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Contra Continuous and Open Maps via Neutrosophic Soft Z – Open Sets

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Abstract: This paper investigates the concepts of contra Z-continuous, contra Z-irresolute, contra Z-open and contra Z-closed maps in neutrosophic soft topological spaces. We also explore the notions of contra Z and Z-C homeomorphisms. Theoretical results are presented with examples and theorems, enhancing the understanding of these mappings within the framework of neutrosophic soft topology.

Keywords: contra z-continuous maps, contra z-irresolute maps, contra z-open maps, contra z-closed maps, contra z homeomorphism and contra z-c homeomorphism.

1. Introduction

The foundational framework of fuzzy sets, introduced by Lofti A.Zadeh [20] in 1965, offers a powerful mathematical framework to handle the complexities that arise from ambiguity in practical, real world scenarios. This concept has been employed across various fields, including economics, sociology and medical science, where researchers frequently encounter vague, imprecise and occasionally incomplete information. These fields utilize fuzzy sets and fuzzy logic to model uncertain data for a range of specialized purposes. Standard fuzzy sets are defined by their membership value or degree of membership, although assigning this value can sometimes be challenging. Chang [6] in 1968 introduced fuzzy sets into topology under the framework known as fuzzy topological spaces.

Building on this foundational idea, in the 1986, K T. Atanassov [2] introduced intuitionstic fuzzy sets, which build on fuzzy sets by including a non-membership degree in addition to degree of membership. Coker [7] in 1997 introduced intuitionistic fuzzy sets into the realm of topology, defining them as intuitionstic fuzzy topological spaces. Intuitionistic fuzzy sets are limited to managing incomplete information by considering both membership and non-membership values. However, they do not address uncertain and contradictory information often found in belief systems. To tackle these issues, Florentin Smarandache [17] introduced the concept of neutrosophic set in 2005, which serves as a mathematical framework for dealing with imprecise, indeterminate, and inconsistent data. In 2012, Salama and Alblowi [14] proposed the concept of neutrosophic topological spaces. In 1999, Molodstov [12] initiated the soft set principle as a versatile mathematical approach that addresses parameterization issues and surpasses the limitations of other uncertainty theories. This theory is highly practical, efficient and widely applicable across different disciplines. Molodstov's implementations of soft set principle include domains such as function analysis, decision making, operational research and integrative mathematics among others. As a result, soft set theory has gained significant traction and continuous to advance rapidly in diverse fields.

In 2011, Shabir and Naz [16] initiated the notion of soft topological spaces. Subsequently, in 2013 Maji [10] introduced the neutrosophic soft set concept, which inspired numerous mathematicians to explore its applications in various mathematical frameworks. Modification by Deli and Broumi [8] further refined this framework, while Bera and Mahapatra [3] explored its algebric structures. In 2011, A. I. EI-Magharabi and A. M. Mubarki [9] introduced Z-open sets in topological spaces. In 2020, A. Vadivel et al [18] proposed Z-open sets in neutrosophic topological spaces.

This paper primarily aims to introduce and explore the concepts of contra Z-continuous maps, contra Z-irresolute maps, contra Z-open maps, and contra Z-closed maps in neutrosophic soft topological spaces, using neutrosophic soft Z-open sets. We analyze and discuss their fundamental properties, along with the notions of contra Z homeomorphisms and Z-C homeomorphisms, providing examples and theorems that contribute to further research in neutrosophic soft topology.

2. Preliminaries

This section offers a summary of essential definitions refers to **neutrosophic sets, soft sets and neutrosophic soft sets** to ensure thorough understanding.

Definition 2.1 [15] Let $\mathbb{W}$ be an underlying universe. A neutrosophic set (in short, NS) D is an object having the form D = $\{\langle w,\ \mu_D(w),\ \sigma_D(w),\ \nu_D(w)\rangle\ :\ w\ \in\ \mathbb{W}\}$ where $\mu_D\ \rightarrow [0,1]$ denote the degree of membership function, $\sigma_D\ \rightarrow [0,1]$ denote the degree of inderterminacy function and $\nu_D\ \rightarrow [0,1]$ denote the degree of non-membership function respectively of each element $w\ \in\ \mathbb{W}$ to the set D and $0 \leq\ \mu_D(w) + \sigma_D(w) + \nu_D(w) \leq 3$ for each $w\ \in\ \mathbb{W}$.

Definition 2.2 [12] Assume that $\mathbb{W}$ is the underlying universe & let ϱ is a parameter set. Let $\mathcal{P}(\mathbb{W})$ represent the collection of all neutrosophic sets within $\mathbb{W}$. A pair (D, ϱ) is known as the soft set(shortly, SS) over $\mathbb{W}$, where D is a mapping D : $\varrho\ \rightarrow \mathcal{P}(\mathbb{W})$. In other terms, a soft set can be viewed as a collection of subsets of the set $\mathbb{W}$, each associated with a specific parameter.

Definition 2.3 [8] Assume that $\mathbb{W}$ is the underlying universe & let ϱ is a parameter set. Let $\mathcal{P}(\mathbb{W})$ represent the collection of all neutrosophic sets within $\mathbb{W}$. Then a neutrosophic soft set (S,ϱ) over $\mathbb{W}$ (shortly, NSS) is characterized by $(S,\varrho) = \{(\varphi,\ \langle\varepsilon,\ \mu_{S(\varphi)}(\varepsilon),\ \sigma_{S(\varphi)}(\varepsilon),\ \nu_{S(\varphi)}(\varepsilon)\rangle\ :\ \varepsilon\ \in\ \mathbb{W})\ :\ \varphi \in \varrho\}$, where $\mu_{S(\varphi)}(\varepsilon),\ \sigma_{S(\varphi)}(\varepsilon),\ \nu_{S(\varphi)}(\varepsilon)\ \in [0,1]$ are respectively called the *degree of membership function, the degree of indeterminacy function and the degree of non-membership function* of $S(\varphi)$. As the maximum value for each of μ,σ,ν is 1.

The inequality $0 \leq\ \mu_{S(\varphi)}(\varepsilon) + \sigma_{S(\varphi)}(\varepsilon) + \nu_{S(\varphi)}(\varepsilon) \leq 3$ naturally holds.

Definition 2.4 [[10], [4]] Assume that $\mathbb{W}$ is an underlying universe & NS sets (S,ϱ) & (D,ϱ) are in the form

$(S,\varrho) = \{(\varphi,\ \langle\varepsilon,\ \mu_{S(\varphi)}(\varepsilon),\ \sigma_{S(\varphi)}(\varepsilon),\ \nu_{S(\varphi)}(\varepsilon)\rangle\ :\ \varepsilon\ \in\ \mathbb{W})\ :\ \varphi \in \varrho\}$ &

$(D,\varrho) = \{(\varphi,\ \langle\varepsilon,\ \mu_{D(\varphi)}(\varepsilon),\ \sigma_{D(\varphi)}(\varepsilon),\ \nu_{D(\varphi)}(\varepsilon)\rangle\ :\ \varepsilon\ \in\ \mathbb{W})\ :\ \varphi \in \varrho\}$, then

1. $0_{(\mathbb{W},\varrho)} = \{(\varphi, \langle \varepsilon, 0, 0, 1\rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}$ and $1_{(\mathbb{W},\varrho)} = \{(\varphi, \langle \varepsilon, 1, 1, 0\rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}$.
2. $(S,\varrho) \subseteq (D,\varrho)$ iff $\mu_{S(\varphi)}(\varepsilon) \le \mu_{D(\varphi)}(\varepsilon)$, $\sigma_{S(\varphi)}(\varepsilon) \le \sigma_{D(\varphi)}(\varepsilon)$ and $\nu_{S(\varphi)}(\varepsilon) \ge \nu_{D(\varphi)}(\varepsilon) : \varepsilon \in \mathbb{W} : \varphi \in \varrho$.
3. $(S,\varrho) = (D,\varrho)$ iff $(S,\varrho) \subseteq (D,\varrho)$ and $(D,\varrho) \subseteq (S,\varrho)$.
4. $(S,\varrho)^c = \{(\varphi, \langle \varepsilon, \nu_{S(\varphi)}(\varepsilon), 1 - \sigma_{S(\varphi)}(\varepsilon), \mu_{S(\varphi)}(\varepsilon)\rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}$.
5. $(S,\varrho) \cup (D,\varrho) = \{(\varphi, \langle \varepsilon, \max(\mu_{S(\varphi)}(\varepsilon), \mu_{D(\varphi)}(\varepsilon)), \max(\sigma_{S(\varphi)}(\varepsilon), \sigma_{D(\varphi)}(\varepsilon)),$
   $\min(\nu_{S(\varphi)}(\varepsilon), \nu_{D(\varphi)}(\varepsilon))\rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}$.
6. $(S,\varrho) \cap (D,\varrho) = \{(\varphi, \langle \varepsilon, \min(\mu_{S(\varphi)}(\varepsilon), \mu_{D(\varphi)}(\varepsilon)), \min(\sigma_{S(\varphi)}(\varepsilon), \sigma_{D(\varphi)}(\varepsilon)),$
   $\max(\nu_{S(\varphi)}(\varepsilon), \nu_{D(\varphi)}(\varepsilon))\rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}$.

Definition 2.5 [4] A neutrosophic soft topology (in short, NSt) on an underlying universe $\mathbb{W}$ is a collection of τ of NS subsets (S,ϱ) of $\mathbb{W}$ where ϱ be the parameters set, satisfying

1. $0_{(\mathbb{W},\varrho)}$, $1_{(\mathbb{W},\varrho)} \in \tau$.
2. $[(S,\varrho) \cap (D,\varrho)] \in \tau$ for any $(S,\varrho), (D,\varrho) \in \tau$.
3. $\bigcup_{k\in K} (S,\varrho)_k \in \tau$ for all $(S,\varrho_k) : k \in K \subseteq \tau$.

Then $(\mathbb{W},\tau,\varrho)$ is known as a *neutrosophic soft topological space* (shortly, NSts) and the elements of τ elements are known as *neutrosophic soft open sets* (shortly, NSOS) in $\mathbb{W}$. A NSS (S,ϱ) is called the *neutrosophic soft closed set* (in short, NSCS) if its complement $(S,\varrho)^c$ is NSOS.

Definition 2.6 [4] Let $(\mathbb{W},\tau,\varrho)$ act as a NSts on $\mathbb{W}$ & let (S,ϱ) is a NSS on $\mathbb{W}$. *The neutrosophic soft interior* of (S,ϱ) (in brief, NSint(S,ϱ)) and *the neutrosophic soft closure* of (S,ϱ) (in brief, NScl(S,ϱ)) are represented as

(i) NSint$(S,\varrho) = \bigcup\{(D,\varrho) : (D,\varrho) \subseteq (S,\varrho)$ and (D,ϱ) is a NSOS in $\mathbb{W}\}$.
(ii) NScl$(S,\varrho) = \bigcap\{(D,\varrho) : (D,\varrho) \supseteq (S,\varrho)$ and (D,ϱ) is a NSCS in $\mathbb{W}\}$.

Definition 2.7 [4] Suppose $(\mathbb{W},\tau,\varrho)$ act as a NSts on $\mathbb{W}$ & let (S,ϱ) is a NSS on $\mathbb{W}$. Then (S,ϱ) is called the NS

(i) regular-open set (in short, NSROS) if (S,ϱ) = NSint(NScl(S,ϱ)).

(ii) pre-open set (briefly, NSPOS) if $(S,\varrho) \subseteq$ NSint(NScl(S,ϱ)).

(iii) semi-open set (briefly, NSSOS) if $(S,\varrho) \subseteq$ NScl(NSint(S,ϱ)).

(iv) α-open set (shortly, NSαOS) if $(S,\varrho) \subseteq$ NSint(NScl(NSint(S,ϱ))).

(v) β −open set (shortly, NSβOS) if $(S,\varrho) \subseteq$ NScl(NSint(NScl(S,ϱ))).

The complement of a NSROS(resp. NSPOS, NSSOS, NSαOS, NSβOS) is called a neutrosophic soft regular (resp. pre, semi, α, β) closed set (shortly, NSRCS(resp. NSPCS, NSSCS, NSαCS, NSβCS)) in $\mathbb{W}$.

The family of all NSROS(resp. NSRCS, NSPOS, NSPCS, NSSOS, NSSCS, NSαOS, NSαCS, NSβOS, NSβCS) of $\mathbb{W}$ is represented by NSROS($\mathbb{W}$) (resp. NSRCS($\mathbb{W}$), NSPOS($\mathbb{W}$) NSPCS($\mathbb{W}$), NSSOS($\mathbb{W}$), NSSCS($\mathbb{W}$), NSαOS($\mathbb{W}$), NSαCS($\mathbb{W}$), NSβOS($\mathbb{W}$), NSβCS($\mathbb{W}$)).

Definition 2.8 [1] Let (D, ϱ) be a NSts. Then

(i) neutrosophic soft δ-interior of (D, ϱ) (in short, NSδint(D, ϱ)) is defined by NSδint(D, ϱ) = $\cup\{(D, \varrho) : (S, \varrho) \subseteq (D, \varrho)$ and (S, ϱ) is a NSROS in $\mathbb{W}\}$

(ii) neutrosophic soft δ-closure of (D, ϱ) (in short, NSδcl(D, ϱ)) is defined by NSδcl(D, ϱ) = $\cap\{(S, \varrho) : (S, \varrho) \supseteq (D, \varrho)$ & (S, ϱ) is a NSRCS in $\mathbb{W}\}$

Definition 2.9 [1] A NSS (D, ϱ) is referred as the neutrosophic soft δ-open set(shortly, NSδOS) if (D, ϱ) = NSδint(D, ϱ).
The complement of NSδOS is called NSδCS.

Definition 2.10 [13] A NSS (D, ϱ) is called the neutrosophic soft

(i) δ-semiopen set (in short, NSδSOS) if $(D, \varrho) \subseteq$ NScl(NSδint(D, ϱ)).

(ii) e-open set (briefly, NSeOS) if $(D, \varrho) \subseteq$ NScl(NSδint(D, ϱ)) ∪ NSint(NSδcl(D, ϱ)).

The complement of NSδSOS and NSeOS is called NSδSCS and NSeCS.

Throughout this paper, Let $(\mathbb{W}, \tau, \varrho)$ be any NSts. Let (S, ϱ) & (D, ϱ) be a neutrosophic soft sets in NSts.

3. Neutrosophic soft contra Z - continuous maps

Definition 3.1 A mapping $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is said to be a neutrosophic soft contra Z-continuous (shortly, NSContraZCts) if the inverse image of each NSOS of $(\mathbb{T}, \sigma, \varrho)$ is NSZCS in $(\mathbb{W}, \tau, \varrho)$.

Example 3.1 Let $\mathbb{W} = \{w_{1,} w_{2,} w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_{1,} e_2\}$ and NS sets (S_1,ϱ), (S_2,ϱ) and (S_3,ϱ) in $\mathbb{W}$ and (V_1, ϱ) in $\mathbb{T}$ are defined as

$$(S_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$$

$$(S_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_2, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(S_2, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.4}\right)\rangle$$

$$(S_3, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.3}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_3, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}\right)\rangle$$

$$(S_4, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(S_4, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.4}\right), \left(\frac{\mu_{w_2}}{0.7}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.3}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.3}\right)\rangle$$

$(V_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.5}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.6}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.6}\right)\rangle$

$(V_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.4}\right), \left(\frac{\mu_{t_2}}{0.7}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.3}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.3}\right)\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then

1) $\mathcal{G}$ is a NSContraZCts function.
2) $\mathcal{G}$ is a NSContraCts but not NSContraδCts, because the set $\mathcal{G}^{-1}(V_1, \varrho) = (S_4, \varrho)$ is a NSCS but not NSδCS.

Preposition 3.1 The statements hold true but not the converse.

a) Each NSContraδCts is a NSContraCts.
b) Each NSContraCts is a NSContraδSCts.
c) Each NSContraCts is a NSContraPCts .
d) Each NSContraδSCts is a NSContraZCts.
e) Each NSContraPCts is a NSContraZCts.
f) Each NSContraZCts is a NSContraeCts.

Proof. Consider the map $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$.

(a) Let (S, ϱ) be a NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraδCts, $\mathcal{G}^{-1}(S, \varrho)$ is a NSδCS in $\mathbb{W}$. Since all NSδCS are NSCS, $\mathcal{G}^{-1}(S, \varrho)$ is NSCS in $\mathbb{W}$. Thus, $\mathcal{G}$ is a NSContraCts.

(b) Let (S, ϱ) be a NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraCts, $\mathcal{G}^{-1}(S, \varrho)$ is a NSCS in $\mathbb{W}$. Since all NSCS are NSδCS, $\mathcal{G}^{-1}(S, \varrho)$ is a NSδSCS in $\mathbb{W}$. Thus, $\mathcal{G}$ is a NSContraδSCts.

(c) Let (S, ϱ) be a NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraCts, $\mathcal{G}^{-1}(S, \varrho)$ is a NSCS in $\mathbb{W}$. Since all NSCS is a NSPCS, $\mathcal{G}^{-1}(S, \varrho)$ is a NSPCS in $\mathbb{W}$. Thus, $\mathcal{G}$ is a NSContraPCts.

(d) Let (S, ϱ) be a NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraδSCts, $\mathcal{G}^{-1}(S, \varrho)$ is a NSδSCS in $\mathbb{W}$. Since all NSδSCS is a NSZCS, $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{W}$. Thus, $\mathcal{G}$ is a NSContraZCts.

(e) Let (S, ϱ) be a NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraPCts, $\mathcal{G}^{-1}(S, \varrho)$ is a NSPCS in $\mathbb{W}$. Since all NSPCS is a NSZCS, $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{W}$. Thus, $\mathcal{G}$ is a NSContraZCts.

(f) Let (S, ϱ) be a NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraZCts, $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{W}$. Since all NSZCS is a NSeCS, $\mathcal{G}^{-1}(S, \varrho)$ is a NSeCS in $\mathbb{W}$. Thus, $\mathcal{G}$ is a NSContraeCts.

Example 3.2 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (S_1,ϱ), (S_2,ϱ) and (S_3,ϱ) in $\mathbb{W}$ and (V_1, ϱ) in $\mathbb{T}$ are defined as

$(S_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$

$(S_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}\right)\rangle$

$(S_2, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$

$(S_2, e_2) = \langle \left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.4}\right) \rangle$

$(S_3, e_1) = \langle \left(\frac{\mu_{w_1,}}{0.3}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}\right) \rangle$

$(S_3, e_2) = \langle \left(\frac{\mu_{w_1,}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}\right) \rangle$

$(S_4, e_1) = \langle \left(\frac{\mu_{w_1,}}{0.7}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.2}\right), \left(\frac{\mu_{w_2}}{0.8}, \frac{\sigma_{w_2}}{0.6}, \frac{\nu_{w_2}}{0.4}\right), \left(\frac{\mu_{w_3}}{0.7}, \frac{\sigma_{w_3}}{0.8}, \frac{\nu_{w_3}}{0.3}\right) \rangle$

$(S_4, e_2) = \langle \left(\frac{\mu_{w_1,}}{0.7}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.2}\right), \left(\frac{\mu_{w_2}}{0.7}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.2}\right), \left(\frac{\mu_{w_3}}{0.9}, \frac{\sigma_{w_3}}{0.6}, \frac{\nu_{w_3}}{0.2}\right) \rangle$

$(V_1, e_1) = \langle \left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.2}\right), \left(\frac{\mu_{t_2}}{0.8}, \frac{\sigma_{t_2}}{0.6}, \frac{\nu_{t_2}}{0.4}\right), \left(\frac{\mu_{t_3}}{0.7}, \frac{\sigma_{t_3}}{0.8}, \frac{\nu_{t_3}}{0.3}\right) \rangle$

$(V_1, e_2) = \langle \left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.2}\right), \left(\frac{\mu_{t_2}}{0.7}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.2}\right), \left(\frac{\mu_{t_3}}{0.9}, \frac{\sigma_{t_3}}{0.6}, \frac{\nu_{t_3}}{0.2}\right) \rangle$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraPCts but not NSContraCts, because the set $\mathcal{G}^{-1}(V_1, \varrho) = (S_4, \varrho)$ is a NSPCS but not NSCS.

Remark 3.1 From the results discussed above, the following diagram is obtained.

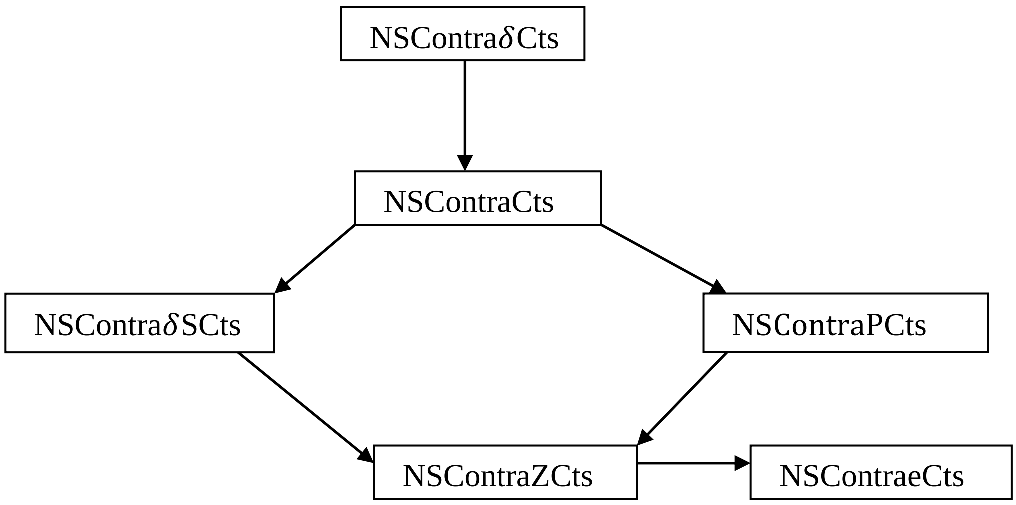

Diagram.1 Neutrosophic soft contra Z – continuous maps

Example 3.3 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (S_1,ϱ), (S_2,ϱ) and (S_3,ϱ) in $\mathbb{W}$ and (V_1, ϱ) in $\mathbb{T}$ are defined as

$(S_1, e_1) = \langle \left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right) \rangle$

$(S_1, e_2) = \langle \left(\frac{\mu_{w_1,}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}\right) \rangle$

$(S_2,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5},\frac{\sigma_{w_1}}{0.5},\frac{\nu_{w_1}}{0.6}\right),\left(\frac{\mu_{w_2}}{0.5},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.5}\right),\left(\frac{\mu_{w_3}}{0.6},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.6}\right)\rangle$

$(S_2,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.4},\frac{\sigma_{w_1}}{0.6},\frac{\nu_{w_1}}{0.6}\right),\left(\frac{\mu_{w_2}}{0.3},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.7}\right),\left(\frac{\mu_{w_3}}{0.3},\frac{\sigma_{w_3}}{0.7},\frac{\nu_{w_3}}{0.4}\right)\rangle$

$(S_3,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.3},\frac{\sigma_{w_1}}{0.4},\frac{\nu_{w_1}}{0.7}\right),\left(\frac{\mu_{w_2}}{0.1},\frac{\sigma_{w_2}}{0.3},\frac{\nu_{w_2}}{0.8}\right),\left(\frac{\mu_{w_3}}{0.2},\frac{\sigma_{w_3}}{0.3},\frac{\nu_{w_3}}{0.8}\right)\rangle$

$(S_3,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.1},\frac{\sigma_{w_1}}{0.3},\frac{\nu_{w_1}}{0.7}\right),\left(\frac{\mu_{w_2}}{0.1},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.8}\right),\left(\frac{\mu_{w_3}}{0.1},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.9}\right)\rangle$

$(S_4,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5},\frac{\sigma_{w_1}}{0.5},\frac{\nu_{w_1}}{0.5}\right),\left(\frac{\mu_{w_2}}{0.6},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.6}\right),\left(\frac{\mu_{w_3}}{0.4},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.5}\right)\rangle$

$(S_4,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.2},\frac{\sigma_{w_1}}{0.5},\frac{\nu_{w_1}}{0.3}\right),\left(\frac{\mu_{w_2}}{0.6},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.4}\right),\left(\frac{\mu_{w_3}}{0.4},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.3}\right)\rangle$

$(V_1,e_1) = \langle\left(\frac{\mu_{t_1,}}{0.5},\frac{\sigma_{t_1}}{0.5},\frac{\nu_{t_1}}{0.5}\right),\left(\frac{\mu_{t_2}}{0.6},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.6}\right),\left(\frac{\mu_{t_3}}{0.4},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.5}\right)\rangle$

$(V_1,e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2},\frac{\sigma_{t_1}}{0.5},\frac{\nu_{t_1}}{0.3}\right),\left(\frac{\mu_{t_2}}{0.6},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.4}\right),\left(\frac{\mu_{t_3}}{0.4},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.3}\right)\rangle$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1,\varrho), (S_2,\varrho), (S_3,\varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1,\varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then

(i) $\mathcal{G}$ is a NSContraZSCts but not NSContraδCts, because the set $\mathcal{G}^{-1}(V_1, \varrho)$ = (S_4, ϱ) is a NSδZCS but not NSCS.

(ii) $\mathcal{G}$ is a NSContraZCts but not NSContrPCts, because the set $\mathcal{G}^{-1}(V_1, \varrho)$ = (S_4, ϱ) is a NSZCS but not NSPCS.

Example 3.4 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (S_1,ϱ), (S_2,ϱ) and (S_3,ϱ) in $\mathbb{W}$ and (V_1, ϱ) in $\mathbb{T}$ are defined as

$(S_1,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.4},\frac{\sigma_{w_1}}{0.5},\frac{\nu_{w_1}}{0.6}\right),\left(\frac{\mu_{w_2}}{0.5},\frac{\sigma_{w_2}}{0.4},\frac{\nu_{w_2}}{0.8}\right),\left(\frac{\mu_{w_3}}{0.4},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.7}\right)\rangle$

$(S_1,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.2},\frac{\sigma_{w_1}}{0.4},\frac{\nu_{w_1}}{0.6}\right),\left(\frac{\mu_{w_2}}{0.2},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.7}\right),\left(\frac{\mu_{w_3}}{0.2},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.8}\right)\rangle$

$(S_2,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5},\frac{\sigma_{w_1}}{0.5},\frac{\nu_{w_1}}{0.6}\right),\left(\frac{\mu_{w_2}}{0.5},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.5}\right),\left(\frac{\mu_{w_3}}{0.6},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.6}\right)\rangle$

$(S_2,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.4},\frac{\sigma_{w_1}}{0.6},\frac{\nu_{w_1}}{0.6}\right),\left(\frac{\mu_{w_2}}{0.3},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.7}\right),\left(\frac{\mu_{w_3}}{0.3},\frac{\sigma_{w_3}}{0.7},\frac{\nu_{w_3}}{0.4}\right)\rangle$

$(S_3,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.3},\frac{\sigma_{w_1}}{0.4},\frac{\nu_{w_1}}{0.7}\right),\left(\frac{\mu_{w_2}}{0.1},\frac{\sigma_{w_2}}{0.3},\frac{\nu_{w_2}}{0.8}\right),\left(\frac{\mu_{w_3}}{0.2},\frac{\sigma_{w_3}}{0.3},\frac{\nu_{w_3}}{0.8}\right)\rangle$

$(S_3,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.1},\frac{\sigma_{w_1}}{0.3},\frac{\nu_{w_1}}{0.7}\right),\left(\frac{\mu_{w_2}}{0.1},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.8}\right),\left(\frac{\mu_{w_3}}{0.1},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.9}\right)\rangle$

$(S_4,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.7},\frac{\sigma_{w_1}}{0.7},\frac{\nu_{w_1}}{0.1}\right),\left(\frac{\mu_{w_2}}{0.8},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.1}\right),\left(\frac{\mu_{w_3}}{0.8},\frac{\sigma_{w_3}}{0.7},\frac{\nu_{w_3}}{0.2}\right)\rangle$

$(S_4,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.7},\frac{\sigma_{w_1}}{0.7},\frac{\nu_{w_1}}{0.1}\right),\left(\frac{\mu_{w_2}}{0.8},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.11}\right),\left(\frac{\mu_{w_3}}{0.9},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.1}\right)\rangle$

$(V_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.3}\right), \left(\frac{\mu_{t_2}}{0.8}, \frac{\sigma_{t_2}}{0.7}, \frac{\nu_{t_2}}{0.1}\right), \left(\frac{\mu_{t_3}}{0.8}, \frac{\sigma_{t_3}}{0.7}, \frac{\nu_{t_3}}{0.2}\right)\rangle$

$(V_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.7}, \frac{\nu_{t_1}}{0.1}\right), \left(\frac{\mu_{t_2}}{0.8}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.1}\right), \left(\frac{\mu_{t_3}}{0.9}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.1}\right)\rangle$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, the $\mathcal{G}$ is a NSContraZCts but not NSContraδSCts, because the set $\mathcal{G}^{-1}(V_1, \varrho) = (S_4, \varrho)$ is a NSZCS but not NSδSCS.

Example 3.5 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (P_1,ϱ), (P_2,ϱ) and (P_3,ϱ) in $\mathbb{W}$ and (Q_1, ϱ) in $\mathbb{T}$ are defined as

$(P_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.3}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.9}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.4}, \frac{\nu_{w_3}}{0.8}\right)\rangle$

$(P_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}\right)\rangle$

$(P_2, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.6}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.6}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.5}\right)\rangle$

$(P_2, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.4}, \frac{\nu_{w_3}}{0.6}\right)\rangle$

$(P_3, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.4}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$

$(P_3, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.7}\right)\rangle$

$(Q_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.4}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.3}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

$(Q_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.4}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (P_1, \varrho), (P_2, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (Q_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraeCts but not NSContraZCts, because the set $\mathcal{G}^{-1}(Q_1, \varrho) = (P_3, \varrho)$ is a NSeCS but not NSZCS.

Theorem 3.1 A map $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSContraZCts iff the inverse image of every NSCS in $\mathbb{T}$ is NSZOS in $\mathbb{W}$.

Proof. Consider a NSCS (S, ϱ) in $\mathbb{T}$. Then $(S, \varrho)^c$ is NSOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraZCts, $\mathcal{G}^{-1}((S, \varrho)^c)$ is NSZCS in $\mathbb{W}$. As $\mathcal{G}^{-1}((S, \varrho)^c) = (\mathcal{G}^{-1}(S, \varrho))^c$, $\mathcal{G}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{W}$.

Conversely, consider a NSCS (S, ϱ) in $\mathbb{T}$. So $(S, \varrho)^c$ is a NSOS in $\mathbb{T}$. By hypothesis, $\mathcal{G}^{-1}((S, \varrho)^c)$ is NSZCS in $\mathbb{W}$. As $\mathcal{G}^{-1}((S, \varrho)^c) = (\mathcal{G}^{-1}(S, \varrho))^c$, $(\mathcal{G}^{-1}(S, \varrho))^c$ is a NSZCS in $\mathbb{W}$. Hence, $\mathcal{G}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{W}$. Hence $\mathcal{G}$ is NSContraZCts.

Theorem 3.2 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a NSContraZCts where every NSZOS in $\mathbb{W}$ is a NSOS in $\mathbb{W}$, then $\mathcal{G}$ is a NSContraCts.

Proof. Let (S, ϱ) be a NSOS in $\mathbb{T}$. Then $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{W}$. By hypothesis, $\mathcal{G}^{-1}(S, \varrho)$ is a NSOS in $\mathbb{W}$, then $\mathcal{G}$ is a NSContraZCts.

Theorem 3.3 Let $\mathcal{G}: (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSContraZCts map and $\mathcal{H}: (\mathbb{T}, \sigma, \varrho) \rightarrow (\mathbb{U}, \rho, \varrho)$ be an NSContraCts, then $\mathcal{H} \circ \mathcal{G}: (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{U}, \rho, \varrho)$ is a NSContraZCts map.

Proof. Let (S, ϱ) be a NSOS in $\mathbb{U}$. Then $\mathcal{H}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{T}$, by hypothesis, Since $\mathcal{G}$ is a NSContraZCts maps, $\mathcal{G}^{-1}(\mathcal{H}^{-1}(S, \varrho))$ is a NSZCS in $\mathbb{W}$. Hence $\mathcal{H} \circ \mathcal{G}$ is a NSContraZCts map.

Theorem 3.4 Let $\mathcal{G}: (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSContraZCts map. Then the following conditions are hold.

(i) $\mathcal{G}(\text{NSZcl}(S, \varrho)) \supseteq \text{NSint}(\mathcal{G}(S, \varrho))$, for all NSS (S, ϱ) in $\mathbb{W}$.

(ii) $\text{NSZcl}(\mathcal{G}^{-1}(D, \varrho)) \supseteq \mathcal{G}^{-1}(\text{NSint}(D, \varrho))$, for all NSS in $\mathbb{T}$.

Proof. (i) As $\text{NSZcl}(\mathcal{G}(S, \varrho))$ is a NSZCS in $\mathbb{T}$ and $\mathcal{G}$ is NSContraZCts, then $\mathcal{G}^{-1}(\text{NSZcl}(\mathcal{G}(S, \varrho))$ is NSZOS in $\mathbb{W}$. Now, as $(S, \varrho) \supseteq \mathcal{G}^{-1}(\text{NSint}(\mathcal{G}(S, \varrho)))$. $\text{NSZcl}(S, \varrho) \supseteq \mathcal{G}^{-1}(\text{NSint}(\mathcal{G}(S, \varrho)))$. Therefore, $\mathcal{G}(\text{NSZcl}(S, \varrho)) \supseteq \text{NSint}(\mathcal{G}(S, \varrho))$.

(ii) By replacing (S, ϱ) by (D, ϱ) in (i), we obtain $\mathcal{G}(\text{NSZcl}(\mathcal{G}^{-1}(D, \varrho))) \supseteq \text{NSint}(\mathcal{G}(\mathcal{G}^{-1}(D, \varrho))) \supseteq \text{NSint}(D, \varrho)$. Hence $\text{NSZcl}(\mathcal{G}^{-1}(D, \varrho)) \supseteq \mathcal{G}^{-1}(\text{NSint}(D, \varrho))$.

4. Neutrosophic soft contra Z – irresolute maps

Definition 4.1 A map $\mathcal{G}: (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ is known as a neutrosophic soft contra Z-irresolute (briefly, NSContraZ-irr) map if $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $(\mathbb{W}, \tau, \varrho)$ for each NSZOS (S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$.

Theorem 4.1 Let $\mathcal{G}: (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSContraZ-irr. Then $\mathcal{G}$ is a NSContraZCts map. But not conversely.

Proof. Assume $\mathcal{G}$ is a NSContraZ-irr map. Consider a NSOS (S, ϱ) in $\mathbb{T}$. As each NSOS is a NSZOS, (S, ϱ) is a NSZOS in $\mathbb{T}$. By hypothesis, $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{W}$. Hence $\mathcal{G}$ is a NSContraZCts map.

Example 4.1 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (S_1,ϱ), (S_2,ϱ), (S_3,ϱ), and (S_4, ϱ) in $\mathbb{W}$ and (V_1,ϱ) and (V_2,ϱ) in $\mathbb{T}$ are defined as

$$(S_1, e_1) = \langle\left(\frac{\mu_{w_1}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$$

$$(S_1, e_2) = \langle\left(\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_2, e_1) = \langle\left(\frac{\mu_{w_1}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(S_2, e_2) = \langle\left(\frac{\mu_{w_1}}{0.4}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.4}\right)\rangle$$

$$(S_3, e_1) = \langle\left(\frac{\mu_{w_1}}{0.3}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_3, e_2) = \langle\left(\frac{\mu_{w_1}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}\right)\rangle$$

$(S_4, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$

$(S_4, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.4}\right), \left(\frac{\mu_{w_2}}{0.7}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.3}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.3}\right)\rangle$

$(V_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.4}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

$(V_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.2}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(V_2, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.5}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.6}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.6}\right)\rangle$

$(V_2, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.4}\right), \left(\frac{\mu_{t_2}}{0.7}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.3}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.3}\right)\rangle$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraZCts but not NSContraZ-irr, because the set (V_2, ϱ) is a NSZCS in $\mathbb{T}$ but $\mathcal{G}^{-1}(V_2, \varrho) = (S_4, \varrho)$ is not NSZOS in $\mathbb{W}$.

Theorem 4.2 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a NSContraZ-irr. If $\mathbb{W}$ is a NSZ$U_{\frac{1}{2}}$ – space, then $\mathcal{G}$ is a NSContraCts map.

Proof. Consider a NSOS (S, ϱ) in $\mathbb{T}$. Then (S, ϱ) is a NSZOS in $\mathbb{T}$. Hence $\mathcal{G}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{W}$. As $\mathbb{W}$ is a NSZ$U_{\frac{1}{2}}$ – space, $\mathcal{G}^{-1}(S, \varrho)$ is a NSCS in $\mathbb{W}$. Thus $\mathcal{G}$ is a NSContraCts map.

Theorem 4. 3 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a NSContraZ-irr and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be a NSZCts maps. Then $\mathcal{H} \circ \mathcal{G}: (\mathbb{W}, \tau, \varrho) \to (\mathbb{U}, \rho, \varrho)$ is a NSContraZCts map.

Proof. Consider a NSOS (S, ϱ) in $\mathbb{U}$. Then $\mathcal{H}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{T}$. As $\mathcal{G}$ is a NSContraZ-irr, $\mathcal{G}^{-1}(\mathcal{H}^{-1}(S, \varrho))$ is a NSZCS in $\mathbb{W}$. Hence $\mathcal{H} \circ \mathcal{G}$ is a NSContraZCts map.

Theorem 4.4 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be mappings. Then $\mathcal{H} \circ \mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{U}, \rho, \varrho)$ is

(i) NSContraZCts if $\mathcal{G}$ is NSZirr and $\mathcal{H}$ is NSContraZCts.

(ii) NSContraZ-irr if $\mathcal{G}$ is NSContraZirr (resp. NSZirr) and $\mathcal{H}$ is NSZ-irr (resp NSContraZ-irr).

Proof. (i) Let (S, ϱ) be a NSOS in $\mathbb{U}$. Then $\mathcal{H}^{-1}(S, \varrho)$ is a NSZCS in $\mathbb{T}$. As $\mathcal{H}$ is a NSZ-irr map, $\mathcal{G}^{-1}(\mathcal{H}^{-1}(S, \varrho))$ is a NSZCS in $\mathbb{W}$. Hence $\mathcal{H} \circ \mathcal{G}$ is a NSContraZCts map.

The other cases are similar.

Theorem 4.5 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a mapping.

(i) If $(\mathbb{W}, \tau, \varrho)$ is NSZ$U_{\frac{1}{2}}$ – space, then the concepts of NSContraCts and NSContraZCts are equivalent.

(ii) If $(\mathbb{T}, \sigma, \varrho)$ is NSZ$U_{\frac{1}{2}}$ – space, then the concepts of NSContraZCts and NSContraZ-irr are equivalent.

(iii) If $(\mathbb{W}, \tau, \varrho)$ and $(\mathbb{T}, \sigma, \varrho)$ are NSZ$U_{\frac{1}{2}}$ – space, then the concepts of NSContraCts, NSContraZCts and NSContraZ-irr are equivalent.

Proof. (i) Let (S, ϱ) be a NSCS in $\mathbb{T}$. Then $\mathcal{H}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{W}$ if $\mathcal{G}$ is NSContraZCts. As $(\mathbb{W}, \tau, \varrho)$ is a NSZ$U_{\frac{1}{2}}$ – space, $\mathcal{H}^{-1}(S, \varrho)$ is a NSOS in $\mathbb{W}$. Hence $\mathcal{G}$ is also NSContraCts map.

The other cases are similar.

Theorem 4.6 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be NSContraZCts mappings and $(\mathbb{T}, \sigma, \varrho)$ be a NSZ$U_{\frac{1}{2}}$ – space. Then $\mathcal{H} \circ \mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is a NSZCts map.

Proof. Let (S, ϱ) be a NSCS in $\mathbb{U}$. Then $\mathcal{H}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{T}$. Since $\mathcal{H}$ is NSContraZCts. As $(\mathbb{T}, \sigma, \varrho)$ is a NSZ$U_{\frac{1}{2}}$ – space, $\mathcal{H}^{-1}(S, \varrho)$ is a NSOS in $\mathbb{T}$. Then, $\mathcal{G}(\mathcal{H}^{-1}(S, \varrho))$ is a NSZCS in $\mathbb{W}$ because $\mathcal{G}$ is NSContraZCts. Hence, $\mathcal{H} \circ \mathcal{G}$ is a NSZCts map.

Theorem 4.7 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a map from a NSts $\mathbb{W}$ into a NSts $\mathbb{T}$. If $\mathbb{W}$ and $\mathbb{T}$ are NSZ$U_{\frac{1}{2}}$ – space, then the following are equivalent.

(i) $\mathcal{G}$ is a NSContraZ-irr map.

(ii) $\mathcal{G}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{W}$ for every NSZCS (S, ϱ) in $\mathbb{T}$.

(iii) NScl($\mathcal{G}^{-1}(S, \varrho)$) $\supseteq \mathcal{G}^{-1}$ (NSint(S, ϱ)) for each (S, ϱ) of $\mathbb{T}$.

Proof. (i) → (ii): Consider a NSZCS (S, ϱ) in $\mathbb{T}$. Then $(S, \varrho)^c$ is a NSZOS in $\mathbb{T}$. As $\mathcal{G}$ is NSContraZ-irr, $\mathcal{G}^{-1}((S, \varrho)^c)$ is a NSZCS in $\mathbb{W}$. We know that, $\mathcal{G}^{-1}((S, \varrho)^c) = (\mathcal{G}^{-1}(S, \varrho))^c$. Thus $\mathcal{G}^{-1}(S, \varrho)$ is a NSZOS in $\mathbb{W}$.

(ii) → (iii) : Consider a NSS (S, ϱ) in $\mathbb{T}$ and NSint$(S, \varrho) \subseteq (S, \varrho)$. Then $\mathcal{G}^{-1}(NSint(S, \varrho)) \subseteq \mathcal{G}^{-1}(S, \varrho)$. As NSint$(S, \varrho)$ is a NSOS in $\mathbb{T}$, NSint(S, ϱ) is a NSZOS in $\mathbb{T}$. Therefore $(NSint(S, \varrho))^c$ is a NSZCS in $\mathbb{T}$. By presumption, $\mathcal{G}^{-1}$(NSint$(S, \varrho))^c$ is a NSZOS in $\mathbb{W}$. As $\mathcal{G}^{-1}((NSint(S, \varrho))^c) = (\mathcal{G}^{-1}(\text{NSint}(S, \varrho)))^c$, $\mathcal{G}^{-1}$(NSint(S, ϱ)) is a NSZOS in $\mathbb{W}$. As $\mathbb{W}$ is a NSZ$U_{\frac{1}{2}}$ – space, $\mathcal{G}^{-1}$ (NSint(S, ϱ)) is a NSOS in $\mathbb{W}$. Thus, NScl($\mathcal{G}^{-1}(S, \varrho)$) $\supseteq$ NScl($\mathcal{G}^{-1}$(NSint(S, ϱ))) $= \mathcal{G}^{-1}$ (NSint(S, ϱ)). That is, NScl($\mathcal{G}^{-1}(S, \varrho)$) $\supseteq \mathcal{G}^{-1}$(NSint$(S, \varrho)$).

(iii) → (i): Consider a NSZCS (S, ϱ) in $\mathbb{T}$. As $\mathbb{T}$ is NSZ$U_{\frac{1}{2}}$ – space, (S, ϱ) is a NSCS in $\mathbb{T}$ and NScl$(S, \varrho) = (S, \varrho)$. Hence $\mathcal{G}^{-1}(S, \varrho) = \mathcal{G}^{-1}$(NScl$(S, \varrho)$) $\supseteq$ NSint($\mathcal{G}^{-1}(S, \varrho)$). But clearly $\mathcal{G}^{-1}(S, \varrho) \supseteq$ NSint($\mathcal{G}^{-1}(S, \varrho)$).

Therefore NSint($\mathcal{G}^{-1}(S, \varrho)$) $= \mathcal{G}^{-1}(S, \varrho)$. So, $\mathcal{G}^{-1}(S, \varrho)$ is a NSOS and hence it is a NSZOS in $\mathbb{W}$. Thus $\mathcal{G}$ is a NSContraZ-irr map.

5. Neutrosophic soft contra Z - open mapping

Definition 5.1 A mapping $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is neutrosophic soft contra Z – open (in short, NSContraZO) if the image of each NSOS of $(\mathbb{W}, \tau, \varrho)$ is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$.

Theorem 5.1 The statements are hold but the converse does not true.

a) Each NSContraδO is a NSContraO.
b) Each NSContraO is a NSContraδSO.
c) Each NSContraO is a NSContraPO.
d) Each NSContraδSO is a NSContraZO.
e) Each NSContraPO is a NSContraZO.
f) Each NSContraZO is a NSContraeO.

Proof.

(a) Let (S,ϱ) be a NSOS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraδO, $\mathcal{G}(S,\varrho)$ is a NSδCS in $\mathbb{T}$. Since all NSδCS are NSCS, $\mathcal{G}(S,\varrho)$ is NSCS in $\mathbb{T}$. Thus, $\mathcal{G}$ is a NSContraO.

(b) Let (S,ϱ) be a NSOS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraO, $\mathcal{G}(S,\varrho)$ is a NSCS in $\mathbb{T}$. Since all NSCS are NSδSCS, $\mathcal{G}(S,\varrho)$ is a NSδSCS in $\mathbb{T}$. Thus, $\mathcal{G}$ is a NSContraδSO.

(c) Let (S,ϱ) be a NSOS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraO, $\mathcal{G}(S,\varrho)$ is a NSCS in $\mathbb{T}$. Since all NSCS are NSPCS, $\mathcal{G}(S,\varrho)$ is a NSPCS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraPO.

(d) Let (S,ϱ) be a NSOS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraδSO, $\mathcal{G}(S,\varrho)$ is a NSδSCS in $\mathbb{T}$. Since all NSδSCS is a NSZCS, $\mathcal{G}(S,\varrho)$ is a NSZCS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraZO.

(e) Let (S,ϱ) be a NSOS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraPO, $\mathcal{G}(S,\varrho)$ is a NSPCS in $\mathbb{T}$. Since all NSPCS are NSZCS, $\mathcal{G}(S,\varrho)$ is a NSZCS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraZO.

(f) Let (S,ϱ) be a NSOS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraZO, $\mathcal{G}(S,\varrho)$ is a NSZCS in $\mathbb{T}$. Since all NSZCS is a NSeCS, $\mathcal{G}(S,\varrho)$ is a NSeCS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraeO.

Example 5.1 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (V_1,ϱ) in $\mathbb{W}$ and (S_1,ϱ), (S_2,ϱ), (S_3,ϱ) and (S_4,ϱ) in $\mathbb{T}$ are defined as

$$(V_1,e_1) = \langle\left(\frac{\mu_{w_1,}}{0.6},\frac{\sigma_{w_1}}{0.5},\frac{\nu_{w_1}}{0.5}\right),\left(\frac{\mu_{w_2}}{0.5},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.5}\right),\left(\frac{\mu_{w_3}}{0.6},\frac{\sigma_{w_3}}{0.5},\frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(V_1,e_2) = \langle\left(\frac{\mu_{w_1,}}{0.6},\frac{\sigma_{w_1}}{0.4},\frac{\nu_{w_1}}{0.4}\right),\left(\frac{\mu_{w_2}}{0.7},\frac{\sigma_{w_2}}{0.5},\frac{\nu_{w_2}}{0.3}\right),\left(\frac{\mu_{w_3}}{0.4},\frac{\sigma_{w_3}}{0.3},\frac{\nu_{w_3}}{0.3}\right)\rangle$$

$$(S_1,\mathrm{e}_1) = \langle\left(\frac{\mu_{t_1,}}{0.4},\frac{\sigma_{t_1}}{0.5},\frac{\nu_{t_1}}{0.6}\right),\left(\frac{\mu_{t_2}}{0.5},\frac{\sigma_{t_2}}{0.4},\frac{\nu_{t_2}}{0.8}\right),\left(\frac{\mu_{t_3}}{0.4},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.7}\right)\rangle$$

$$(S_1,\mathrm{e}_2) = \langle\left(\frac{\mu_{t_1,}}{0.2},\frac{\sigma_{t_1}}{0.4},\frac{\nu_{t_1}}{0.6}\right),\left(\frac{\mu_{t_2}}{0.2},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.7}\right),\left(\frac{\mu_{t_3}}{0.2},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.8}\right)\rangle$$

$$(S_2,e_1) = \langle\left(\frac{\mu_{t_1,}}{0.5},\frac{\sigma_{t_1}}{0.5},\frac{\nu_{t_1}}{0.6}\right),\left(\frac{\mu_{t_2}}{0.5},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.5}\right),\left(\frac{\mu_{t_3}}{0.6},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.6}\right)\rangle$$

$$(S_2,e_2) = \langle\left(\frac{\mu_{t_1,}}{0.4},\frac{\sigma_{t_1}}{0.6},\frac{\nu_{t_1}}{0.6}\right),\left(\frac{\mu_{t_2}}{0.3},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.7}\right),\left(\frac{\mu_{t_3}}{0.3},\frac{\sigma_{t_3}}{0.7},\frac{\nu_{t_3}}{0.4}\right)\rangle$$

$$(S_3,e_1) = \langle\left(\frac{\mu_{t_1,}}{0.3},\frac{\sigma_{t_1}}{0.4},\frac{\nu_{t_1}}{0.7}\right),\left(\frac{\mu_{t_2}}{0.1},\frac{\sigma_{t_2}}{0.3},\frac{\nu_{t_2}}{0.8}\right),\left(\frac{\mu_{t_3}}{0.2},\frac{\sigma_{t_3}}{0.3},\frac{\nu_{t_3}}{0.8}\right)\rangle$$

$$(S_3,e_2) = \langle\left(\frac{\mu_{t_1,}}{0.1},\frac{\sigma_{t_1}}{0.3},\frac{\nu_{t_1}}{0.7}\right),\left(\frac{\mu_{t_2}}{0.1},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.8}\right),\left(\frac{\mu_{t_3}}{0.1},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.9}\right)\rangle$$

$$(S_4,e_1) = \langle\left(\frac{\mu_{t_1,}}{0.6},\frac{\sigma_{t_1}}{0.5},\frac{\nu_{t_1}}{0.5}\right),\left(\frac{\mu_{t_2}}{0.5},\frac{\sigma_{t_2}}{0.5},\frac{\nu_{t_2}}{0.5}\right),\left(\frac{\mu_{t_3}}{0.6},\frac{\sigma_{t_3}}{0.5},\frac{\nu_{t_3}}{0.6}\right)\rangle$$

$(S_4, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.4}\right), \left(\frac{\mu_{t_2}}{0.7}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.3}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.3}\right)\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraO but not NSContraδO, because the set $\mathcal{G}$ (V_1, ϱ) = (S_4, ϱ) is a NSCS but not NSδCS.

Example 5.2 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (V_1, ϱ) in $\mathbb{W}$ and (S_1,ϱ), (S_2,ϱ), (S_3,ϱ) and (S_4,ϱ) in $\mathbb{T}$ are defined as

$(V_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.7}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.2}\right), \left(\frac{\mu_{w_2}}{0.8}, \frac{\sigma_{w_2}}{0.6}, \frac{\nu_{w_2}}{0.4}\right), \left(\frac{\mu_{w_3}}{0.7}, \frac{\sigma_{w_3}}{0.8}, \frac{\nu_{w_3}}{0.3}\right)\rangle$

$(V_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.7}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.2}\right), \left(\frac{\mu_{w_2}}{0.7}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.2}\right), \left(\frac{\mu_{w_3}}{0.9}, \frac{\sigma_{w_3}}{0.6}, \frac{\nu_{w_3}}{0.2}\right)\rangle$

$(S_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.4}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

$(S_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.2}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(S_2, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.6}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.6}\right)\rangle$

$(S_2, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.3}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.3}, \frac{\sigma_{t_3}}{0.7}, \frac{\nu_{t_3}}{0.4}\right)\rangle$

$(S_3, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.3}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.1}, \frac{\sigma_{t_2}}{0.3}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(S_3, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.1}, \frac{\sigma_{t_1}}{0.3}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.1}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.1}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.9}\right)\rangle$

$(S_4, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.2}\right), \left(\frac{\mu_{t_2}}{0.8}, \frac{\sigma_{t_2}}{0.6}, \frac{\nu_{t_2}}{0.4}\right), \left(\frac{\mu_{t_3}}{0.7}, \frac{\sigma_{t_3}}{0.8}, \frac{\nu_{t_3}}{0.3}\right)\rangle$

$(S_4, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.2}\right), \left(\frac{\mu_{t_2}}{0.7}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.2}\right), \left(\frac{\mu_{t_3}}{0.7}, \frac{\sigma_{t_3}}{0.6}, \frac{\nu_{t_3}}{0.2}\right)\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraPO but not NSContraO, because the set $\mathcal{G}$ (V_1, ϱ) = (S_4, ϱ) is a NSPCS but not NSCS.

Example 5.3 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets (V_1, ϱ) in $\mathbb{W}$ and (S_1,ϱ), (S_2,ϱ), (S_3,ϱ) and (S_4,ϱ) in $\mathbb{T}$ are defined as

$(V_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{w_2}}{0.6}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.6}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.5}\right)\rangle$

$(V_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.2}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.3}\right), \left(\frac{\mu_{w_2}}{0.6}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.9}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.3}\right)\rangle$

$(S_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.4}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

$(S_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.2}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(S_2, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.6}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.6}\right)\rangle$

$(S_2, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.3}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.3}, \frac{\sigma_{t_3}}{0.7}, \frac{\nu_{t_3}}{0.4}\right)\rangle$

$(S_3, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.3}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.1}, \frac{\sigma_{t_2}}{0.3}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(S_3, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.1}, \frac{\sigma_{t_1}}{0.3}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.1}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.1}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.9}\right)\rangle$

$(S_4, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.5}\right), \left(\frac{\mu_{t_2}}{0.6}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.6}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.5}\right)\rangle$

$(S_4, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.3}\right), \left(\frac{\mu_{t_2}}{0.6}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.9}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.3}\right)\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping. Then

(i) $\mathcal{G}$ is a NSContraδSO but not NSContraO, because the set $\mathcal{G}$ (V_1, ϱ) = (S_4, ϱ) is a NSδSCS but not NSCS.

(ii) $\mathcal{G}$ is a NSContraZO but not NSContraPO, because the set $\mathcal{G}$ (V_1, ϱ) = (S_4, ϱ) is a NSZCS but not NSPCS.

Example 5.4 Let $\mathbb{W} = \{ w_1, w_2, w_3 \} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{ e_{1,} e_2\}$ and NS sets (V_1, ϱ) in $\mathbb{W}$ and (S_1,ϱ), (S_2,ϱ), (S_3,ϱ) and (S_4,ϱ) in $\mathbb{T}$ are defined as

$(V_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.7}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.3}\right), \left(\frac{\mu_{w_2}}{0.8}, \frac{\sigma_{w_2}}{0.7}, \frac{\nu_{w_2}}{0.1}\right), \left(\frac{\mu_{w_3}}{0.8}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.2}\right)\rangle$

$(V_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.7}, \frac{\sigma_{w_1}}{0.7}, \frac{\nu_{w_1}}{0.1}\right), \left(\frac{\mu_{w_2}}{0.8}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.1}\right), \left(\frac{\mu_{w_3}}{0.9}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.1}\right)\rangle$

$(S_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.4}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

$(S_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.2}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(S_2, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.6}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.6}\right)\rangle$

$(S_2, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.3}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.3}, \frac{\sigma_{t_3}}{0.7}, \frac{\nu_{t_3}}{0.4}\right)\rangle$

$(S_3, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.3}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.1}, \frac{\sigma_{t_2}}{0.3}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.8}\right)\rangle$

$(S_3, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.1}, \frac{\sigma_{t_1}}{0.3}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.1}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.1}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.9}\right)\rangle$

$(S_4, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.6}, \frac{\nu_{t_1}}{0.3}\right), \left(\frac{\mu_{t_2}}{0.8}, \frac{\sigma_{t_2}}{0.7}, \frac{\nu_{t_2}}{0.1}\right), \left(\frac{\mu_{t_3}}{0.8}, \frac{\sigma_{t_3}}{0.7}, \frac{\nu_{t_3}}{0.2}\right)\rangle$

$(S_4, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.7}, \frac{\sigma_{t_1}}{0.7}, \frac{\nu_{t_1}}{0.1}\right), \left(\frac{\mu_{t_2}}{0.8}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.1}\right), \left(\frac{\mu_{t_3}}{0.9}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.1}\right)\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1,\varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1,\varrho),(S_2,\varrho),(S_3,\varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraZO but not NSContraδSO, because the set $\mathcal{G}$ (V_1, ϱ) = (S_4, ϱ) is a NSZCS but not NSδSCS.

Remark 5.1 From the results discussed above, the following diagram is obtained.

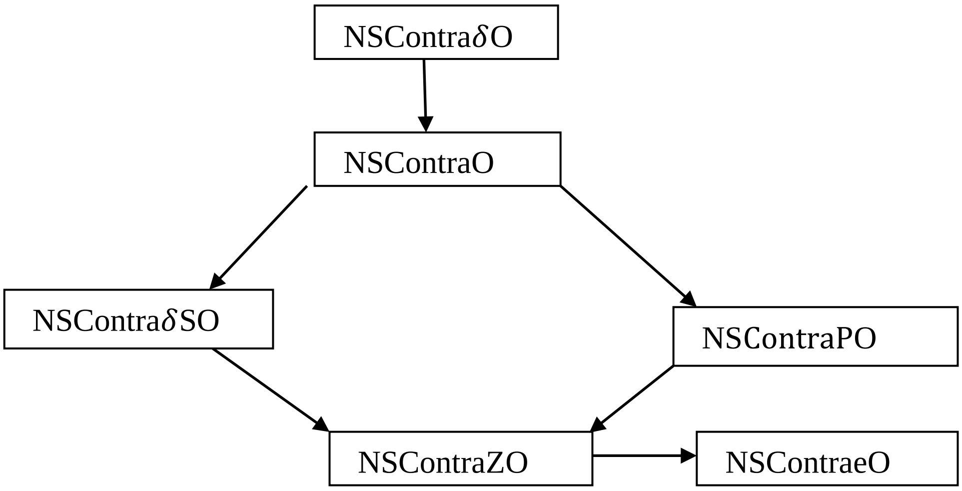

Diagram.2 Neutrosophic soft contra Z – open maps

Example 5.5 Let $\mathbb{W}$ = $\{ w_{1,} w_{2,} w_3 \}$ = $\{t_1 , t_2, t_3\}$ = $\mathbb{T}$, $\varrho = \{ e_{1,} e_2\}$ and NS sets (Q_1, ϱ) in $\mathbb{W}$ and (P_1,ϱ), (P_2,ϱ) and (P_3,ϱ) in $\mathbb{T}$ are defined as

$$(Q_1, e_1) = \langle\left(\frac{\mu_{w_{1,}}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.4}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$$

$$(Q_1, e_2) = \langle\left(\frac{\mu_{w_{1,}}}{0.6}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.7}\right)\rangle$$

$$(P_1, e_1) = \langle\left(\frac{\mu_{t_{1,}}}{0.3}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.4}, \frac{\sigma_{t_2}}{0.3}, \frac{\nu_{t_2}}{0.9}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.4}, \frac{\nu_{t_3}}{0.8}\right)\rangle$$

$$(P_1, e_2) = \langle\left(\frac{\mu_{t_{1,}}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{t_2}}{0.3}, \frac{\sigma_{t_2}}{0.4}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.1}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.8}\right)\rangle$$

$$(P_2, e_1) = \langle\left(\frac{\mu_{t_{1,}}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.6}, \frac{\sigma_{t_2}}{0.3}, \frac{\nu_{t_2}}{0.6}\right), \left(\frac{\mu_{t_3}}{0.3}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.5}\right)\rangle$$

$$(P_2, e_2) = \langle\left(\frac{\mu_{t_{1,}}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{t_2}}{0.4}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.4}, \frac{\nu_{t_3}}{0.6}\right)\rangle$$

$$(P_3, e_1) = \langle\left(\frac{\mu_{t_{1,}}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{w_1}}{0.4}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.3}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$$

$(P_3, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.4}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.7}\right)\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (Q_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (P_1, \varrho), (P_2, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is a NSContraeO but not NSContraZO, because the set $\mathcal{G}(Q_1, \varrho) = (P_3, \varrho)$ is a NSeCS but not NSZCS.

Theorem 5.2 A mapping $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSContraZO iff for every NSS (S, ϱ) of $(\mathbb{W}, \tau, \varrho)$, $\mathcal{G}(\text{NSint}(S, \varrho)) \supseteq \text{NSZcl}(\mathcal{G}(S, \varrho))$.

Proof. Necessity: Assume $\mathcal{G}$ is a NSContraZO mapping and (S, ϱ) is a NSOS $(\mathbb{W}, \tau, \varrho)$. Now, $\mathcal{G}(\text{NSint}(S, \varrho)) \subseteq (S, \varrho)$ implies $\mathcal{G}(\text{NSint}(S, \varrho)) \subseteq \mathcal{G}(S, \varrho)$. Since $\mathcal{G}$ is a NSContraZO mapping, $\mathcal{G}(\text{NSint}(S, \varrho))$ is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$ such that $\mathcal{G}(\text{NSint}(S, \varrho)) \supseteq \mathcal{G}(S, \varrho)$. Therefore, $\mathcal{G}(\text{NSint}(S, \varrho)) \supseteq \text{NSZcl}(\mathcal{G}(S, \varrho))$.

Sufficiency: Assume (S, ϱ) is a NSOS $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}(S, \varrho) = \mathcal{G}(\text{NSint}(S, \varrho)) \supseteq \text{NSZcl}(\mathcal{G}(S, \varrho))$. But $\text{NSZcl}(\mathcal{G}(S, \varrho)) \supseteq \mathcal{G}(S, \varrho)$. So, $\mathcal{G}(S, \varrho) = \text{NSZcl}(\mathcal{G}(S, \varrho))$ which implies $\mathcal{G}(S, \varrho)$ is a NSZCS of $(\mathbb{T}, \sigma, \varrho)$ and hence $\mathcal{G}$ is a NSContraZO.

Theorem 5.3 If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSContraZO mapping, then $\text{NSint}\,(\mathcal{G}^{-1}(S, \varrho) \subseteq \mathcal{G}^{-1}(\text{NSZcl}(S, \varrho))$ for every NSS (S, ϱ) of $(\mathbb{T}, \sigma, \varrho)$.

Proof. Consider a NSS (S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$. Then, $\text{NSint}\,(\mathcal{G}^{-1}(S, \varrho)$ is a NSOS in $(\mathbb{W}, \tau, \varrho)$. Since $\mathcal{G}$ is NSContraZO, $\mathcal{G}(\text{NSint}\,(\mathcal{G}^{-1}(S, \varrho))$ is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$ and hence $\mathcal{G}(\text{NSint}\,(\mathcal{G}^{-1}(S, \varrho)) \subseteq \text{NSZcl}\,(\mathcal{G}\,(\mathcal{G}^{-1}(S, \varrho))) \subseteq \text{NSZcl}(S, \varrho)$. Thus, $\text{NSint}\,(\mathcal{G}^{-1}(S, \varrho) \subseteq \mathcal{G}^{-1}(\text{NSZcl}(S, \varrho))$.

Theorem 5.4 A mapping $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSContraZO iff for every NSS (S, ϱ) of $(\mathbb{T}, \sigma, \varrho)$, and for each NSOS (B, ϱ) of $(\mathbb{W}, \tau, \varrho)$ containing $\mathcal{G}^{-1}(S, \varrho)$, there is a NSZOS (K, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ such that $(S, \varrho) \subseteq (B, \varrho)$ and $\mathcal{G}^{-1}(K, \varrho) \subseteq (B, \varrho)$.

Proof. Necessity: Assume $\mathcal{G}$ be a NSContraZO mapping. Let a NSCS (S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$ and a NSOS (B, ϱ) in $(\mathbb{W}, \tau, \varrho)$ such that $\mathcal{G}^{-1}(S, \varrho) \subseteq (B, \varrho)$. Then, $(K, \varrho) = (\mathcal{G}(B, \varrho)^c)^c$ is NSZOS of $(\mathbb{T}, \sigma, \varrho) \ni \mathcal{G}^{-1}(K, \varrho) \subseteq (B, \varrho)$.

Sufficiency: Assume (B, ϱ) is a NSOS $(\mathbb{W}, \tau, \varrho)$. So, $\mathcal{G}^{-1}(\mathcal{G}(B, \varrho)^c) \subseteq (B, \varrho)^c$ and $(B, \varrho)^c$ is NSCS in $(\mathbb{W}, \tau, \varrho)$. By presumption, there is a NSZOS (K, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ such that $(\mathcal{G}(B, \varrho))^c \subseteq (K, \varrho)$ and $\mathcal{G}^{-1}(K, \varrho) \subseteq (B, \varrho)^c$. Therefore, $(B, \varrho) \subseteq \left(\mathcal{G}^{-1}(K, \varrho)\right)^c$. Hence $(K, \varrho)^c \subseteq \mathcal{G}(B, \varrho) \subseteq \mathcal{G}(\left(\mathcal{G}^{-1}(K, \varrho)\right)^c) \subseteq (K, \varrho)^c$ which implies $\mathcal{G}(B, \varrho) = (K, \varrho)^c$. As $(K, \varrho)^c$ is NSZCS of $(\mathbb{T}, \sigma, \varrho)$. Hence $\mathcal{G}(B, \varrho)$ is NSZCS in $(\mathbb{T}, \sigma, \varrho)$ and thus $\mathcal{G}$ is NSContraZO mapping.

Theorem 5.5 A mapping $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSContraZO iff $\mathcal{G}^{-1}(\text{NScl}(S, \varrho)) \supseteq \text{NSint}(\mathcal{G}^{-1}(S, \varrho))$ for every NSS (S, ϱ) of $(\mathbb{T}, \sigma, \varrho)$.

Proof. Necessity: Let $\mathcal{G}$ be a NSContraZO mapping. For any NSS (S, ϱ) of $(\mathbb{T}, \sigma, \varrho)$, $\mathcal{G}^{-1}(S, \varrho) \subseteq \text{NScl}(\mathcal{G}^{-1}(S, \varrho))$. Therefore, by Theorem 5.4 there exists a NSZOS (B, ϱ) in $(\mathbb{T}, \sigma, \varrho) \ni (S, \varrho) \supseteq (B, \varrho)$ & $\mathcal{G}^{-1}(B, \varrho) \supseteq NSint(\mathcal{G}^{-1}(S, \varrho))$. Hence $\mathcal{G}^{-1}(\text{NSZcl}(S, \varrho)) \supseteq \mathcal{G}^{-1}(B, \varrho) \supseteq \text{NSint}(\mathcal{G}^{-1}(S, \varrho))$.

Sufficiency: Let (S,ϱ) be a NSS in $(\mathbb{T}, \sigma, \varrho)$ and (B,ϱ) be a NSCS of $(\mathbb{W}, \tau, \varrho)$ containing $\mathcal{G}^{-1}(\mathrm{S},\varrho)$. Put $(\mathrm{K},\varrho) = \mathrm{NScl}(\mathrm{S},\varrho)$, then $(\mathrm{S},\varrho) \subseteq (\mathrm{K},\varrho)$ and (K,ϱ) is NSZC and $\mathcal{G}^{-1}(\mathrm{S},\varrho) \subseteq \mathrm{NSint}(\mathcal{G}^{-1}(\mathrm{S},\varrho)) \subseteq (\mathrm{B},\varrho)$. Thus by Theorem 5.4, $\mathcal{G}$ is NSZO mapping.

Theorem 5.6 If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U}, \rho, \varrho)$ be two neutrosophic soft mappings and $\mathcal{H} \circ \mathcal{G}: (\mathbb{W}, \tau, \varrho) \to (\mathbb{U},\rho,\varrho)$ is NSContraZO. If $\mathcal{H}: (\mathbb{T},\sigma,\varrho) \to (\mathbb{U}, \rho, \varrho)$ is NSContraZ-irr then $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSZO mapping.

Proof. Let (S,ϱ) be a NSOS in $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{H} \circ \mathcal{G}(\mathrm{S},\varrho)$ is NSZCS of $(\mathbb{U}, \rho, \varrho)$ because $\mathcal{H} \circ \mathcal{G}$ is NSContraZO mapping. As $\mathcal{H}$ is NSContraZ-irr and $\mathcal{H} \circ \mathcal{G}(\mathrm{S},\varrho)$ is NSZCS of $(\mathbb{U}, \rho, \varrho)$ therefore $\mathcal{H}^{-1}(\mathcal{H} \circ \mathcal{G}(\mathrm{S},\varrho)) = \mathcal{G}(\mathrm{S},\varrho)$ is NSZOS in $(\mathbb{T},\sigma,\varrho)$. Hence $\mathcal{G}$ is NSZO mapping.

Theorem 5.7 If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSO and $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U}, \rho, \varrho)$ is NSContraZO mappings, then $\mathcal{H} \circ \mathcal{G}: (\mathbb{W}, \tau, \varrho) \to (\mathbb{U},\rho,\varrho)$ is NSContraZO.

Proof. Let (S,ϱ) be a NSOS in $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}(\mathrm{S},\varrho)$ is a NSOS of $(\mathbb{T}, \sigma, \varrho)$ because $\mathcal{G}$ is a NSO mapping. Since $\mathcal{H}$ is NSContraZO, $\mathcal{H}\big(\mathcal{G}(\mathrm{S},\varrho)\big) = (\mathcal{H} \circ \mathcal{G})(\mathrm{S},\varrho)$ is NSZCS of $(\mathbb{U}, \rho, \varrho)$. Hence $\mathcal{H} \circ \mathcal{G}$ is NSContraZO mapping.

6. Neutrosophic soft contra Z - closed mapping

Definition 6.1 A mapping $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is neutrosophic soft contra Z – closed (briefly, NSContraZC) if image of every NSCS of $(\mathbb{W},\tau,\varrho)$ is a NSZOS in $(\mathbb{T},\sigma,\varrho)$.

Theorem 6.1 The statements are hold but the converse does not true. Every

a) NSContraδC is a NSContraC.
b) NSContraC is a NSContraδSC.
c) NSContraC is a NSContraPC.
d) NSContraδSC is a NSContraZC.
e) NSContraPC is a NSContraZC.
f) NSContraZC is a NSContraeC.

Proof. Consider the map $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$

(a) Let (S,ϱ) be a NSCS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraδC, $\mathcal{G}(S,\varrho)$ is a NSδOS in $\mathbb{T}$. Since all NSδOS are NSOS, $\mathcal{G}(S,\varrho)$ is NSOS in $\mathbb{T}$. Then, $\mathcal{G}$ is a NSContraC.

(b) Let (S,ϱ) be a NSCS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraC, $\mathcal{G}(S,\varrho)$ is a NSOS in $\mathbb{T}$. Since all NSOS are NSδSOS, $\mathcal{G}(S,\varrho)$ is a NSδSOS in $\mathbb{T}$. Then, $\mathcal{G}$ is a NSContraδS.

(c) Let (S,ϱ) be a NSCS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraC, $\mathcal{G}(S,\varrho)$ is a NSOS in $\mathbb{T}$. Since all NSOS are NSPOS, $\mathcal{G}(S,\varrho)$ is a NSPOS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraPC.

(d) Let (S,ϱ) be a NSCS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraδSC, $\mathcal{G}(S,\varrho)$ is a NSδSOS in $\mathbb{T}$. Since all NSδSOS is a NSZOS, $\mathcal{G}(S,\varrho)$ is a NSZOS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraZC.

(e) Let (S,ϱ) be a NSCS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraPC, $\mathcal{G}(S,\varrho)$ is a NSPOS in $\mathbb{T}$. Since all NSPOS are NSZOS, $\mathcal{G}(S,\varrho)$ is a NSZOS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraZC.

(f) Let (S, ϱ) be a NSCS in $\mathbb{W}$. As $\mathcal{G}$ is NSContraZC, $\mathcal{G}(S, \varrho)$ is a NSZOS in $\mathbb{T}$. Since all NSZOS is a NSeOS, $\mathcal{G}(S, \varrho)$ is a NSeOS in $\mathbb{T}$. Hence, $\mathcal{G}$ is a NSContraeC

Example 6.1 In example 5.1, $\mathcal{G}$ is a NSContraC but not NSContraδC mapping because the set $(V_1, \varrho)^c$ is NSCS in $\mathbb{W}$ and $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is NSOS but not NSδOS in $\mathbb{T}$.

Example 6.2 In example 5.2, $\mathcal{G}$ is a NSContraPC but not NSContraC mapping because the set $(V_1, \varrho)^c$ is NSCS in $\mathbb{W}$ and $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is NSPOS but not NSOS in $\mathbb{T}$.

Example 6.3 In example 5.3,

(i) $\mathcal{G}$ is a NSContraδSC but not NSContraC mapping because the set $(V_1, \varrho)^c$ is NSCS in $\mathbb{W}$ and $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is NSδSOS but not NSOS in $\mathbb{T}$.

(ii) $\mathcal{G}$ is a NSContraZC but not NSContraPC mapping because the set $(V_1, \varrho)^c$ is NSCS in $\mathbb{W}$ and $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is NSZOS but not NSPOS in $\mathbb{T}$.

Example 6.4 In example 5.4, $\mathcal{G}$ is a NSContraZC but not NSContraδSC mapping because the set $(V_1, \varrho)^c$ is NSCS in $\mathbb{W}$ and $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is NSZOS but not NSδSOS in $\mathbb{T}$.

Example 6.5 In example 5.5, $\mathcal{G}$ is a NSContraeC but not NSContraZC mapping because the set $(Q_1, \varrho)^c$ is NSCS in $\mathbb{W}$ and $\mathcal{G}(Q_1, \varrho)^c = (P_3, \varrho)^c$ is NSeOS but not NSZOS in $\mathbb{T}$.

Remark 6.1 The diagram shows NSConraZO mappings in NSts.

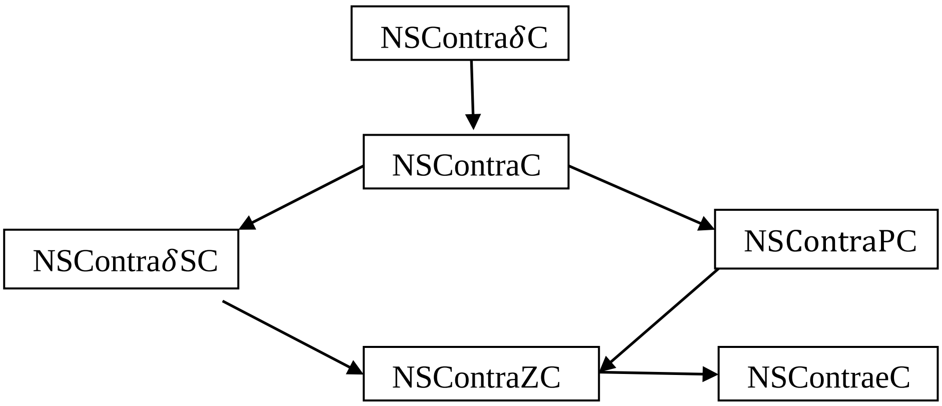

Diagram.3 Neutrosophic soft contra Z – closed maps

Theorem 6.2 A mapping $\mathcal{G}: (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is NSContraZC iff for each NSS (S, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ and for each NSCS (B, ϱ) of $(\mathbb{W}, \tau, \varrho)$ containing $\mathcal{G}^{-1}(S, \varrho)$ there is a NSZCS (K, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ such that (S, ϱ) ⊆ (K, ϱ) and $\mathcal{G}^{-1}$ (K, ϱ) ⊆ (B, ϱ).

Proof. Necessity: Assume $\mathcal{G}$ be a NSContraZC mapping. Let a NSOS (S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$ and a NSCS (B, ϱ) in $(\mathbb{W}, \tau, \varrho)$ such that $\mathcal{G}^{-1}(S, \varrho) \subseteq$ (B, ϱ). Then (K, ϱ) = $\mathbb{T} - \mathcal{G}((B, \varrho)^c)^c$ is NSZCS of $(\mathbb{T}, \sigma, \varrho)$ such that $\mathcal{G}^{-1}(K, \varrho) \subseteq$ (B, ϱ).
Suffciency: Assume (B, ϱ) is a NSCS of $(\mathbb{W}, \tau, \varrho)$. Then, $((\mathcal{G}(B, \varrho))^c$ is a NSS of $(\mathbb{T}, \sigma, \varrho)$ and $(B, \varrho)^c$ is NSOS in $(\mathbb{W}, \tau, \varrho)$ such that $\mathcal{G}^{-1}((\mathcal{G}(B, \varrho))^c \subseteq (B, \varrho)^c$. By presumption, there is a NSZCS (K, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ such that $(\mathcal{G}(B, \varrho))^c \subseteq$ (K, ϱ) and $\mathcal{G}^{-1}$ (K, ϱ) ⊆ $(B, \varrho)^c$. Therefore,

(B, ϱ) ⊆ $(\mathcal{G}^{-1}(K,\varrho))^c$. Hence $(K,\varrho)^c \subseteq \mathcal{G}(B,\varrho) \subseteq \mathcal{G}((\mathcal{G}^{-1}(K,\varrho))^c)$ which implies $\mathcal{G}$(B, ϱ) = $(K,\varrho)^c$. As $(K,\varrho)^c$ is NSZOS of $(\mathbb{T},\sigma,\varrho)$, $\mathcal{G}$(B, ϱ) is NSZOS in $(\mathbb{T},\sigma,\varrho)$ and hence $\mathcal{G}$ is NSContraZC mapping.

Theorem 6.3 If $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is NSC and $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U},\rho,\varrho)$ is NSContraZC. Then $\mathcal{H} \circ \mathcal{G}: (\mathbb{W},\tau,\varrho) \to (\mathbb{U},\rho,\varrho)$ is NSContraZC.

Proof. Let (S, ϱ) be a NSCS in $(\mathbb{W},\tau,\varrho)$. As $\mathcal{G}$ is NSC mapping, $\mathcal{G}$ (S, ϱ) is NSCS in $(\mathbb{T},\sigma,\varrho)$. As $\mathcal{H}$ is NSContraZC mapping ($\mathcal{H} \circ \mathcal{G}$) (S, ϱ) = $\mathcal{H}(\mathcal{G}$ S, ϱ)) is NSZOS in $(\mathbb{U},\rho,\varrho)$. Hence $\mathcal{H} \circ \mathcal{G}$ is NSContraZC mapping.

Theorem 6.4 If $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is NSContraZCmap, then NSZint($\mathcal{G}$(S, ϱ)) ⊇ $\mathcal{G}$ (NSint(S,ϱ)).

Proof. The proof is obvious from Defnition 6.1 and Defnition neutrosophic soft Z-interior.

Theorem 6.5 Let $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ and $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U},\rho,\varrho)$ be NSContraZC mappings. If every NSZOS of $(\mathbb{T},\sigma,\varrho)$ is NSOS, then $\mathcal{H} \circ \mathcal{G}: (\mathbb{W},\tau,\varrho) \to (\mathbb{U},\rho,\varrho)$ is NSZC.

Proof. Let (S, ϱ) be a NSCS in $(\mathbb{W},\tau,\varrho)$. As, $\mathcal{G}$ is NSContraZC mapping, $\mathcal{G}$(S, ϱ) is NSZOS in $(\mathbb{T},\sigma,\varrho)$. By presumption, $\mathcal{G}$(S, ϱ) is NSOS of $(\mathbb{T},\sigma,\varrho)$. As $\mathcal{H}$ is NSContraZC mapping, $\mathcal{H}$ ($\mathcal{G}$ (S, ϱ)) = ($\mathcal{H} \circ \mathcal{G}$) (S, ϱ) is NSZCS in $(\mathbb{U},\rho,\varrho)$. Hence $\mathcal{H} \circ \mathcal{G}$ is NSZC mapping.

Theorem 6.6 Let $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ be a bijective mapping. Then, the following statements are equivalent.

(i) $\mathcal{G}$ is a NSContraZO mapping.

(ii) $\mathcal{G}$ is a NSContraZC mapping.

(iii) $\mathcal{G}^{-1}$ is NSZCts mapping.

Proof. (i) ⟹ (ii) : Assume $\mathcal{G}$ is a NSContraZO mapping. If NSOS (S, ϱ) in $(\mathbb{W},\tau,\varrho)$, by presumption $\mathcal{G}$(S, ϱ) is a NSZCS in $(\mathbb{T},\sigma,\varrho)$. But now, (S, ϱ) is NSCS in $(\mathbb{W},\tau,\varrho)$. So, $1_{(\mathbb{W},\varrho)} -$ (S, ϱ) is a NSOS in $(\mathbb{W},\tau,\varrho)$. By assumption, $\mathcal{G}(1_{(\mathbb{T},\varrho)} -$ (S, ϱ)) is a NSZCS in $(\mathbb{T},\sigma,\varrho)$. Hence $1_{(\mathbb{W},\varrho)} - \mathcal{G}(1_{(\mathbb{T},\varrho)}) -$ (S, ϱ) is a NSZOS in $(\mathbb{T},\sigma,\varrho)$. Thus, $\mathcal{G}$ is a NSContraZC mapping.

(ii) ⟹ (iii) : Consider a NSCS in $(\mathbb{W},\tau,\varrho)$. By assumption, $\mathcal{G}$(S, ϱ) is a NSZOS in $(\mathbb{T},\sigma,\varrho)$. Hence, $\mathcal{G}$(S, ϱ) = $(\mathcal{G}^{-1})^{-1}$ (S, ϱ). So, $\mathcal{G}^{-1}$ is a NSZOS in $(\mathbb{T},\sigma,\varrho)$. Thus, $\mathcal{G}^{-1}$ is NSZCts.

(iii) ⟹ (i) : Consider a NSOS (S, ϱ) in $(\mathbb{W},\tau,\varrho)$. By assumption, $(\mathcal{G}^{-1})^{-1}$ (S, ϱ) = $\mathcal{G}$ (S, ϱ) is a NSContraZO mapping.

7 Neutrosophic soft contra Z-homeomorphism

Defnition7.1 A bijection $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is called a neutrosophic soft contra Z-homeomorphism (briefly, NSContraZHom) if $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraZCts mappings.

Theorem7.1 Each NSContraHom is a NSContraZHom. But the converse not true.

Proof.: Assume $\mathcal{G}$ is NSContraHom. Then $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraCts. We know that each NSContraCts function is NSContraZCts. So, $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraZCts. Thus, $\mathcal{G}$ is a NSContraZHom.

Example7.1 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets $(S_1, \varrho), (S_2, \varrho)$ (S_3, ϱ) and (S_4, ϱ) in $\mathbb{W}$ and (V_1, ϱ) in $\mathbb{T}$ are defined as

$$(S_1, e_1) = \langle\left(\frac{\mu_{w_1}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$$

$$(S_1, e_2) = \langle\left(\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_2, e_1) = \langle\left(\frac{\mu_{w_1}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(S_2, e_2) = \langle\left(\frac{\mu_{w_1}}{0.4}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.4}\right)\rangle$$

$$(S_3, e_1) = \langle\left(\frac{\mu_{w_1}}{0.3}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_3, e_2) = \langle\left(\frac{\mu_{w_1}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}\right)\rangle$$

$$(S_4, e_1) = \langle\left(\frac{\mu_{w_1}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{w_2}}{0.6}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.6}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.5}\right)\rangle$$

$$(S_4, e_2) = \langle\left(\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.3}\right), \left(\frac{\mu_{w_2}}{0.6}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.4}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.3}\right)\rangle$$

$$(V_1, e_1) = \langle\left(\frac{\mu_{t_1}}{0.5}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.5}\right), \left(\frac{\mu_{t_2}}{0.6}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.6}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.5}\right)\rangle$$

$$(V_2, e_2) = \langle\left(\frac{\mu_{t_1}}{0.2}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.3}\right), \left(\frac{\mu_{t_2}}{0.6}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.4}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.3}\right)\rangle$$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping. Then $\mathcal{G}$ is a NSContraZHom because $(S_1, \varrho), (S_2, \varrho)$ and (S_3, ϱ) are NSOS in $\mathbb{W}$ and $\mathcal{G}$ (S_1, ϱ), $\mathcal{G}(S_2, \varrho)$ and $\mathcal{G}(S_3, \varrho)$ are NSZCS in $\mathbb{T}$ and $\mathcal{G}^{-1}$ $(V_1, \varrho) = (S_4, \varrho)$ is NSZCS in $\mathbb{W}$. But $\mathcal{G}$ is not NSContraHom because $\mathcal{G}(S_1, \varrho)$, $\mathcal{G}(S_2, \varrho)$ and $\mathcal{G}(S_3, \varrho)$ are not NSCS in $\mathbb{T}$ and $\mathcal{G}^{-1}(V_1, \varrho) = (S_4, \varrho)$ is a NSCS in $\mathbb{W}$.

Theorem7.2 Consider a bijective mapping $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$. The following statements are equivalent if $\mathcal{G}$ is NSContraZCts.

(i) $\mathcal{G}$ is a NSContraZC mapping.

(ii) $\mathcal{G}$ is a NSContraZO mapping.

(iii) $\mathcal{G}^{-1}$ is a NSContraZHom.

Proof.: (i) $\Longrightarrow$ (ii): Let $\mathcal{G}$ be a bijective mapping and a NSContraZC mapping. Therefore, $\mathcal{G}^{-1}$ is a NSContraZCts mapping. As each NSOS in $(\mathbb{W}, \tau, \varrho)$ is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$, $\mathcal{G}$ is a NSContraZO mapping.

(ii) $\Longrightarrow$ (iii): Assume $\mathcal{G}$ is a bijective and NSConraO mapping. Also, $\mathcal{G}^{-1}$ is a NSContraZCts mapping. Therefore, $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraZCts. Thus, $\mathcal{G}$ is a NSContraZHom.

(iii) $\Longrightarrow$ (i): Assume $\mathcal{G}$ is a NSContraZHom. So, $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraZCts. As every NSCS in $(\mathbb{W}, \tau, \varrho)$ is a NSZOS in $(\mathbb{T}, \sigma, \varrho)$, $\mathcal{G}$ is a NSContraZC mapping.

Theorem 7.3 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a NSContraZHom. If $(\mathbb{W}, \tau, \varrho)$ and $(\mathbb{T}, \sigma, \varrho)$ are NSZT$_{\frac{1}{2}}$ - spaces, then $\mathcal{G}$ is a NSContraHom.

Proof. Consider a NSCS (S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$. So, $\mathcal{G}^{-1}$ (S, ϱ) is a NSZOS in $(\mathbb{W}, \tau, \varrho)$. As, $(\mathbb{W}, \tau, \varrho)$ is a NSZT$_{\frac{1}{2}}$ -space, $\mathcal{G}^{-1}$ (S, ϱ) is a NSOS in $(\mathbb{W}, \tau, \varrho)$. Therefore, $\mathcal{G}$ is NSContraCts. By hypothesis, $\mathcal{G}^{-1}$ is NSContraZCts. Let (B, ϱ) be a NSCS in $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}$(B, ϱ) is a NSZOS in $(\mathbb{T}, \sigma, \varrho)$, by presumption. Since $(\mathbb{T}, \sigma, \varrho)$ is a NSZT$_{\frac{1}{2}}$ - space, $\mathcal{G}$(B, ϱ) is a NSOS in $(\mathbb{T}, \sigma, \varrho)$. Therefore, $\mathcal{G}^{-1}$ is NSContraCts. Thus $\mathcal{G}$ is a NSContraHom.

Theorem 7.4 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a NSCts. If $(\mathbb{T}, \sigma, \varrho)$ are NSZT$_{\frac{1}{2}}$ - space, then the following are equivalent.

1. $\mathcal{G}$ is NSContraZC mapping.

2. If (B, ϱ) is a NSOS in $(\mathbb{W}, \tau, \varrho)$, then $\mathcal{G}$ (B, ϱ) is NSZCS in $(\mathbb{T}, \sigma, \varrho)$.

3. $\mathcal{G}$ (NSint(B, ϱ)) ⊆ NScl(NSint($\mathcal{G}$ (B, ϱ))) for every NSS (B, ϱ) in $(\mathbb{W}, \tau, \varrho)$.

Proof. (i) $\Longrightarrow$ (ii) : Obvious.

(ii) $\Longrightarrow$ (iii) Consider a NSS (B, ϱ) in $(\mathbb{W}, \tau, \varrho)$. We know that, NSint(B, ϱ) is a NSOS in $(\mathbb{W}, \tau, \varrho)$. Then, $\mathcal{G}$(NSint((B, ϱ)) is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$. Since $(\mathbb{T}, \sigma, \varrho)$ is a NSZT$_{\frac{1}{2}}$ -space $\mathcal{G}$(NSint(B, ϱ)) is a NSCS in $(\mathbb{T}, \sigma, \varrho)$. Therefore, $\mathcal{G}$(NSint(B, ϱ)) = NScl($\mathcal{G}(NSint$((B, ϱ))) ⊆ NScl(NSint($\mathcal{G}$((B, ϱ))). NScl(NSint(G(B))).

(iii) $\Longrightarrow$ (i) Let (B, ϱ) be a NSCS in $(\mathbb{W}, \tau, \varrho)$. Then, $(B, \varrho)^c$ is a NSOS in $(\mathbb{W}, \tau, \varrho)$. As, $\mathcal{G}$ (NSint($(B, \varrho)^c$) ⊆ NScl(NSint($\mathcal{G}(B, \varrho)^c$)), we get $\mathcal{G}((B, \varrho)^c)$ ⊆ NScl(NSint($\mathcal{G}(B, \varrho)^c$)). Therefore, $\mathcal{G}((B, \varrho)^c)$ is NSZCS in $(\mathbb{T}, \sigma, \varrho)$. Thus, $\mathcal{G}$(B, ϱ) is a NSZOS in $(\mathbb{W}, \tau, \varrho)$. Hence, $\mathcal{G}$ is a NSContraZC mapping.

Theorem 7.5 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be a NSContraZC, where $(\mathbb{W}, \tau, \varrho)$ and $(\mathbb{U}, \rho, \varrho)$ are two NSts's and $(\mathbb{T}, \sigma, \varrho)$ a NSZT$_{\frac{1}{2}}$ - space, then the composition $\mathcal{H} \circ \mathcal{G}$ is NSZC.

Proof. Consider a NSCS(B, ϱ) in $(\mathbb{W}, \tau, \varrho)$. As $\mathcal{G}$ is NSContraZC and $\mathcal{G}(B, \varrho)$ is a NSZOS in $(\mathbb{T}, \sigma, \varrho)$, by assumption, $\mathcal{G}$(B, ϱ) is a NSOS in $(\mathbb{T}, \sigma, \varrho)$. Since $\mathcal{H}$ is NSContraZC, then $\mathcal{H}$ ($\mathcal{G}$(B, ϱ)) is NSZCS in $(\mathbb{U}, \rho, \varrho)$ and $\mathcal{H}$ ($\mathcal{G}$(B, ϱ)) = ($\mathcal{H} \circ \mathcal{G}$) (B, ϱ). Thus, $\mathcal{H} \circ \mathcal{G}$ is NSZC.

Theorem 7.6 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be two NSts's, then the following hold.

1. If $\mathcal{H} \circ \mathcal{G}$ is NSContraZO and $\mathcal{G}$ is NSCts, then $\mathcal{H}$ is NSContraZO.

2. If $\mathcal{H} \circ \mathcal{G}$ is NSO and $\mathcal{H}$ is NSContraZCts, then $\mathcal{H}$ is NSContraZO.

Proof. The proof is obvious from Definition 5.1, Definitions of neutrosophic soft Z continuous function, Definition of neutrosophic soft Z open mapping and Definitions 3.1.

8 NeutrosophicsoftcontraZ-Chomeomorphism

Definition 8.1 A bijection $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is called a neutrosophic soft contraZ-Chomeomorphism (briefly, NSContraZCHom) if $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraZ-irr mappings.

Theorem 8.1 Each NSContraZCHom is a NSContraZHom. But the converse not true.

Proof.: Consider a NSOS(S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$. Then, (S, ϱ) is a NSZOS in $(\mathbb{T}, \sigma, \varrho)$. By presumption, $\mathcal{G}^{-1}$ (S, ϱ) is a NSZCS in $(\mathbb{W}, \tau, \varrho)$. Therefore, $\mathcal{G}$ is a NSContraZCts mapping. So $\mathcal{G}$ and $\mathcal{G}^{-1}$ are NSContraZCts mapping. Thus, $\mathcal{G}$ is a NSContraZHom.

Example 8.1 Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}$, $\varrho = \{e_1, e_2\}$ and NS sets $(S_1, \varrho), (S_2, \varrho)$ (S_3, ϱ) and (S_4, ϱ) in $\mathbb{W}$ and (V_1, ϱ) in $\mathbb{T}$ are defined as

$$(S_1, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}\right)\rangle$$

$$(S_1, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_2, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(S_2, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.4}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.6}\right), \left(\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}\right), \left(\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.4}\right)\rangle$$

$$(S_3, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.3}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}\right)\rangle$$

$$(S_3, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}\right), \left(\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}\right), \left(\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}\right)\rangle$$

$$(S_4, e_1) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}\right), \left(\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}\right), \left(\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}\right)\rangle$$

$$(S_4, e_2) = \langle\left(\frac{\mu_{w_1,}}{0.6}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.4}\right), \left(\frac{\mu_{w_2}}{0.7}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.3}\right), \left(\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.3}\right)\rangle$$

$$(V_1, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.4}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.4}, \frac{\nu_{t_2}}{0.8}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.7}\right)\rangle$$

$$(V_1, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.2}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.6}\right), \left(\frac{\mu_{t_2}}{0.2}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.7}\right), \left(\frac{\mu_{t_3}}{0.2}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.8}\right)\rangle$$

$$(V_2, e_1) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.5}, \frac{\nu_{t_1}}{0.5}\right), \left(\frac{\mu_{t_2}}{0.5}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.5}\right), \left(\frac{\mu_{t_3}}{0.6}, \frac{\sigma_{t_3}}{0.5}, \frac{\nu_{t_3}}{0.6}\right)\rangle$$

$$(V_2, e_2) = \langle\left(\frac{\mu_{t_1,}}{0.6}, \frac{\sigma_{t_1}}{0.4}, \frac{\nu_{t_1}}{0.4}\right), \left(\frac{\mu_{t_2}}{0.7}, \frac{\sigma_{t_2}}{0.5}, \frac{\nu_{t_2}}{0.3}\right), \left(\frac{\mu_{t_3}}{0.4}, \frac{\sigma_{t_3}}{0.3}, \frac{\nu_{t_3}}{0.3}\right)\rangle$$

Here, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (V_1, \varrho)\}$. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping. Then $\mathcal{G}$ is a NSContraZHom because $(S_1, \varrho), (S_2, \varrho)$ and (S_3, ϱ) are NSOS in $\mathbb{W}$ and $\mathcal{G}$ (S_1, ϱ), $\mathcal{G}(S_2, \varrho)$ and $\mathcal{G}(S_3, \varrho)$ are NSZCS in $\mathbb{T}$.

Also(V_1, ϱ) is NSOS in $\mathbb{T}$ and $\mathcal{G}^{-1}$ (V_1, ϱ) = (S_1, ϱ) is NSZCS in $\mathbb{W}$. But $\mathcal{G}$ is not NSContraHom because $\mathcal{G}(V_2, \varrho)$ is NSZCS in $\mathbb{T}$ but (V_2, ϱ) is not NSZOS in $\mathbb{W}$.

Theorem 8.2 If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ is a NSContraZCHom, then NSint($\mathcal{G}^{-1}$ (S, ϱ)) ⊆ $\mathcal{G}^{-1}$ (NScl(B, ϱ)) for every NSS(S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$.

Proof.: Consider a NSS(S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$. Since NScl(S, ϱ) is a NSCS in $(\mathbb{T}, \sigma, \varrho)$ and every NSCS is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$. As $\mathcal{G}$ is NSContraZ-irr, $\mathcal{G}^{-1}$(NScl(S, ϱ)) is a NSZOS in $(\mathbb{W}, \tau, \varrho)$. Then, NSint($\mathcal{G}^{-1}$ (NScl(S, ϱ))) = $\mathcal{G}^{-1}$ (NScl(S, ϱ)). Here, NSZint($\mathcal{G}^{-1}$ (S, ϱ)) ⊆ NSZint($\mathcal{G}^{-1}$ (NScl(S, ϱ))) = $\mathcal{G}^{-1}$ (NScl(S, ϱ)). Therefore NSZint($\mathcal{G}^{-1}$(S, ϱ)) ⊆ $\mathcal{G}^{-1}$ (NScl(S, ϱ)) for every NSS(S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$.

Theorem 8.3 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSContraZCHom. Then NSZint($\mathcal{G}^{-1}$ (S, ϱ)) ⊆ $\mathcal{G}^{-1}$ (NSZcl(S, ϱ)) for every NSS(S, ϱ) in $(\mathbb{T}, \sigma, \varrho)$.

Proof.: As $\mathcal{G}$ is a NSContraZCHom, $\mathcal{G}$ is a NSContraZ-irr mapping. Consider a NSS(S, ϱ). It is obvious that, NSZcl(S, ϱ) is a NSZCS in $(\mathbb{T}, \sigma, \varrho)$. As $\mathcal{G}^{-1}$(S, ϱ) ⊆ $\mathcal{G}^{-1}$(NSZcl(S, ϱ)), we have NSZint($\mathcal{G}^{-1}$(S, ϱ)) ⊆ NSZint($\mathcal{G}^{-1}$ (NSZcl(S, ϱ)) ⊆ $\mathcal{G}^{-1}$ (NSZcl(S, ϱ)). Thus, NSZint($\mathcal{G}^{-1}$ (S, ϱ)) ⊆ $\mathcal{G}^{-1}$ (NSZcl(S, ϱ)).

Theorem 8.4 Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \rightarrow (\mathbb{U}, \rho, \varrho)$ be a NSContraZCHom's. Then $\mathcal{H} \circ \mathcal{G}$ is a NSZCHom.

Proof.: Assume that $\mathcal{G}$ and $\mathcal{H}$ are two NSContraZCHom's. Let (S, ϱ) be a NSZCS in $(\mathbb{U}, \rho, \varrho)$. Then, $\mathcal{H}^{-1}$(S, ϱ) is a NSZOS in $(\mathbb{T}, \sigma, \varrho)$. By presumption, $\mathcal{G}^{-1}$(($\mathcal{H}^{-1}$((S, ϱ)) is a NSZCS in $(\mathbb{W}, \tau, \varrho)$. Therefore,$(\mathcal{H} \circ \mathcal{G})^{-1}$ is a NSZ-irr mapping. Assume (B, ϱ) is NSZCS in $(\mathbb{W}, \tau, \varrho)$. Then, by hypothesis, $\mathcal{G}$ ($\mathcal{H}$) is a NSZOS in $(\mathbb{T}, \sigma, \varrho)$. Hence, $\mathcal{H}(\mathcal{G}$ (B, ϱ)) is a NSZCS in $(\mathbb{U}, \rho, \varrho)$. This implies that $\mathcal{H} \circ \mathcal{G}$ is NSZ-irr mapping. Thus, $\mathcal{H} \circ \mathcal{G}$ is a NSZCHom.

9. Conclusion

In this paper, we have introduced and explored contra Z-continuous, contra Z-irresolute, contra Z-open, and contra Z-closed maps in neutrosophic soft topological spaces. Additionally, we have investigated contra Z and Z-C homeomorphisms, with relevant theorems and examples, thereby contributing to the expansion of neutrosophic soft topology. These results pave the way for future research and potential applications in this emerging field.

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