(y,\Gamma x), \frac{d(x,\Gamma y)d(y,\Gamma x)}{1+d(x,y)}, \frac{d(x,\Gamma x)d(y,\Gamma y)}{1+d(x,y)}\right\}.$$

Then Γ has a unique fixed point $x^\star \in X$.

Corollary 2.5. *Let (X,d) be a metric space, $\Gamma : X \to X$ is a generalized α-admissible modified almost z- contraction self mapping. There exists $\zeta \in Z$ and $\alpha : X \times X \to [0,\infty)$ be a function with $\alpha(x,\Gamma x) = 1$, $\alpha(y,\Gamma y) = 1$,and $Q = 0$. for all $x, y \in X$ such that*

$$\zeta(d(\Gamma x, \Gamma y)K(x,y) \geq 0, forall\, x,y \in X, andL \geq 0,$$

Also where

$$K(x,y) = max\left\{d(x,y), \frac{[1+d(x,\Gamma x)]d(y,\Gamma y)}{1+d(x,y)}, \frac{[1+d(x,\Gamma y)]d(y,\Gamma x)}{1+d(x,y)}\right\}$$

Then Γ has a unique fixed point $x^\star \in X$.

Example 2.6. Let $[0,4]$ be endowed with metric space $d(x,y)=|x-y|$ for all $x,y\in X$ and $\Gamma : X\to X$ be defined by $\Gamma x = 4-x$. Consider $\zeta(t,s)=\alpha s-t$, where $\alpha\in[0,1)$, for all $t\geq 0$ and $L\geq 0$ and $\alpha : X\times X\to[0,\infty)$ be defined by

$$\alpha(x,y)=\left\{\begin{array}{cc} 1 & if,\ x,y\in[0,1] \\ 0, & otherwise \end{array}\right\}$$

Note that Γ is triangular α-orbital admissible if $\alpha(x,\Gamma x)\geq 1 \Rightarrow \alpha(\Gamma x,\Gamma^2 x)\geq 1$, and $\alpha(x,y)\geq 1$ and $\alpha(y,\Gamma y)\Rightarrow\alpha(x,\Gamma y)\geq 1$. Since $\alpha(x,y)>1$ and $x,y\in[0,1]$. Then ,we have $\alpha(x,\Gamma x)=(x,4)=1$ and $\alpha(y,\Gamma y)=1$ for all $x,y\in X$.
In fact, for all $x\neq y$, then we have

$$\begin{aligned}\zeta(d(\Gamma x,\Gamma y)d(x,y)) &= \alpha\,|x-y|-|4-x-(4-y)| \\ &= \alpha\,|x-y|-|x-y| \\ &< |x-y|-|x-y| \\ &= 0\end{aligned}$$

Now we show that Γ is a generalized α admissible modified almost z -contractionwith respect to $\zeta\in Z$ .Now

$$\begin{aligned}\zeta(d(\Gamma x,\Gamma y)K(x,y))+LQ(x,y) &= K(x,y)+LQ(x,y)-d(\Gamma x,\Gamma y) \\ &= \alpha\left[|K(x,y)+LQ(x,y)|\right]-|4-x-(4-y)|\,, \\ &= \alpha\left[|K(x,y)+LQ(x,y)|\right]-|x-y|\,,\end{aligned}$$

wher

$$\begin{aligned}K(x,y) &= max\left\{|x-y|\,,\frac{[1+|x-(4-x)|]\,|y-(4-y)|}{1+|x-y|},\frac{[1+|x-(4-y)|]\,|y-(4-x)|}{1+|x-y|}\right\} \\ &= max\left\{|x-y|\,,\frac{[1+|2x-4|]\,|2y-4|}{1+|x-y|},\frac{[1+|x-4+y|]\,|y-4-x|}{1+|x-y|}\right\}\end{aligned}$$

and,

$$\begin{aligned}Q(x,y) &= min\{|x-(4-x)|\,,|y-(4-y)|\,,|x-(4-y)|\,,|y-(4-x)|\,,\frac{|x-(4-y)|\,.\,|y-(4-x)|}{1+|x-y|} \\ &\quad \frac{|x-(4-y)|\,.\,|y-(4-x)|}{1+|x-y|}\} \\ &= min\left\{|2x-4|\,,|2y-4|\,,|x+y-4|\,,\frac{|x+y-4|}{1+|x-y|},\frac{|2x-4|\,.\,|2y-4|}{1+|x-y|}\right\}.\end{aligned}$$

We deduce that

$$\begin{aligned}\zeta(d(\Gamma x,\Gamma y)K(x,y))+LQ(x,y) &= \alpha[max\left\{|x-y|\,,\frac{[1+|2x-4|]\,|2y-4|}{1+|x-y|},\frac{[1+|x+y-4|]\,|x+y-4|}{1+|x-y|}\right\} \\ &\quad + Lmin\left\{|2x-4|\,,|2y-4|\,,|x+y-4|\,,\frac{|x+y-4|}{1+|x-y|},\frac{|2x-4|\,.\,|2y-4|}{1+|x-y|}\right\}] \\ &\quad -|x-y|\,.\end{aligned}$$

Hence, we get two cases:
Case(i): If $x=y$, then,

$$\begin{aligned}\zeta(d(\Gamma x,\Gamma y)K(x,y))+LQ(x,y) &= \alpha[\{[1+|2x-4|]\,|2x-4|\,,[1+|2x-4|]\,|2x-4|\} \\ &\quad + L\{|2x-4|\,,|2y-4|\,,|2x-4|\,,|2x-4|\,,|2x-4|\,.\,|2y-4|\}]; \\ &= \alpha\{[1+|2x-4|]\,|2x-4|+L\,|2x-4|\}\geq 0.\end{aligned}$$

Case(ii): Without loss of generality, suppose that$x > y$, then

$$\begin{aligned}\zeta(d(\Gamma x, \Gamma y)K(x,y)) + LQ(x,y) &= \alpha[\left\{\frac{[1+|2x-4|]\,|2y-4|}{1+|x-y|}, \frac{[1+|2x-4|]\,|2y-4|}{1+|x-y|}\right\} \\ &+ L\left\{|2y-4|\,, |2y-4|\,, |2y-4|\,, \frac{|2y-4|}{1+|x-y|}, \frac{|2y-4|\,.\,|2y-4|}{1+|x-y|}\right\}] \\ &- |x-y|\,. \\ &= \alpha\frac{[1+|2x-4|]\,|2y-4|}{1+|x-y|} + \alpha L\,|2y-4| - |x-y|\,.\end{aligned}$$

If $\alpha = \frac{1}{2}$ and $L = 10$, then we get

$$\zeta(d(\Gamma x, \Gamma y)K(x,y)) + LQ(x,y) = \frac{1}{2}\frac{[1+|2x-4|]\,|2y-4|}{1+|x-y|} + 5\,|2y-4| - |x-y|\,. \tag{59}$$

Therefore, Γ is generalized α admissible modified almost$z$-contraction with respect to $\zeta \in Z$. Hence, all the assumptions of Theorem 2.2 with Corollary 2.4 and Corollary 2.5 are satisfied. hence, Γ has a unique fixed point.

References

1. T. Abdeljawad :Meir -Keeler α contractive fixed and common fixed point theorems, Fixed Point Theory and Appl. 18 (2013).
2. T. Abdeljawad and D. Gopal: Erratum to meir -Keeler α contractive fixed and common fixed point theorems, Fixed Point Theory and Appl.110 (2013).
3. A.S. Alharbi, H. Alsulami and E. Karapinar: On the power of simulation and admissible functions in metric fixed point theory, Jour. Func. Spaces, Vol. 2017, Article ID 2068163, 7 pages.
4. H.Argoubi, B.Samet, and C. Vetro: Nonlinear contractions involving simulation functions in a metric space with a partial order, Jour. Nonlinear Sci. Appl., 8(2015), 1082-1094.
5. H. Aydi, A.FelhiI, E. Karapinar and F.A. Alojail: Fixed points on quasi-metric spaces via simulation functions and conesequences, Jour. Math. Anal., 9 (2) (2018). 10-24.
6. G.V.R. Babu, M.L. Sandhya and M.V.R. Kameshwari: note on a fixed point theorems of berinde on weak contractions, Carpathian J. Math. 24 (2008), 8-12.
7. S.Banach: Sur less operattions dans les ensembles abstaits et leur application aux equations integrales, Fund. Math., 2(1922), 133-181.
8. M. Berzing, and M. D. Rus: Fixed point theorems for α-contractive mappings of Meir-Keeler type and application, Non linear Anal. Model. Control.,19(2),(2014),178-198.
9. V. Berinde: Approximating fixed points of weak contractions using the Picard iteration, Nonlinear Anal. Forum , 9(1) (2004), 43-53.
10. V. Berinde: General constructive fixed point theorems for Ciric type almost contractions in metric spaces, Carpathian J. Math. 24(2) (2008), 10-19.
11. P. Bunpatcharacharoen, S. Saelee and P.Sipara: Modified almost type $z-$ contraction, Thai Journal of Mathematics, 18(1)(2020), 252-260.
12. A. Chanda, B. Damjanovic, and L.K.Dey: Fixed point results on θ- metric spaces via simulation functions, Filomat, 31(11),(2017), 3365-3375.
13. S-H, Cho: Fixed point theorem for $(\alpha, \beta) - z$-contractions in metric spaces, Int. J. OF Math. Anal. 13(4), (2019), 161-174.
14. A. Dewangan, A.K.Dubey, M.D. Pandey and R.P.Pandey: Fixed point for (α, β)- admissible mapping via simulation functions, Communications in mathematics and applications, 12(4),(2021), 1101-1111.
15. A. Felhi, H. Adyi and D. Zhang: Fixed points for α admissible contraction mapping via simulation functions, Journal of Nonlinear Sciences and applications,9(10), (2016), 5544-5560.

16. D.Gopal:Fixed points of α-type F -contractive mappingss with an application to nonlinear fractional differential equation, Acta Mathematica Scientica,36(2016),957-970.
17. N. Hussain, P. Salimi , and A. Latif: Fixed point results for single and set valued $\alpha-\eta-\psi$ contraction mappings, Fixed Pont Theory Application, 212(2013).
18. N.Hussain, E.Karapinar, P.Salimi, and F.Akbar:α- admissible mappings and related fixed point theorem, Fixed Point Theory appl., 2012 (2013).
19. N. Hussain, M. Arshad, A. Shoaib ,and Fabimuddin: Common fixed point results for $\alpha-\psi$-contractions on a metric space endowed with a graph, J. Inequality appl., 136(2014).
20. H. Isik, N.B.Gungor, C. Park, and S.Y.Jang: Fixe point theorems for almost Z-contractions with an application, $\sum$ mathematics,6(208), 37,1-8.doi:10.3390/math6030037
21. E. Karapinar, and B.Samet: Generalized (α, ψ) Contractive type mappings and related fixed point theorems with applications, Abstra.Appl. Anal. 2012,(2012) Article ID 793486.
22. E. Karapinar, P. Kumam, and P. Salimi: On $\alpha-\psi$- Meir -Keeler contractive mappings, Fixed point theory and Application, (2013), Article ID 94 (2013).
23. E.Karapinar: Fixe point results via simulation functions, Filomat 30(8)(2016),2343-2350.
24. E. Karapinar: $\alpha-\psi$-Geraghty contraction type mappings and some related fixed point results, Filomat 28(1),(2014),37-48.
25. E. Karapinar V.M.L. Hima Bindu: Discussion on the almost z contraction, Open Mathematics, 18(2020), 448-457.
26. A. Dewangan, A.K.Dubey U.Mishra and R.P.Dubey: Fixed point results for $(\alpha-\beta)$ admissible almost z- contractions in metyric like space via simulation function, Facta Universitats(NIS) Ser. Math. Inform. ,37(3), (2022), 529-540.
27. S. Melliani, A. Moussaoui, and L..S. Chadli: Admissible almost type contraction an fixed point results, Int. J. ot Mathematics and Mathematical Sciences, Vol.2020, article id :9104909 (2020),1-7.
28. S.G. Teweldemedhin and K.K.Tola: Fixe point results for an almost generalized α- admissible z -contraction in the settinmg of Partially ordered b- metric spaces, Abstract and applied analysis, Vol. 2021, Article id:5988007, (2021),1-11.
29. F. Khojasteh, S. Shukla, S. Radenovic, A New approach to the study of fixed point theory for simulation functions, Filomat 29(6),(2015), 1189-1194.
30. S. Komal, P. Kumam, D. Gopal, Best proximity point z-contraction and Suzuki type z contraction mappings with an application to fractional calculus, Appl. Gen. Topol. 17(2) (2016), 185-198.
31. P.Kumam, D. Gopal, L. Budhiya: A new fixed point theorem under Suzuki type z- contraction mappings, J. Math. Anal. 8(1), (2017), 113-119.
32. M.Olgun, O. bICER, AND t. Alyildiz: Anew aspect to Picard operators with simulation functions, Turk J. Math. 40(2016),832-837.
33. A. Padcharoen, P. Kumam, P. Saipara, P. Chaipunya: Generalized Suzuki type z -contraction in complete metric space, Kragujevac Journal of Mathematics ,43(3)(2018),419-430.
34. S.Radenovic, and S. Chandok: Simulation type functions and coicidence points, Filomat, 32(1)(2018),141-147.
35. S. Radenovic, F. Vetro, J. Vujakovic: An alternative and easy approach to fixed point results via Simulation functions, Demonstrate . Math. 50(1)(2017),223-230.
36. A.F.Roldan-Lopez-de-Hierro, E. Karapinar, C. Roldan-Lopez-de-Hierro, and J. Martinez Moreno: Coicidence point theorem on metric spaces via simulation functions, J. Comput.Appl. Math. 275(2015),345-355.
37. A.F.Roldan-Lopez-de-Hierro, and B.Samet: ψ- admissibility results via extended simulation functions, J. FixedvPoint Theory Appl.,19(3),(2017), 1997-2015.
38. V.L. Rosa and P.Vetro: Common fixed points for α, φ, ϕ- contracttions in generalized metric space,Nnlinear Anal. Model. Control, 19(1)(2014), 43-54.
39. B.Samet , C. Vetro ,and P. Vetro : Fixed point theorems for $\alpha-\psi$ contractive type mappings, Nonlinear Anal. 75, (20120, 2154-2165.
40. P.Salimi, A.Latif and N.Hussain: Modified $\alpha-\psi$- contractive mappings with applications, Fixed point theory Aplli.,15 (2015)
41. P.Shahi,J. Kaur, and S.S. Bhatia: Coicedence and Common fixed point results for generalized $\alpha-\psi$ contractive mappings with applications, Bull. Belg. Math. Soc. Simon Stevin, 22(2)(2015),299-318.

42. H. Qawagneh, M.S.M Noorani, W. Shatanawi, and H. Alsamir: Common fixed points for pair of triangular α -admissible mappings,Journal of Nonlinear Sciences and applications, 10(12)(2017),6192-6204.
43. Dipti, A.K.Dubey and U.Mishra: Generalized α- admissible almost z- contractions involving simulation function in a metric space, Communications on Applied Nonlinear Analysis, Vol. 32 No. 1 , (2025),354-362.

[1]Department of Mathematics, Dr. C.V. Raman University, Bilaspur, chhattisgarh-India.
Email address: sk10tiwari@gmail.com

[2] Teacher,School Education Department , Takhatpur, Bilaspur, Chhattisgarh, -India.
Email address: dubeyanandmohan767@gmail.com

[2] TDepartment of MCA ,Hidustan College of Arts and Science, Coimbatore Tamilnadu, -India.
Email address: avsenthilkumar@yahoo.com