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Applying Differential Transform Method (DTM) to get Series Solution of Sixth Order Boundary Value Problems

Dnyaneshwar Kadam[1], Avinash Khambayat[2]

[1]Research Scholar, Department of Mathematics, School of Science, Sandip University, Nasik, Maharashtra, India

[1]Assistant Professor, Xavier Institute of Engineering, Mumbai, Maharashtra, India

[2]Professor, Department of Mathematics, School of Science, Sandip University, Nasik, Maharashtra, India

[1]dskadam7@gmail.com, [2]avinash.khambayat@sandipuniversity.edu.in

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Abstract: The Differential Transform Method (DTM) is an efficient and versatile analytical technique for obtaining series solutions to various Differential and Integral Equations. By significantly reducing computational effort while preserving accuracy, the DTM offers an attractive alternative to conventional methods. This paper demonstrates the efficacy and simplicity of the DTM through illustrative examples, focusing on higher-order Differential Equations with boundary value problems to derive precise series solutions. Furthermore, the DTM's adaptability and ease of implementation make it particularly suited for solving complex mathematical models. Its applications span diverse fields, including engineering, physics, and applied sciences, where it has been effectively utilized for analysing heat transfer, fluid dynamics, wave propagation, and electrical circuit problems. The findings underscore the potential of the DTM as a powerful tool for tackling challenging problems in theoretical and applied mathematics.

Keywords: Differential Transform Method(DTM), Higher order Boundary Value Problems

1. Introduction

The Differential Transform Method (DTM), first introduced by J. K. Zhou in 1986 for solving problems in electric circuits, has since evolved into a versatile analytical tool for addressing a wide range of mathematical problems [2]. Farshid Mirzaee extended its application to ordinary linear and nonlinear differential equations [1], while Narhari Patil and Avinash Khambayat utilized it for solving linear differential equations [3]. Shawagfeh N. and Kaya D used DTM for solving the Euler equidimensional equation.[4]. Y. Keskin and G. Oturance further advanced the method by employing the Reduced Differential Transform Method (RDTM) to tackle partial differential equations [5]. In recent years, DTM has gained significant attention for its ability to solve fractional differential equations, as highlighted in studies focusing on its application to boundary value problems, optimal control, and calculus of variations [2]. Additionally, advancements in fractional DTM have opened new avenues for addressing problems in engineering and applied sciences, further solidifying its role as a powerful semi-analytical method [1]. DTM's adaptability and computational efficiency make it a valuable tool for solving both linear and nonlinear ordinary and partial differential equations, offering precise solutions with reduced effort compared to traditional numerical methods.

2. Objectives

This study aims to explore the efficiency and applicability of the Differential Transform Method (DTM) in deriving series solutions for sixth-order boundary value problems. By systematically analysing its computational advantages, the research seeks to establish DTM as a reliable approach for tackling complex differential equations with boundary constraints. The study will demonstrate the effectiveness of DTM through practical examples, highlighting its role in simplifying problem-solving while maintaining precision. Additionally, the research intends to investigate the adaptability of DTM across various scientific fields, including engineering and physics, reinforcing its potential as a valuable analytical tool for mathematical modelling.

3. Methods

The Differential Transform Method:

The rth derivative of a function is transformed as described below.

$$U(r) = \frac{1}{r!}\left(\frac{d^r u(x)}{dx^r}\right) \; at \; x = x_0$$

…(1)

Let u(x) represent the original function and U(r) denote the transformed function. The inverse differential transformation of U(r) is defined as follows:

$$u(x) = \sum_{r=0}^{\infty} U(r)(x - x_0)^r$$

…(2)

If $x_0 = 0$, the function $u(x)$ in (2) is expressed as

$$u(x) = \sum_{r=0}^{\infty} U(r)\, x^r$$

…(3)

We utilize the following core theorems of the differential transform method given by J. K. Zhou [1] :

Theorem 1] If $u(x) = \alpha g(x) \pm \beta h(x) \;\; then \;\; U(r) = \alpha G(r) \pm \beta H(r)$

Theorem 2] If $u(x) = x^m \;\; then \;\; U(r) = \delta(r - m) \;\; where \;\; \delta(r - m) = \begin{cases} 1, \; if \; r = m \\ 0 \, , \; if \; r \neq m \end{cases}$

Theorem 3] If $u(x) = e^x \;\; then \;\; U(r) = \frac{1}{r!}$

Theorem 4] If $u\,(x) = g\,(x)\,h\,(x)$ $then \;\; U(r) = \sum_{l=0}^{r} G(l)H(r - l)$

Theorem 5] If $u(x) = u_1(x)\, u_2(x) \; then \; U(r) = \sum_{r_1=0}^{r} U_1(r_1)\, U_2(r - r_1)$

Theorem 6] If $u(x) = \frac{d^n u_1(x)}{dx^n}$, $then\ U(r) = \frac{(r+n)!}{r!} U_1(r+n)$

Theorem 7] If $u(x) = e^{\lambda x}\ then\ U(r) = \frac{\lambda^r}{r!}$, $\lambda\ is\ constant$

Theorem 8] If $u(x) = sin(wx + \alpha)\ then\ U(r) = \frac{w^r}{r!} sin(\frac{r\pi}{2} + \alpha)$, where α, w are constants.

Theorem 9] If $u(x) == cos(wx + \alpha)\ then\ U(r) = \frac{w^r}{r!} cos(\frac{r\pi}{2} + \alpha)$, where α, w are constants.

4. Results

Example 1 **:** Consider the following equation

5. $u^6(x) = e^{-x} + u(x)$ *, 0 < x <1*
…(4)

Subjected to the conditions,

$u(0) = u'(0) = u''(0) = 1, u(1) = u'(1) = 2$
…(5)

Now we apply DTM on example 1, using above theorems we get,

$$r^6 U(r) = \frac{1}{r+1} + U(r)$$

Rearrange the equation to solve for U(r),$U(r) = \frac{1}{(r+1)(r^6-1)}$

With the boundary conditions

U(0) = U(1) = U(2) = 1

By applying inverse differential transform, we obtain the series solutions,

$$u(x) = \sum_{r=0}^{\infty} \frac{1}{(r+1)(r^6-1)} \frac{x^r}{r!}$$

Since U(1) is undefined, the series representation would be,

$$u(x) = -1 + \frac{x^2}{189\,.\,2!} + \frac{x^3}{2912\,.\,3!} + \frac{x^4}{20475\,.\,4!} + \frac{x^5}{93744\,.\,5!} + \frac{x^6}{326585\,.\,6!} + \frac{x^7}{941184\,.\,7!} + \frac{x^8}{2359287\,.\,8!} + \frac{x^9}{5314400\,.\,9!}$$

The exact solution for this example is

$u(x) = -e^{-x}$ is

Maple output graph is shown in the figure (1) below :

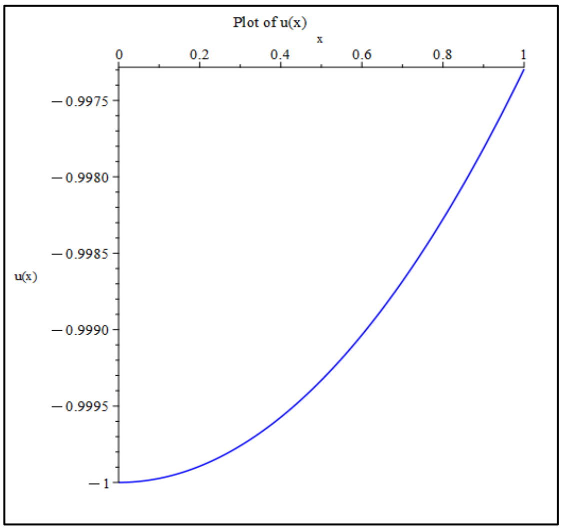

Figure 1 : Maple Output graph

Example 2 : Consider the following equation

6. $u^6(x) + u(x) = e^{-x}$, $0 < x < 1$ …(6)

Subjected to the conditions,

$u(0) = u'(0) = u''(0) = 1, u(1) = u'(1) = 2$ …(7)

Now we apply DTM on example 1, using above theorems we get,

$$r^6 U(r) + U(r) = \frac{1}{r+1}$$

Rearrange the equation to solve for U(r),

$$U(r) = \frac{1}{(r+1)(r^6+1)}$$

By applying inverse differential transform, we obtain the series solutions,

$$u(r) = \sum_{r=0}^{\infty} \frac{1}{(r+1)(r^6+1)} \frac{x^r}{r!}$$

The series representation would be,

$$u(x) = 1 + \frac{x}{4\,.1!} + \frac{x^2}{195\,.2!} + \frac{x^3}{2920\,.3!} + \frac{x^4}{20485\,.4!} + \frac{x^5}{93756\,.5!} + \frac{x^6}{326599\,.6!} + \frac{x^7}{941192\,.7!} + \frac{x^8}{2359350\,.8!} + \frac{x^9}{5314410\,.9!}$$

The exact solution for this example is

$u(x) = \frac{1}{2}e^{-x}$ is

Maple output graph is shown in the figure (2) below :

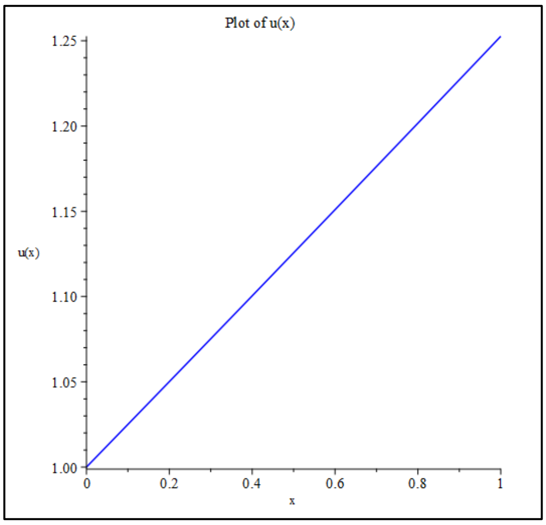

Figure 2 : Maple Output graph

5. Discussion

In this study, we effectively use DTM to solve differential equations with boundary conditions. DTM proves to be a reliable and efficient technique for addressing boundary value problems. The precision of the results can be improved by incorporating additional terms into the series. This approach is highly effective for tackling differential equations involving boundary conditions.

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