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Analytical Study of Perfect Insulation in a Vertical Wall Based on Internal Energy Generation

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Abstract:
In this paper, the creation of perfect insulation in a vertical wall using internal energy generation has analyzed. The heat conduction Fourier law has solved analytically in the presence of internal energy generation. Two types of internal energy generation (uniform and linear) have considered. Solving the problem, the desired value of internal energy generation to create perfect insulation has obtained. The analytical solution of present study is in good agreement with the solution of previous researches. The results showed that linear internal energy generation is better for creating perfect insulation because the wall temperature is lower and the cost of creating internal energy generation is lower. The research results provide a general theoretical framework for achieving basic knowledge for creating perfect insulation.

Keywords: Perfect insulation; Internal energy generation; Analytical study

Introduction

Nowadays, the issue of insulation has become a major challenge in various industries and applications. In conventional insulation, it is trying to minimize heat loss. However, there is always some heat loss. In some specific applications, the perfect insulation (zero heat loss) is needed. The engineering sciences can definitely have the scientific answer for it. Perfect thermal insulation refers to a system that completely prevents heat transfer between two areas. In a perfect insulation, the heat loss is completely prevented by paying expenditure. One way to create perfect insulation is to use internal energy generation. In this way, the value of internal energy generation should be found in such a way that the temperature slope is zero at the desired boundary. According to Fig. 1, the present study problem is that a biological species is kept inside a cubic tank. For the growth of this biological species, the fluid temperature inside the tank must remain constant or, in other words, the heat loss from the tank should be zero. Therefore, perfect insulation should be made in the wall of the tank. In order to achieve the purpose of the problem, internal energy generation is used to create perfect insulation in the tank wall.

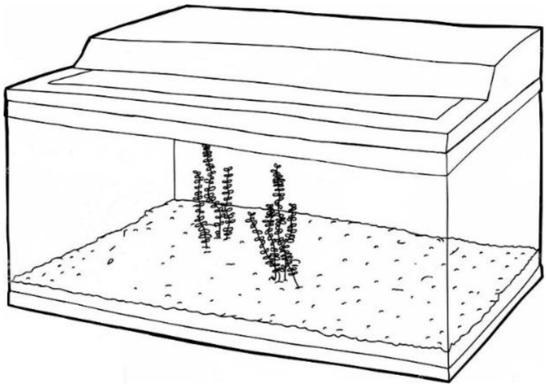

Fig. 1. Schematic view of the present study problem [1]

The application of insulation has been examined in many scientific research, which is discussed below.

Yari et al. (2024) numerically investigated the effect of simultaneously using insulation and phase change material in building walls to reduce heat loss [2]. The numerical results of their research showed that the combined system of insulation and phase change material reduces heat loss from the building by about 70%.

Janetti et al. (2024) developed an insulation material for buildings from a combination of clay, biochar, and natural fiber [3]. Their measurements showed that the thermal conductivity of the proposed insulation is about 0.06 W/m.K.

Feng et al. (2024) investigated the thermophysical properties of inorganic insulations [4]. They obtained an insulating material from a combination of glass fiber and kaolinite that had high compressive strength and a thermal conductivity of about 0.075 W/m.K.

Yue et al. (2024) numerically investigated the thermal performance of composite insulation materials [5]. They presented correlations for the effective thermal conductivity of composite insulation materials using ANSYS software.

Liang et al. (2025) developed a transient thermal model to investigate the insulation of a hydrogen storage tank [6]. The insulation consisted of a multi-layer material of variable density connected to a vapor-cooled shield. Their research results showed that the insulation reduced heat loss by up to 77%.

Wang et al. (2025) investigated the thermal performance of a multilayer insulation for application in hypersonic vehicles [7]. Their thermal model considered both conduction and radiation losses. Their results showed that if the radiative emissivity of the upper layers of multilayer insulation is lower than the radiative emissivity of the lower layers, the thermal performance of the multilayer insulation will be better.

Zhang et al. (2025) presented an improved design of a metamaterial with high mechanical properties and strong insulation for use in hypersonic aircrafts [8]. They optimized their design with the node optimization strategy and introduced a metamaterial with high strength and strong insulation.

Yu et al. (2025) introduced and modeled a novel insulation system for liquid hydrogen storage tanks [9]. Their insulation system includes a heat shield and multi-layer insulation. Their novel design reduces heat loss from the tank by about 90% compared to conventional insulations.

Pagoni et al. (2025) investigated the use of various insulation materials to reduce heat loss in historic buildings [10]. Their results showed that a combination of four materials of glass foam, autoclaved aerated concrete, aerogel, hemp with lime can effectively reduce heat loss.

Previous studies [2-10] have focused on the design, construction, and the thermal performance evaluation of various types of conventional insulation. However, the thermal performance of perfect insulation has not been investigated. In this research, a general theoretical framework for creating perfect insulation in a vertical wall using internal energy generation is developed and solved.

Formulation of the problem

The geometry of a vertical wall is given in Fig. 2.

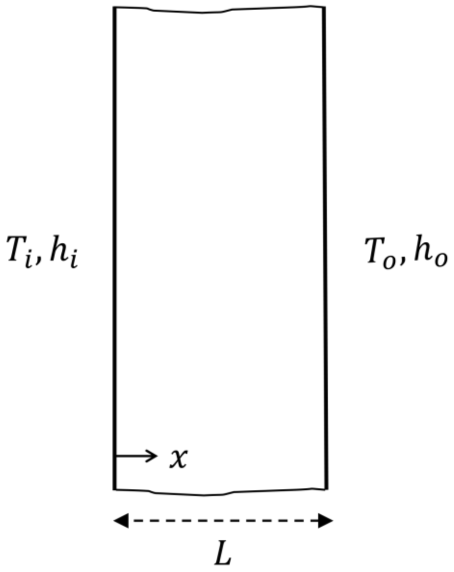

Fig. 2. Geometry of the problem

Where T_i, T_o, h_i, h_o, and L, are inside fluid temperature, outside fluid temperature, inside convective heat transfer coefficient, outside convective heat transfer coefficient and wall thickness, respectively. In order to derive the problem governing equation, the energy balance and the Fourier conduction law for the differential control volume of Fig. 3 is written.

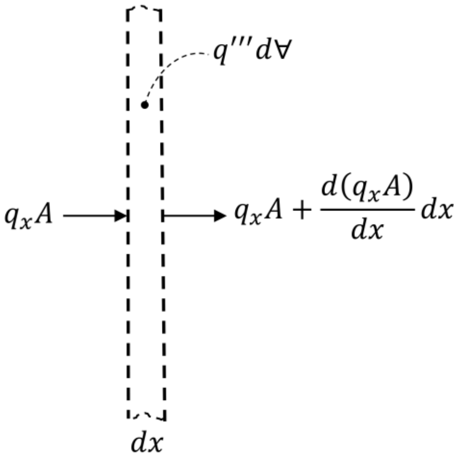

Fig. 3. Differential control volume of the wall

Where q_x, A, dx, $d\forall$, and q''', are conductive heat flux, wall area, thickness of differential control volume, volume of differential control volume and internal energy generation per unit volume, respectively. The energy balance for the control volume of Fig. 3 is written as follows [11]

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \underbrace{\dot{E}_{st}}_{=0} \tag{1}$$

Where $\dot{E}_{in}$, $\dot{E}_{out}$, $\dot{E}_{gen}$, and $\dot{E}_{st}$, are rate of input energies to the control volume, rate of output energies from the control volume, rate of energy generation in the control volume, and rate of energy change in the control volume, respectively. Due to the steady state condition, $\dot{E}_{st} = 0$.

Substituting the energy rates of Fig. 3 into Eq. (1) yields

$$q_x A - \left(q_x A + \frac{d(q_x A)}{dx} dx\right) + q''' d\forall = 0 \tag{2}$$

The conductive heat flux is defined by heat conduction Fourier law as follows [11]

$$q_x = -k \frac{dT}{dx} \tag{3}$$

Here, k is thermal conductivity of the wall. Also $d\forall = Adx$. By substituting Eq. (3) into Eq. (2) and simplifying, the governing equation for the problem is obtained as follows

$$\begin{cases} \dfrac{d^2T}{dx^2} + \dfrac{q'''}{k} = 0 \\ T(0) = T_{s1} \\ T(L) = T_{s2} \end{cases} \tag{4}$$

Where T_{s1} and T_{s2} are temperature of inner surface of the wall and temperature of outer surface of the wall, respectively. In the present study, the internal energy generation is considered to be uniform and linear as follows

$$q''' = q_0''' \tag{5}$$

$$q''' = q_0''' \left(1 - \frac{x}{L}\right) \tag{6}$$

The temperatures of T_{s1} and T_{s2} and the internal energy generation of q_0''' are obtained from the following boundary conditions

$$\frac{dT(0)}{dx} = 0 \tag{7}$$

$$-k \frac{dT(L)}{dx} = h_o[T(L) - T_o] \tag{8}$$

$$T(0) = T_i \tag{9}$$

The analytical solution of Eq. (4) for uniform and linear internal energy generation is given as follows

Uniform internal energy generation:

$$T(x) = -\frac{q_o'''}{k}\frac{x^2}{2} + \left(\frac{T_{s2} - T_{s1}}{L} + \frac{q_o'''}{k}\frac{L}{2}\right)x + T_{s1} \tag{10}$$

$$T_{s1} = T_i \tag{11}$$

$$T_{s2} = T_i - \frac{q_o'''}{k}\frac{L^2}{2} \tag{12}$$

$$q_o''' = \frac{h_o(T_i - T_o)}{L + \frac{h_o}{k}\frac{L^2}{2}} \tag{13}$$

Linear internal energy generation:

$$T(x) = -\frac{q_o'''}{k}\left(\frac{x^2}{2} - \frac{x^3}{6L}\right) + \left(\frac{T_{s2} - T_{s1}}{L} + \frac{q_o'''}{k}\frac{L}{3}\right)x + T_{s1} \tag{14}$$

$$T_{s1} = T_i \tag{15}$$

$$T_{s2} = T_i - \frac{q_o'''}{k}\frac{L^2}{3} \tag{16}$$

$$q_o''' = \frac{h_o(T_i - T_o)}{\frac{L}{2} + \frac{h_o}{k}\frac{L^2}{3}} \tag{17}$$

The rate of conduction heat transfer along the length of the wall is calculated as follows

Uniform internal energy generation:

$$Q = -kA\frac{dT}{dx} = kA\left(\frac{q_o'''}{k}x - \frac{T_{s2} - T_{s1}}{L} - \frac{q_o'''}{k}\frac{L}{2}\right) \tag{18}$$

Linear internal energy generation:

$$Q = -kA\frac{dT}{dx} = kA\left(\frac{q_o'''}{k}\left(x - \frac{x^2}{2L}\right) - \frac{T_{s2} - T_{s1}}{L} - \frac{q_o'''}{k}\frac{L}{3}\right) \tag{19}$$

Validation

In order to validate the wall temperature distribution of the present study in the case of zero internal energy generation $(q''' = 0)$ and the surfaces temperature of $T_{s1} = T_i$ and $T_{s2} = T_o$, it has compared with the temperature distribution of Incropera et al. [11]. The temperature distribution of Incropera et al. [11] is as follows

$$T(x) = \left(\frac{T_{s2} - T_{s1}}{L}\right)x + T_{s1} \tag{20}$$

The design parameters of vertical wall are given in Table 1.

Table 1. Design parameters of vertical wall

Parameter	**Value**
Wall thickness	$L = 0.01$ m
Wall area	$A = 1$ m^2
Inside fluid temperature	$T_i = 50$ ℃
Outside fluid temperature	$T_o = 25$ ℃
Wall conductivity	$k = 0.5$ W/m. ℃
Inside convective heat transfer coefficient	$h_i = 100$ W/m^2. ℃
Outside convective heat transfer coefficient	$h_o = 5$ W/m^2. ℃

The validation of the temperature is shown in Fig. 4.

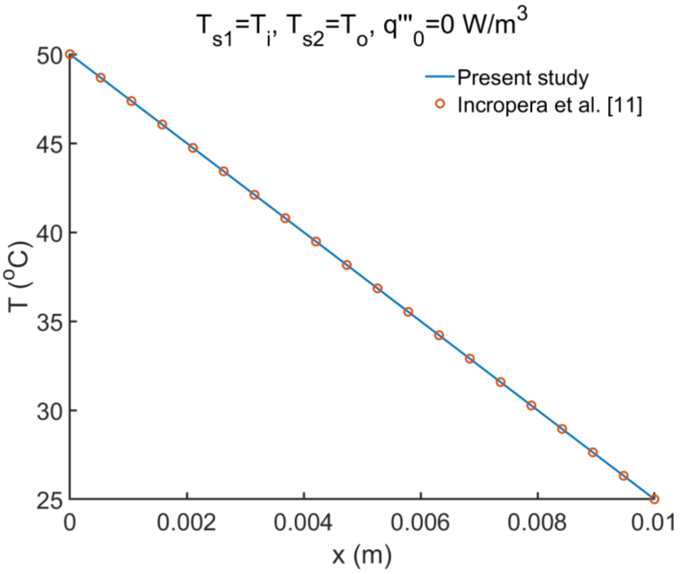

Fig. 4. Validation of the temperature

It is observed from Fig. 4 that there is a good agreement between the temperature of present study and the temperature of Incropera et al. [11].

Results and discussion

In this section, the effect of uniform and linear internal energy generation on the wall temperature is examined. The main purpose is to create perfect insulation at $x = 0$.

Fig. 5 shows the wall temperature distribution for uniform and linear internal energy generation.

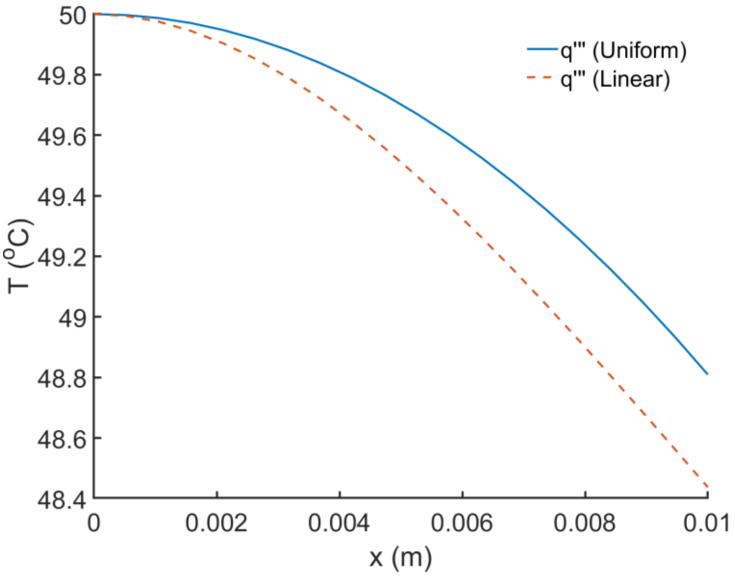

Fig. 5. Wall temperature distribution for uniform and linear internal energy generation

It can be seen from Fig. 5 that the temperature gradient for uniform and linear internal energy generation is zero at $x = 0$. In other words, the condition of perfect insulation holds at location $x = 0$. On the other hand, the wall temperature for linear internal energy generation is lower than uniform internal energy generation. This can be desirable because it costs less to create a boundary condition of perfect insulation. It can be concluded that linear internal energy generation is better than uniform internal energy generation because it has a lower production cost.

Fig. 6 shows the heat transfer rate in the wall as a function of the wall length.

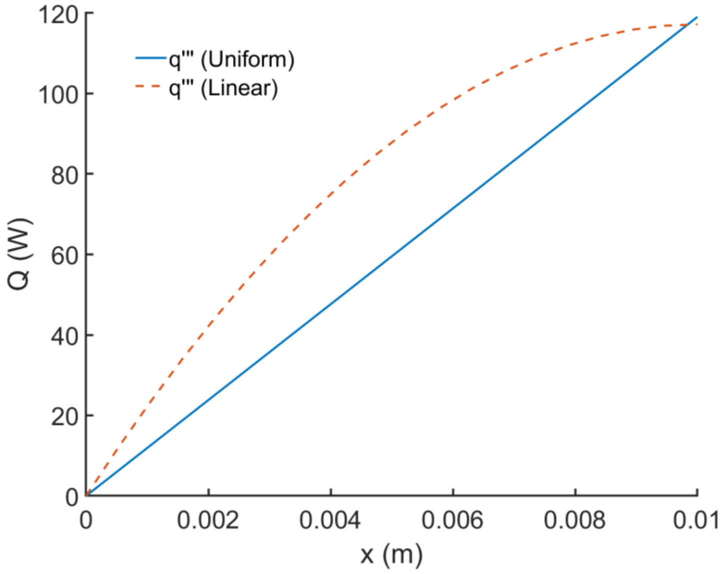

Fig. 6. Heat transfer rate in the wall as a function of the wall length

It can be seen from Fig. 6 that the heat transfer rate is higher for linear internal energy generation. Therefore, the wall temperature is lower for it. This subject shows that linear internal energy generation is better for creating perfect insulation condition because it results in lower costs.

Fig. 7 shows the contours of the heat transfer rate in the wall for different values of internal energy generation.

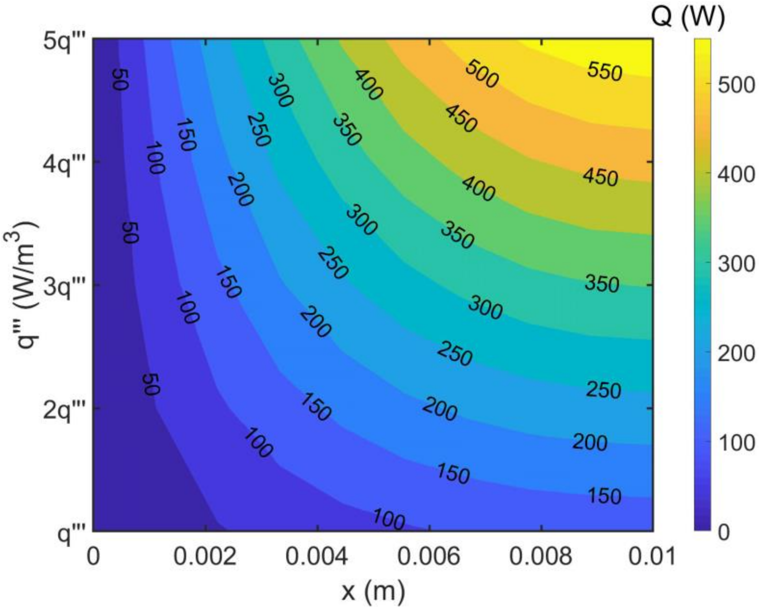

Fig. 7. Contours of the heat transfer rate in the wall for different values of internal energy generation

It can be seen from Fig. 7 that the heat transfer rate in the wall for unit internal energy generation is zero at the boundary $x = 0$, or in other words, the boundary condition of perfect insulation has been established.

Conclusion

In conventional insulation, there is always some heat loss. In the present study, by creating a desirable internal energy generation in the vertical wall, the perfect insulation boundary condition was achieved. Of course, the cost of internal energy generation must be paid. The analytical results of the present study provide a general theoretical framework for creating the perfect insulation boundary condition.

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