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The Non Split Eccentric Domination Number of Corona Product and Join of Some Standard Graphs

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Abstract: A subset D of the vertex set V(G) of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is said to be an eccentric dominating set if for every , there exists at least one eccentric point of v in D. An eccentric dominating set D of G is a non split eccentric dominating set if the induced sub graph < V- D > is connected. The minimum of the cardinalities of the non split eccentric dominating sets of G is called the non split eccentric domination number of G. This paper evaluates the non split eccentric domination number of Corona product and join of some standard graphs.

Keywords: Domination, Eccentric Domination, Non Split Eccentric Domination, Corona product, Join.

1. Introduction

Let G be a finite, simple undirected graph on p vertices and q edges with vertex set V(G) and edge set E(G). For graph theoretic terminology refer Harary [8] Buckley and Harary [5]. In 2010 T.N. Janakiraman M. Bhanumathi and S. Muthammai defined an eccentric domination in graph [9]. V.R. Kulli and Janakiram introduced the concept of split and nonsplit domination number of a graph in 1997 [11] and in 2000 [12] M. Bhanumathi and Sudhasenthil introduced the cocept of split and nonsplit eccentric domination number of a graphs in 2014 [4]. Motivated by these, we have defined Nonsplit eccentric domination number of Corona product and Join of some standard graphs.

Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v): u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices whereas the diameter diam(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. v is a central vertex if e(v) = r(G). The center C(G) is the set of all central vertices. The central subgraph <C(G)> of a graph G is the subgraph induced by the center v is a peripheral vertex if e(v) = d(G). The periphery P(G) is the set of all peripheral vertices.

For a vertex v, each vertex at a distance e(v) from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G) / d(u, v) = e(v)\}$.

The open neighbourhood N(v) of a vertex v is the set of all vertices adjacent to v in V. $N[v] = N(v) \cup \{v\}$ is called the closed neighbourhood of v. For a $v \in V(G)$. $N_i(v) = \{v \in V(G); d(u, v) = i\}$ is defined to be the i^{th} neigborhood of v in G.

A dominating set D of a graph G is a nonsplit dominating set if the induced subgraph $<V - D>$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ of a graph G is the minimum cardinality of a nonsplit dominating set.

A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists atleast one eccentric point of v in D. The eccentric domination number $\gamma_{ed}(G)$ of a graph G is the minimum cardinality of an eccentric dominating set. An eccentric dominating set with cardinality $\gamma_{ed}(G)$ is known as γ_{ed}-set.

Let $S \subseteq V(G)$. Then S is known as an eccentric point set of G if for every $v \in V - S$, S has atleast one vertex u such that $u \in E(v)$. An eccentric point set S of G is a minimal eccentric point set if no proper subset S' of S is an eccentric point set of G. S is known as a minimum eccentric point set if S is an eccentric point set with minimum cardinality. The minimum cardinality of an eccentric point set of G denoted as e(G) is known as eccentric number of G.

2. Prior Results

Theorem 2.1. [4]

(i) $\gamma_{nsed}(K_{1,n}) = n, n \geq 2$

(ii) $\gamma_{nsed}(W_n) = 3$, for $n \geq 4$

(iii) $\gamma_{nsed}(P_n) = n-2$, for $n \geq 4$

(iv) $\gamma_{nsed}(C_n) = n-2$, for $n \geq 3$

(v) $\gamma_{nsed}(K_n) = 1$, for $n \geq 3$

(vi) $\gamma_{nsed}(K_{m,n}) = 2$, for $n \geq 2$

Observation 2.1.

1. For any connected graph,

   $\gamma(G) \leq \gamma_{ns}(G) \leq \gamma_{nsed}(G)$.

2. For any connected graph G,

   $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{nsed}(G)$.

3. There are graphs with

   $\gamma_{ns}(G) = \gamma_{ed}(G)$ and $\gamma_{nsed}(G) = \gamma_{ns}(G)$.

4. There are graphs with

   $\gamma(G) = \gamma_{ed}(G) = \gamma_{nsed}(G)$.

Theorem 2.2. [3]

For a connected graph G with even number of vertices p and $\gamma_{ed}(G) = \frac{p}{2}$ if and only if G is for some connected graph H.

3. Main Results

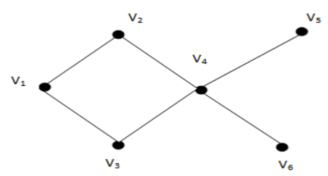
In this section, we determine the exact values of nonsplit eccentric domination number of corona product of graph

Definition 3.1.

An eccentric dominating set D of G is a nonsplit eccentric dominating set if the induced subgraph <V – D> is connected.

The nonsplit eccentric domination number $\gamma_{nsed}(G)$ of a graph G equals the minimum cardinality of a nonsplit eccentric dominating set. That is $\gamma_{nsed}(G)$ = min |D|, where the minimum is taken over D in D, where D is the set of all minimal nonsplit eccentric dominating sets of G. V(G) is a nonsplit eccentric dominating set for any graph G. Hence $\gamma_{nsed}(G)$ is a well defined parameter.

Example:3.1



Here

D = $\{v_1, v_4\}$ is a dominating set.

D′ = $\{v_1, v_5, v_6\}$ is an eccentric dominating set and

also nonsplit eccentric dominating set.

$\gamma_{nsed}(G) = 3$, $\gamma_{ns}(G) = 3$, $\gamma_{ed}(G) = 3$, $\gamma(G) = 2$

$\gamma(G) \le \gamma_{ed}(G) \le \gamma_{nsed}(G)$.

Definition 3.2

Let G and H be two graphs on n and m vertices respectively. The corona of the graphs G and H denoted by $G \circ H$ and is defined as the graph obtained by taking one copy of G and n copies of H and then joining the i[th] vertex of G to every vertex in the i[th] copy of H.

Example:3.2

$P_4 \circ K_3$

Theorem 3.1:

For $n \ge 3,\ m \ge 4,\ \gamma_{nsed}(C_n \circ W_m) = n$, where $W_m = K_1 + C_{m-1}$.

Proof:

Let $V(C_n) = \{v_1, v_2, v_3, \ldots, v_n\}$, $V(W_m) = \{w_i, u_{ij}\ /1 \le i \le n, 1 \le j \le m\}$ and $V(C_n \circ W_m) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{w_i, u_{ij}\ /1 \le i \le n, 1 \le j \le m\}$. Let $D = \{W_i / 1 \le i \le n\}$ where W_i are the central vertices of W_m. Then D is a minimum eccentric dominating set and <V-D> is connected. Therefore D is a minimum nonsplit eccentric dominating set and $|D| = n$ .Therefore $\gamma_{nsed}(C_n \circ W_m) = n$.

Theorem 3.2:

For $m \ge 2,\ \gamma_{nsed}(G_1 \circ K_m) = |V(G_1)|$, where G_1 be any connected graph with n vertices.

Proof:

Let $V(G_1) = \{v_1, v_2, v_3, \ldots, v_n\}$and let $\{u_{i1}, u_{i2}, u_{i3}, \ldots, u_{im}\}$ be the i[th] copy of K_m adjacent to v_i. Then $V(G_1 \circ K_m) = \{v_1, v_2, v_3, \ldots, v_n, u_{11}, u_{12}, \ldots, u_{1m}, u_{21}, u_{22}, \ldots, u_{2m}, \ldots u_{n1}, u_{n2}, \ldots, u_{nm}\}$. Let $D_j = \{u_{1j}, u_{2j}, u_{3j}, \ldots, u_{nj}\ /1 \le j \le m\}$ are some nonsplit dominating set. Further every vertex of < V-D_j> has an eccentric vertex in D_j. Therefore D_j is a nonsplit eccentric dominating set

and $\left|D_j\right| = n$. Hence each D_j is a minimum nonsplit eccentric dominating set of $G_1 \circ K_m$.
Hence $\gamma_{nsed}(G_1 \circ K_m) = \left|D_j\right| = n$.

Theorem 3.3:

If H is any self centred unique eccentric point graph with m vertices and $G = H \circ 2K_1$ then $\gamma_{nsed}(G) = 2m$.

Proof:

If H is any self-centred unique eccentric point graph. Then every vertex of H is an eccentric vertex. Hence m is even and G has 3m vertices.
Let $v_1, v_2, v_3, ..., v_m$ be the vertices of H and $\{V_i', V_i''\}$ for i = 1, 2, ...,m be the vertices of m copies of 2k, then in G, V_i', V_i'' are adjacent to v_i and if v_j is the eccentric vertex of v_i in H. Then V_i', V_i'' are the eccentric vertices of v_j in G and V_j', V_j'' are the eccentric vertices of v_i. It is clear that $D = \{V_1', V_2', ..., V_m'\} \cup \{V_1'', V_2'', ..., V_m''\}$ is a minimum eccentric dominating set of G. Further <V-D> is connected and $|D| = 2m$. Therefore D is a nonsplit eccentric dominating set of G. Hence $\gamma_{nsed}(G) = 2m$.

Theorem 3.4:

For $n \geq 3,\ m \geq 2,\ \gamma_{nsed}(C_n \circ K_{1,m}) = n$.

Proof:

Let $V(C_n) = \{v_1, v_2, ..., v_n\}$ and $V(K_{1,m}) = \{w_1, u_1, u_2, ..., u_m\}$, then $V(C_n \circ K_{1,m}) = \{v_i / 1 \leq i \leq n\} \cup \{w_i,\ u_{ij} /\ 1 \leq i \leq n, 1 \leq j \leq m\}$. By choosing a vertex set $D = \{w_1, w_2, ..., w_n\}$ which dominates all the vertices of $C_n \circ K_{1,m}$ and <V-D> is connected. Further every vertex of <V-D> has an eccentric vertex in D and $|D| = n$. Therefore D is a nonsplit eccentric dominating set of $C_n \circ K_{1,m}$. Hence $\gamma_{nsed}(C_n \circ K_{1,m}) = n$.

Theorem 3.5:

For $n \geq 3,\ m \geq 4,\ \gamma_{nsed}(C_n \circ P_m) = n \cdot \gamma(P_m)$.

Proof:

Let $V(C_n) = \{v_1, v_2, ..., v_n\}$ and the set $u_{i1}, u_{i2}, ..., u_{im}$ be the i^{th} copy of P_n adjacent to the vertex v_i then $V(C_n \circ P_m) = \{v_i / 1 \leq i \leq n\} \cup \{u_{ij} / 1 \leq i \leq n, 1 \leq j \leq m\}$. Let D be the n copies of dominating sets of P_m which dominates all the vertices of $C_n \circ P_m$. Further every vertex of <V-D> has an eccentric vertex in D and <V-D> is connected. Therefore D is a nonsplit eccentric dominating set and $|D| = n \cdot \gamma(P_m)$. Hence $\gamma_{nsed}(C_n \circ P_m) = n \cdot \gamma(P_m)$.

Theorem 3.6:

For a connected graph G with even number of vertices p, $\gamma_{nsed}(G) = \frac{p}{2}$ if and only if G is $H \circ K_1$ for some connected graph H.

Proof:

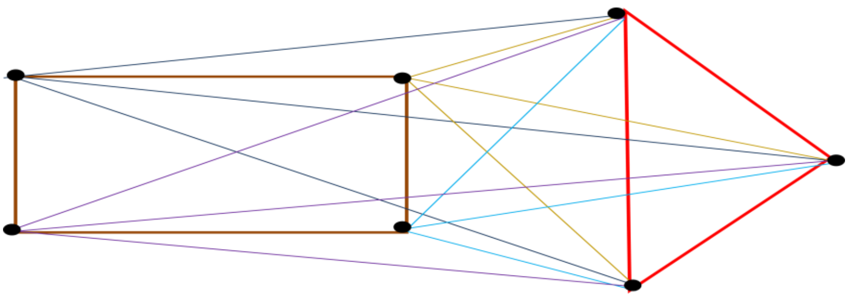
Let $G = H \circ K_1$, where H is a connected graph on $p/2$ vertices. V(H) is a $\gamma - set$ of G and D is the set of all pendent vertices in G is a minimum eccentric dominating set . Further <V-D> is connected. Therefore the set of all pendent vertices in G is a minimum nonsplit eccentric dominating set. Hence $\gamma_{nsed}(G) = p/2$. Conversely assume that $\gamma_{nsed}(G) = p/2$. Since G is a graph with even number of vertices p. By theorem 2.2, we get G is $H \circ K_1$ for some connected graph H.

4. Non Split Eccentric Domination in join of graphs

In this section we determine the exact values of non split eccentric domination number of Join graph G + H

Definition 4.1.

The join G + H of two graphs G and H is the the graph with vertex set $V(G+H) = V(G) \cup V(H)$ and the edge set $E(G+H) = E(G) \cup E(H) \cup \{uv / u \in V(G), v \in V(H)\}$.



$C_4 + K_3$

Theorem 4.1:

For $n \geq 4,\ m \geq 1,\ \gamma_{nsed}(P_n + K_m) = 3$.

Proof:

Let Let $V(P_n)=\{v_1,v_2,...,v_n\}$ and $V(K_m)=\{u_j /1\le j\le m\}$ then $V(P_n+K_m)=\{v_i \cup u_j /1\le i\le n, 1\le j\le m\}$. Let $D=\{v_1,v_n,u\}$, where v_1 and v_n are the end vertices of P_n and u is any vertex of K_m. Then D is a minimum nonsplit dominating set. Further every vertex of <V-D> has an eccentric vertex in D. Therefore D is a minimum nonsplit eccentric dominating set of P_n+K_m and $|D|=3$. Therefore $\gamma_{nsed}(P_n+K_m)=3$.

Theorem 4.2:

For $n\ge 4,\ m\ge 2,\ \gamma_{nsed}(P_n+K_{1,m})=3$.

Proof:

Let $V(P_n)=\{v_1,v_2,...,v_n\}$ and $V(K_{1,m})=\{w, u_j /1\le j\le m\}$ then $V(P_n+K_{1,m})=\{v_i /1\le i\le n\}\cup\{w, u_j /1\le j\le m\}$. Let $D=\{w,v_1,v_n\}$, where w is the root vertex of $K_{1,n}$, v_1 and v_n are the end vertices of P_n. Then D is a minimum nonsplit dominating set and <V-D> is connected. Further every vertex of <V-D> has an eccentric vertex in D. Therefore D is a nonsplit eccentric dominating set and $|D|=3$. Hence $\gamma_{nsed}(P_n+K_{1,m})=3$.

Theorem 4.3:

For $n\ge 4,\ m\ge 4,\ \gamma_{nsed}(P_n+W_m)=4$.

Proof:

Let $V(P_n)=\{v_1,v_2,...,v_n\}$ and $V(W_m)=\{w,u_j /1\le j\le m\}$ then $V(P_n+W_m)=\{v_i /1\le i\le n\}\cup\{w, u_j /1\le j\le m\}$. Let $D=\{v_1,v_n,u_1,u_2\}$, where v_1, v_n are the vertices of P_n and u_1, u_2 are adjacent vertices of W_n. Every vertex of <V-D> has an eccentric vertex in D. Therefore D is an eccentric dominating set. Further <V-D> is connected. Hence D is a minimum eccentric dominating set and $|D|=4$. Hence $\gamma_{nsed}(P_n+W_m)=4$.

Conclusion

Here we have evaluated the results on non split eccentric domination number of corona product and join of some standard graphs and also studied some bounds for non split eccentric domination number of a graph.

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