Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

Magneto Thermo-Elasticity in Rotating Functionally Graded Shell

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Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract:

Introduction: This study describes the magneto-thermo-elastic behavior of a rotational functionally graded material (FGM) shell comprising composite materials under the combined effects of magnetic, thermal, and mechanical loads. The research address how do functionally graded material (FGM) shell response under the influence of magnetic, thermal, and mechanical loads, and what are the effects of various material parameters on their mechanical responses. T he goal of this research paper is to analyze a magneto- thermo-elastic theoretical solution for functionally graded material (FGM) shell under magnetic, thermal, and mechanical loads, enhancing the understanding of their mechanical responses. It aims to provide support for practical engineering applications in diverse fields such as aerospace and electronics. The study determine the mechanical responses of an FGM shell under magnetic, thermal, and mechanical loads are significantly influenced by material parameters such as thermal expansion coefficient [4], volume fraction [3], and Poisson's ratio [3]. The Voigt method [3]is used to analyze the stress-strain relation, and the results are compared with existing research by setting values for some parameters, simplifying the analysis to coupled loads.The findings provide valuable insights for the design and optimization of GM tubes [3] in applications subjected to combined magnetic, thermal,and mechanical loads, offering support for improved performance and reliability.

Objectives: Magneto- Thermo elasticity use to analyze the behavior of material and analyze the changes on material with the temperature changes, magnetic field and temporary deformation. I t includes concept of electromagnetism, thermodynamics and elasticity to analyze the behavior of materials in different-different applications

Methods: In the modelling of composite material in Rotating Functionally Graded Shell. It is less complex method to evaluate the equivalent the material parameter of a composite material made up of two or more different materials.In Voigt method [3], composite material act as individual material property distributed uniformly. Each material contribute the overall property according to its volume fraction.

Results: It formulates a mathematical equation for identify the relationship between nonlinear thermo elastic and effect of arious controlling parameter on dynamics.Voigt method [3] use for evaluate the equivalent material parameter of a Rotating Functionally Graded Shell in the presence of magnetic, mechanical, thermal conditions.

Conclusions: The critical role of Poisson’s ratio in determining the stress distribution within functionally graded thick-walled tubes under various load conditions. Notably, radial stress exhibits considerable variation across the tube’s radius, with the greatest difference observed at the mid-radius. Poisson’s ratio notably influences circumferential and axial stresses, particularly near the inner radius, with axial stress displaying a significant variation between the inner and outer radii.

Keywords: Functionally Graded Materials (FGM) [2], Magneto-thermo-elastic behavior, Voigt method [3], Volume fraction, Multi-field interactions.

1. Introduction

Magneto-thermo elasticity [1] is a field that combines the principle of electromagnetic effect, thermal changes and elasticity. The concept of Magneto- thermo elasticity use to analyze the behavior of material and analyze the changes on material with the temperature changes, magnetic field and temporary deformation. This field assemble effect of elasticity, conduction of heat and effect of electromagnetic field on material. This field describe how magnetic field affected on stress of material in form of heating conduction. The mathematical modeling of Magneto-thermo elasticity [1] shows the interaction between temporary deformation, magnetic field and heat conduction. These mathematical modeling describes with the help of partial differential equation. The partial differential equation (PDEs) shows dynamic property of stress, magnetic property and temperature property of the material. The mathematical modeling of Magneto-thermo elasticity [1] includes elasticity equation, Maxwell equation and conduction equation [1]. In magneto-thermo elasticity [1] relation between mechanical and thermal field is complex in the case of magnetic field. It depend upon nature of material and also depend on how they interact with each other. The main challenges in the magneto-thermo elasticity [1] are thermal shock on material in the presence of magnetic field. When a material experience sudden change of temperature, the material leading to the distribution of stress. This incident becomes more complex in the presence of magnetic field and effects of thermal property. It is important to understand how thermal shock interact with magnetic field and how they affect displacement field and stress in material. The field of Magneto-thermo elasticity [1] faced challenges in domain of dynamic interaction, coupling effect, non-linearity, thermal shock. This study delves intricate relation between stress in mechanical field, thermal gradient and magnetic fields in solid types of material. The aim of this study to progress a framework for analyze the way of acting of magneto-thermo-elastic systems under conditions of external magnetic field and heat source. By analyzing all interaction this study used for enhance engineering applications and materials performance in the presence of magnetic thermal environment. Magneto-thermo elasticity [1] represents a complex area of study that includes concept of electromagnetism, thermodynamics and elasticity to analyze the behavior of materials in different-different applications. As technology advances, the importance of understanding this interaction will grow.

2. Equations Of Generalized Thermoelasticity

The basic equations of linear thermoelasticity [4] for different models for isotropic and

Homogeneous thermo-elastic solid are as follows:

Classical Thermoelasticity (CTE):

Constitutive relations:

$\tau_{ij} = \lambda \, \Delta \, \delta_{ij} + 2\mu \, e_{ij} - \beta \, T \, \delta_{ij}$;   ($i$, = 1,2,3)   (1)

where λ, μ are Lame constants, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of the material's linear thermal expansion, τ_{ij} is the stress tensor, T is the increase in temperature above the reference temperature T_0 and $\Delta = u_{i,i}$ (dilatation).

Strain – Displacement relations:

$e_{ij} = (u_{i,j} + u_{j,i})/2$ (2)

Classical Fourier law:

Equations (1) and (2) are to be augmented by **Classical Fourier law** connecting the heat flux

vector $\vec{q}$ with the temperature gradient ∇ T through the equation

$q_i = -K \, T_{,i}$,   i = 1, 2, 3   (3)

i.e., the heat flux vector is the immediate result of a temperature gradient. Here K > 0 is the

thermal conductivity of the solid material.

Law of conservation of internal energy:

$-q_{i,i} + \rho Q = \rho c_e \; T$,   i = 1, 2, 3   (4)

here Q is the heat source and c e is the specific heat of the solid at constant strain. A

superposed dot denotes the partial derivative with respect to time.

Classical heat transport equation:

Equation (3) and (4) together give the parabolic type of heat transport equation as:

$K\nabla^2 T + \rho Q = \rho c_e T$ (5)

Equations of motion:

Stress equations of motion:

$\tau_{ij,j} + \rho f_i = \rho u_i$, $(i, j = 1,2,3)$ (6)

Where f_i are the body force components and τ_{ij} are given by equations (1).

Displacement equations of motion:

$(\lambda + \mu)u_{j,ij} + \mu u_{i,jj} - \beta T_{,i} + \rho f_i = \rho u_i$ (7)

Equations (1), (5) and (6) [or (7)] constitute the complete mathematical model of the classical theory of thermoelasticity (CTE) [4].

3. Rotating Functionally Graded Shell

Designing:

Rotating Functionally Graded Shell design using combination of material. The FGM shell under magnetic field, mechanical load, thermal conditions are shown in Figure 1. The structure of FGM shell is cylindrical. The inner and outer radius is denoted, showing the dimensions of the tube [2].

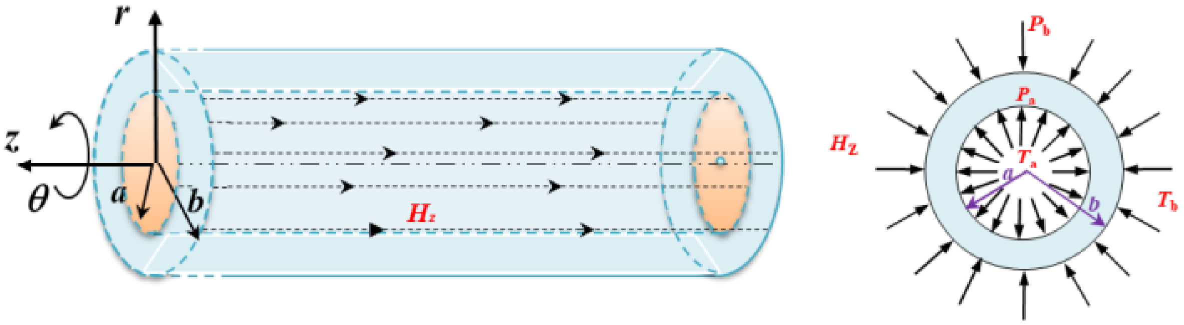

Figure 1: A diagram of a long FGM tube subjected to mechanical loads, thermal loads and magnetic field.

Load conditions:

1. Thermal Load - This load affect the performance of structure and Variations in temperature can increase the thermal stress.

2. Mechanical Load - This load also affect the performance of structure.

3. Magnetic Field - For FGM shell magnetic field affect the behavior of material. Different response for magnetic forces.

FGM shell material consist Material A and Material B. Material A and B are isotropic linear elastic. But Material B has different properties [3].

Volume fraction describe the proportion of material A at radius r in the FGM shell.

4. Magneto-Thermo Elastic Nonlinear Dynamic Modeling Of A Rotating Functionally Graded Shell

The Nonlinear dynamic modeling of magneto thermo elastic in a Rotating functionally graded shell include analyzing the interaction of mechanical forces, thermal effect and magnetic effect on rotating shell. This rotating shell made by FGMs (Functionally graded material) [3]. This modelling help for analyze the behavior of material when rotation motion, thermal shock, magnetic fields experience on surface of material which is mainly used in mechanical, energy system and aerospace etc.

This research mainly focuses on mechanism of coupling of functionally graded material in the presence of physical field for rotating FGM shell. This paper introduces a new method for analyzing the dynamic behavior of rotating shell made by FGMs (Functionally graded material) in the presence of electromagnetic, thermal, and rotational fields.

It formulates a mathematical equation for identify the relationship between nonlinear thermo elastic and effect of various controlling parameter on dynamics.

In this research, functionally graded shell is a combination of magnetic load, thermal load and mechanical load.

Some assumptions applied to different problem scenarios within the research.

Loads & Fields	Parameters
Mechanical Loads	Poisson’s ratio, elastic modulus
Thermal loads	Thermal conductivity, thermal expansion coefficient
Magnetic fields	Magnetic permeability

In previous research, Assumed Poisson’s ratio as a constant and other material properties like elastic modulus, thermal conductivity, thermal expansion coefficient, and magnetic permeability as power functions and exponential functions.

In the term of power function,

Elastic modulus $E(r) = E_0 r^n$

Thermal expansion coefficient $\alpha(r) = \alpha_0 r^n$,

Thermal conductivity $k(r) = k_0 r^n$,

and magnetic permeability $\mu(r) = \mu_0 r^n$.

In the term of exponential function,

Elastic modulus $E(r) = E_0 e^{rn}$,

Thermal expansion coefficient $\alpha(r) = \alpha_0 e^{rn}$,

Thermal conductivity $k(r) = k_0 e^{rn}$ and magnetic permeability $\mu(r) = \mu_0 e^{rn}$

5. Challenges in the Previous Studies

1. In rotating FGM shell [2] the indexes m and n for material parameters are assumed to be the same, but this assumption is not realistic. For simple analyze of numerical value many studies assume exponent same.
2. In rotating FGM shell Poisson's ratio consider as constant across different layers.
3. FGMs are design using homogeneous materials. This is challenging to adjust material parameters according to a simple rule in real manufacturing [3].

To overcome those challenges, Voigt method [3] use for evaluate the equivalent material parameter of a Rotating Functionally Graded Shell in the presence of magnetic, mechanical, thermal conditions. Voigt method [3] based on volume fraction.

6. VOIGT Method

This method used in the modelling of composite material in Rotating Functionally Graded Shell. It is less complex method to evaluate the equivalent the material parameter of a composite material made up of two or more different materials.

In Voigt method [3], composite material act as individual material property distributed uniformly. Each material contribute the overall property according to its volume fraction.

The formula for calculating the equivalent property is given as:

$$Peq(r) = i\sum Pi(r) \cdot ci(r)$$

Where:

- Peq(r) is the equivalent property of the composite at position r.
- Pi(r) is the property of material i at position r.
- ci(r) is the volume fraction of material i at position r.

Following graphs shows the evolution of volume fraction of Material A with different parameter n.

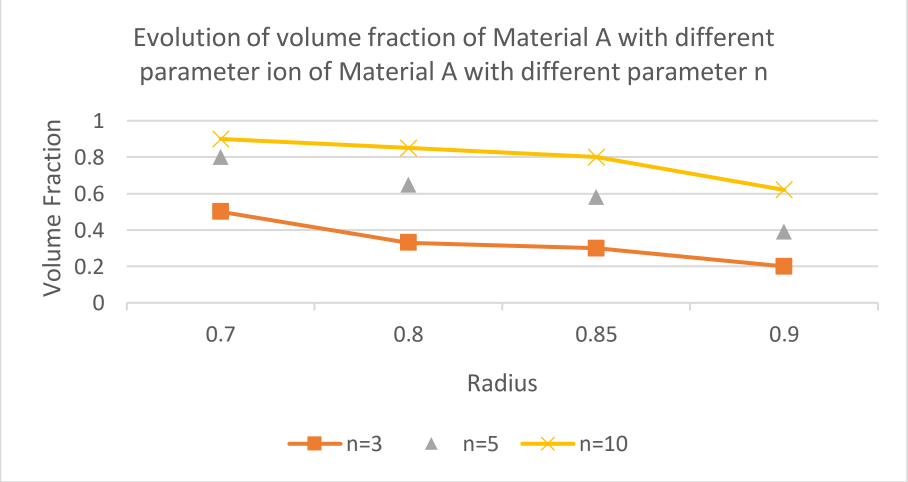

Figure 2: Evolution of volume fraction of Material A with different parameter ion of Material A with different parameter n

This graph helps to understand the interaction of thermal effects, magnetic effect and mechanical forces on Voigt method [3] in a rotating FGM shell.

X axis represent: Radius (inner and outer) of Rotating FGM shell.

Y axis represent: Volume fraction of Material A.

n represent: Different parameter.

As n increase C(r) increase non-linearly.

If we consider particular value of n then,

Let n = 10

At n = 10, volume fraction approaches 1 at the r = 0.7 (inner radius).

It means material A dominates in inner region of shell.

For different values of **n**, the volume fraction of Material A decreases from the inner radius to the outer radius. At the outer radius (**r = 1.0**), the volume fraction of Material A becomes **zero**, indicating that Material B dominates at the outer region. This variation in the volume fraction plays an important role in influencing the shell’s material properties, such as thermal conductivity, and magnetic behavior, all of which depend on the distribution of the materials.

Please see the table 2, this table shown effect of parameter n on the radial displacement u, the radial stress, the circumferential stress, the axial stress and the perturbation of magnetic field, respectively [3].

Table 2: The extremum values corresponding to different parameter n.

n		**1.5**	**3**	**5**	**10**
$\bar{u}$	$\bar{r} = 0.7$	1.2608	1.0163	0.8946	0.8150
	$\bar{r} = 1.0$	1.0410	0.7409	0.7409	0.6756
$\bar{\sigma}_r$	$\bar{r} = 0.7$	-1	-1	-1	-1
	$\bar{r} = 1.0$	0	0	0	0
$\bar{\sigma}_\theta$	$\bar{r} = 0.7$	3.1908	3.2635	3.3125	3.3376
	$\bar{r} = 1.0$	1.1239	0.9039	0.7942	0.7225
$\bar{\sigma}_z$	$\bar{r} = 0.7$	0.6572	0.6790	0.6937	0.7012
	$\bar{r} = 1.0$	0.3231	0.2571	0.2242	0.2027
$\bar{h}_z$	$\bar{r} = 0.7$	-0.6229	-0.5086	-0.4514	-0.4129
	$\bar{r} = 1.0$	-0.6287	-0.5064	-0.4494	-0.4121

7. Interaction Effect

For mechanical Forces: Analyze the pattern of volume fraction changes with respect to radius, which indicate how mechanical forces affect the material distribution.

For Thermal effects: Compare the value of volume fraction for different value of n. If n represent thermal condition then analyze how temperature variations influence material distribution.

For magnetic effects: Compare the value of volume fraction for different value of n. If n represent magnetic condition then analyze how magnetic field variations affect volume fraction.
Volume fraction helps to analyze the distribution of material A and Material B in rotating Functionally Graded Material (FGM) shell [3]. FGM shell made by composite material where material property change with respect to thickness of shell. This variation is analyze and controlled by volume fraction of each material at each point.

In this research, the volume fraction is used to represent the material properties, such as elastic modulus, thermal conductivity, and Poisson's ratio [3], changes from the inner radius to the outer radius of the rotational Functionally Graded Material (FGM) shell [3]. These changes are important because they directly influence the shell mechanical, thermal, and magnetic behaviors.

The volume fraction, which vary with the radial direction from the inner radius to the outer radius, affects the distribution of stresses, displacements, and temperature gradients in FGM shell.

8. Effect of parameter on radial displacement, stress, axial stress and magnetic field

1. Effect of Parameter n on Radial Displacement:

This curve has shown the changes in the parameter n on the radial displacement [3]. Here n parameter related material properties, thermal expansion and loading conditions of the rotational Functionally Graded Material (FGM) shell.

The curve shown the displacement from the inner to outer radius at different radial positions.

As the parameter **n** increases, the **radial displacement [3]** of the rotational Functionally Graded Material (FGM) shell also increases. When **n** becomes larger, the rotational Functional Graded Material (FGM) shell experiences more **deformation or displacement** from its original shape.

Higher values of **n** could make the tube less stable and reduce ability to maintain its structural integrity.

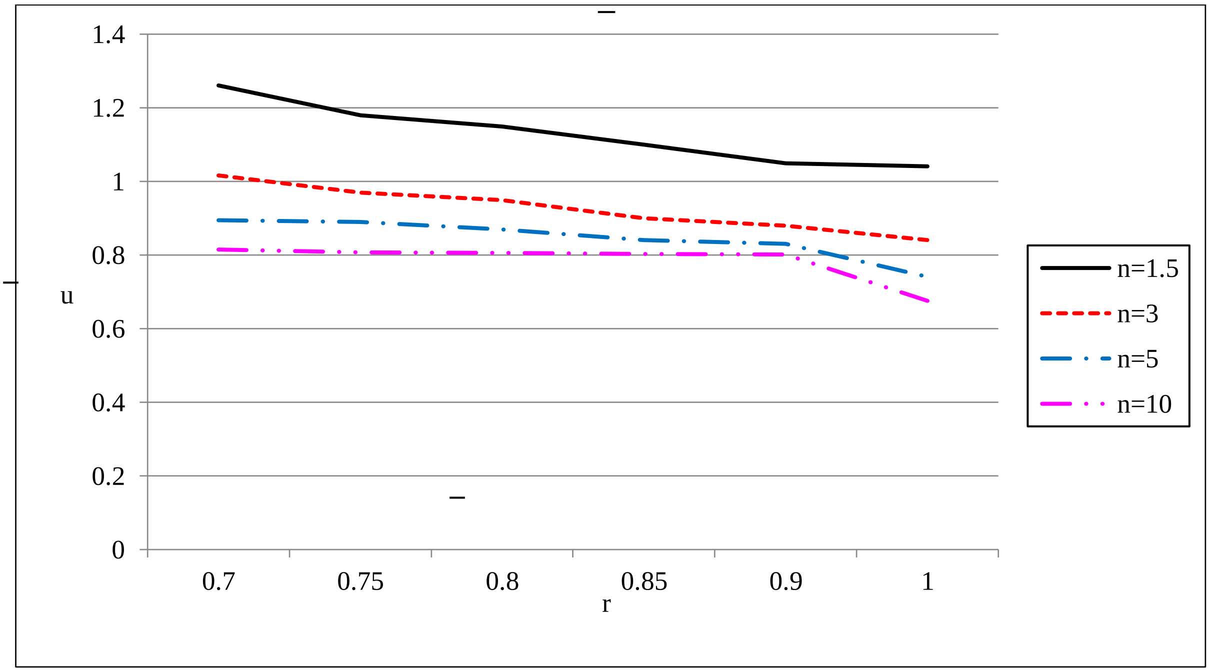

Figure 3: Effects of n on the radial displacement u.

2. Effects of Parameter n on Radial Stress:

This curve shown changes in radial stress with the parameter n in FGM shell.

As **n** increases, radial stress may either **increase or decrease**, which can affect the shell load-**bearing capacity** and **failure characteristics**.

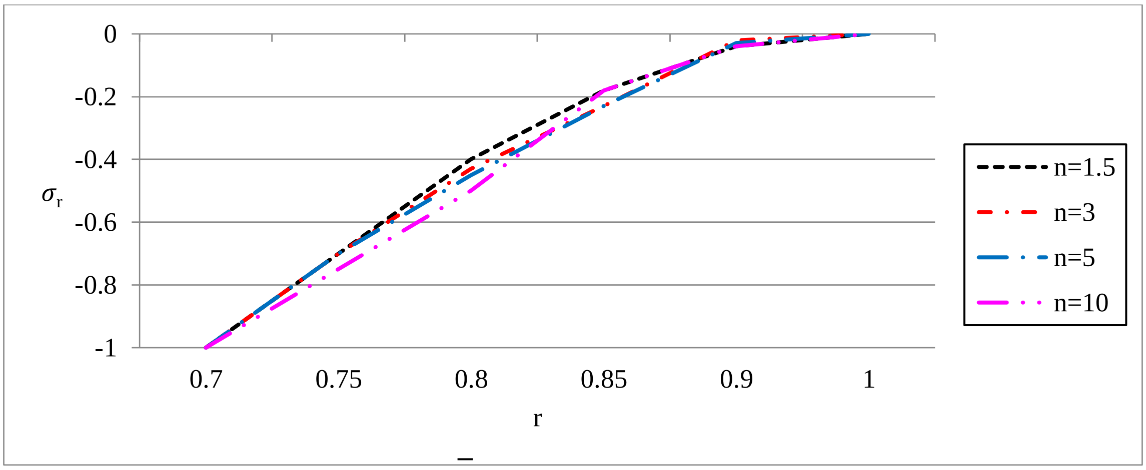

Figure 4: Effects of n on the radial stress σ_r.

3. Effects of Parameter n on circumferential stress:

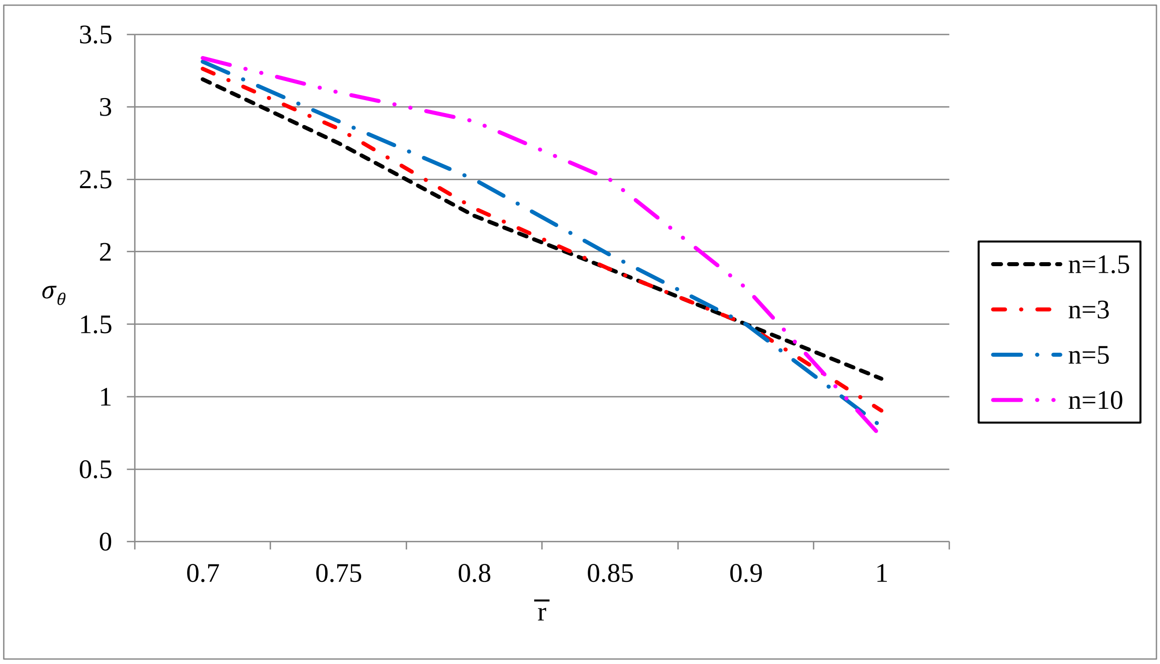

Figure 5: Effects of n on the circumferential stress σ_θ.

This curve shown the effect of parameter n on circumferential stress at various location in FGM shell. With the help of these relationships is crucial for designing FGM tubes that can effectively manage stresses, thus preventing **premature failure [3]** under operational conditions.

4. Effects of Parameter n on Magnetic field

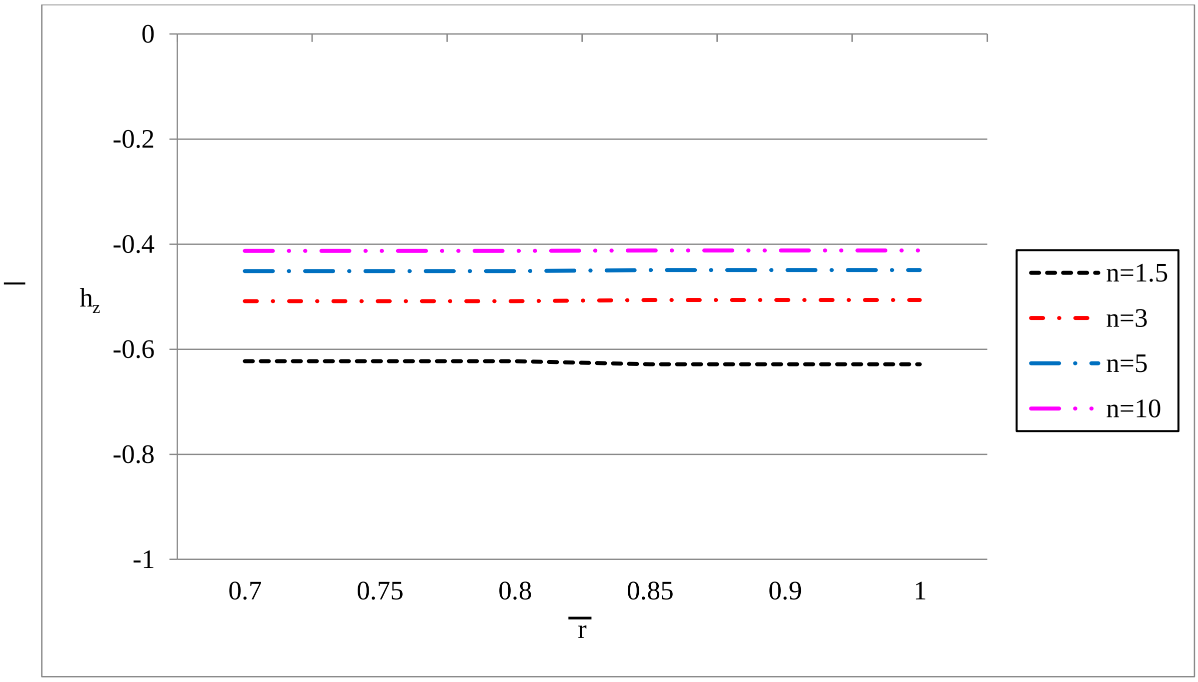

Figure 6: Effects of n on the perturbation of magnetic field $\bar{h}_z$.

This curve shows **perturbation of the magnetic field [3]** changes with the parameter **n** at different radial position.

In the figure 6, the distribution of the magnetic field perturbation across the radial positions remains relatively **constant.**

9. Conclusion

In conclusion, the study demonstrates that the behaviour of functionally graded material (FGM) shells is significantly influenced by factors such as volume fraction, thermal expansion coefficient, and Poisson's ratio. The Voigt model provides an accurate representation of the stress-strain relationship for FGM shells subjected to combined mechanical, thermal, and magnetic loads, confirming the validity of the results when specific parameter values are chosen. These insights are valuable for the engineering design and application of FGM tubes in multi-field environments, enhancing both their performance and structural integrity.

The research further highlights the critical role of Poisson's ratio in determining the stress distribution within functionally graded thick-walled tubes under various load conditions. Notably, radial stress exhibits considerable variation across the tube's radius, with the greatest difference observed at the mid-radius. Poisson's ratio notably influences circumferential and axial stresses, particularly near the inner radius, with axial stress displaying a significant variation between the inner and outer radii. Furthermore, an increase in Poisson's ratio leads to substantial changes in the magnetic field perturbation, though it has minimal effect on horizontal distribution. These findings emphasize the importance of carefully considering Poisson's ratio when analysing FGM tubes under multi-field loading conditions.

10. Abbreviations

τ_{ij}: Stress tensor

λ, μ: Lame constants

α_t: Coefficient of the material’s linear thermal expansion

T: Absolute temperature

T_0: Reference temperature

$\vec{\nabla}$ T: Temperature gradient

$\vec{q}$: Heat flux vector

K: Thermal conductivity

Q: Heat source

C_e: Specific heat of the medium

f_i: Body force components

t: Time variable

u_i: Displacement vector

e_{ij}: Strain tensor

ρ: Density of the medium

e: Cubical dilatation

μ_0: Magnetic permeability

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