

Multi-Criteria Decision Analysis for Wastewater Management using Neutrosophic Hypersoft Sets

Mythili S¹ and Arokialancy A²

¹Asst Prof, Department of Mathematics, Nirmala college for women, Coimbatore (TN), India.,
mythiliaugustin@gmail.com

²Asst Prof, Department of Mathematics, Nirmala college for women, Coimbatore (TN), India.,
aarokia.lancy@gmail.com

Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract: The Neutrosophic Hypersoft Set (NHSS) introduces a parameterized family structure capable of handling intricate sub-attributes of parameters, thereby extending the expressive power of the Neutrosophic Soft Set. This study applies the NHSS framework to a multi-criteria decision-making problem in the context of wastewater management. By incorporating uncertainty, indeterminacy, and sub-parameter dependencies, the proposed approach provides a more robust and flexible decision-making tool compared to existing models. The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is integrated with NHSS to provide a structured ranking of alternatives. A case study involving three management strategies Recycling System, Advanced Treatment Plant, and Waste Reduction Program is used to demonstrate the applicability of the approach.

Keywords: Neutrosophic hypersoft set, wastewater management, multi-criteria decision-making, TOPSIS, neutrosophic topology.

1. Introduction

In 1999, Molodtsov [3] introduced the concept of soft sets, establishing a novel mathematical framework for handling uncertainties in decision-making. Building upon this foundation, Abbas, Murtaza, and Smarandache [1] introduced fundamental operations on hypersoft sets and the concept of a hypersoft point. Subsequently, Sagvan Y. Musa and Baravan A. [6] advanced the theory by proposing the notion of hypersoft topological spaces.

Further extending this concept, Smarandache [8] generalized soft sets into hypersoft sets by replacing single-argument functions with multi-argument functions defined over the Cartesian product of parameter sets. This enhancement significantly improves the flexibility and applicability of soft sets, particularly in complex decision-making scenarios. Saeed, Ahsan, and Siddique [4] conducted an in-depth study of the foundational elements of hypersoft set theory. To address decision problems involving uncertainty and vagueness, Zadeh [10] introduced fuzzy sets. Building on this, Shu-Jen Chen and Ching-Lai Hwang [7] extended the concept of fuzzy multiple attribute decision-making (MADM) and proposed an enhanced version of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. Their contributions are crucial for accurately capturing uncertainty and incompleteness in decision processes.

By utilizing a newly developed score function, a structured decision-making framework was introduced to address the complexities inherent in MADM problems. To validate the effectiveness of the proposed approach, a numerical example is provided, applying the TOPSIS method to evaluate and select a wastewater management system. This application demonstrates the model's capacity to support critical

decisions in environmental management. The TOPSIS technique, originally developed by Hwang and Yoon in 1987, was theoretically presented by Yoon [9] and later refined by Hwang et al. in 1993. This method identifies the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) to evaluate alternatives based on their shortest Euclidean distance from the ideal. Each criterion is considered for either maximization or minimization depending on its nature. TOPSIS ranks alternatives according to their closeness to the ideal solution. The best alternative receives a rank of one, indicating optimality, while the least preferred alternative receives a rank near zero. All other alternatives are assigned intermediate values, ensuring a comprehensive and relative assessment. This systematic approach enables decision-makers to effectively identify the most suitable option among multiple alternatives.

2 Preliminaries

Definition 2.1 ([2]). Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U . Let $A \subseteq E$. A pair (F_A, E) is called a soft set over U , where F_A is a mapping given by, $F_A : A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U .

Definition 2.2. [6]. A pair $(F, A_1 \times A_2 \times \dots \times A_n)$ is called a hypersoft set over U , where F is a mapping given by $F : A_1 \times A_2 \times \dots \times A_n \rightarrow P(U)$. Simply, we write the symbol E for $E_1 \times E_2 \times \dots \times E_n$, and for the subsets of E : the symbols A for $A_1 \times A_2 \times \dots \times A_n$, and B for $B_1 \times B_2 \times \dots \times B_n$. Clearly, each element in A , B and E is an n -tuple element. We can represent a hypersoft set (F, A) as an ordered pair, $(F, A) = \{(\alpha, F(\alpha)) : \alpha \in A\}$.

Definition 2.3. [6]. Fuzzy Set (FS) $A_{FS} = \{x(T_A(x)), x \in U\}$, where $T_A : U \rightarrow P([0, 1])$ is the membership degree of the generic element x with respect to the set A , and $P([0, 1])$ is the powerset of $[0, 1]$, is called a Fuzzy Set.

Definition 2.4. [6]. Given a universal set U , a Neutrosophic set A is characterized by a truth-membership function T_A , an indeterminacy-membership function I_A , and a falsity-membership function F_A . For an element x in U : $A(x) = \{T_A(x), I_A(x), F_A(x)\}$, where:

- $T_A(x)$ represents the degree of truth of x in A ,
- $I_A(x)$ represents the degree of indeterminacy of x in A ,
- $F_A(x)$ represents the degree of falsity of x in A .

Definition 2.5 [1]: A neutrosophic hypersoft set (NHSS) in a universe U with respect to a set of attributes A and their sub-attributes B_i is defined as:

$$NHSS = \{x, \{(E_i, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x) \mid E_i \in B_i)\} / x \in U\},$$

where x is an element of the universe U , E_i represents a sub-attribute in the set of sub-attributes B_i of the attribute A and $T_{E_i}(x)$, $I_{E_i}(x)$ and $F_{E_i}(x)$ denote the degrees of truth, indeterminacy, and falsity, respectively, for the element x with respect to the sub-attribute E_i .

3. Methodology

Step 1: Define alternatives: R (Recycling System), T (Advanced Treatment Plant), W (Waste Reduction Program)

Step 2: Define criteria: E (Environmental Impact), C (Cost), Ef (Efficiency), A (Community Acceptance)

Step 3: Construct the NHSS decision matrix with values of the form (T, I, F) for each alternative under each criterion.

Step 4: Aggregate the neutrosophic values into scalar scores using a linear model

Step 5: Use scalar scores in the TOPSIS method:

- Normalize the decision matrix
- Construct the weighted normalized matrix
- Identify ideal and anti-ideal solutions
- Compute distances to ideal solutions
- Calculate relative closeness
- Rank alternatives

4. Case study

Waste water management is an essential aspect of environmental sustainability and public health. Decision-making in Waste water management involves evaluating multiple criteria, such as environmental impact, cost, efficiency, and community acceptance. Traditional decision-making methods often fall short in addressing the multifaceted nature of water waste management, which involves multiple criteria and inherent uncertainties. In Waste water management, decision-makers must consider various factors, such as environmental impact, cost, efficiency, and community acceptance. Each of these criteria can be subjective and influenced by uncertainty, making it challenging to arrive at an optimal decision. To overcome these challenges, advanced decision-making frameworks that can handle complexity and uncertainty are required.

Neutrosophic sets extend the concept of fuzzy sets by introducing three components: truth (T), indeterminacy (I), and falsity (F). This extension allows for a more nuanced representation of uncertainty, accommodating the inherent vagueness and indeterminacy in real-world scenarios. Hypersoft sets further generalize soft sets by considering multi-attribute evaluations, providing a robust framework for decision-making in complex environments.

In this study, we apply the TOPSIS method within the Neutrosophic hypersoft set framework to evaluate and rank different waste water management alternatives. The alternatives considered are:

1. Recycling System (R)
2. Advanced Treatment Plant (T)
3. Waste Reduction Program (W)

These alternatives are assessed based on four key criteria:

1. Environmental Impact (E)
2. Cost (C)
3. Efficiency (Ef)
4. Community Acceptance (A)

Consider three alternatives for waste water management as mentioned in the previous section.

Step 1: Define membership functions:

Define the neutrosophic membership functions for each alternative with respect to each criterion. We have 3 values for each criterion truth, indeterminacy and falsity.

Table 4.1

Alternative	Environmental Impact (E) (T,I,F)	Cost (C) (T,I,F)	Efficiency (T,I,F)	Community Acceptance (T,I,F)
Recycling System (R)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.9, 0.05, 0.05)	(0.6, 0.3, 0.2)
Advanced Treatment Plant (T)	(0.9, 0.05, 0.05)	(0.5, 0.3, 0.2)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)
Waste Reduction Program (W)	(0.7, 0.2, 0.1)	(0.9, 0.05, 0.05)	(0.6, 0.3, 0.2)	(0.8, 0.1, 0.1)

Step 2:

Here we define criteria weights, that use AHP to determine the weights for each criterion. After pairwise comparisons and consistency assume the weights as,

- Environmental Impact: 0.4
- Cost: 0.2
- Efficiency: 0.3
- Community Acceptance: 0.1

Step 3:

Next, we construct the neutrosophic decision matrix. The decision matrix with membership values and weight is

Table 4.2

Alternative	Environmental Impact (E) (0.4)	Cost (C) (0.2)	Efficiency (0.3)	Community Acceptance (0.1)
Recycling System (R)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.9, 0.05, 0.05)	(0.6, 0.3, 0.2)
Advanced Treatment Plant (T)	(0.9, 0.05, 0.05)	(0.5, 0.3, 0.2)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)
Waste Reduction Program (W)	(0.7, 0.2, 0.1)	(0.9, 0.05, 0.05)	(0.6, 0.3, 0.2)	(0.8, 0.1, 0.1)

Step 4:

Now, apply TOPSIS for neutrosophic hypersoft sets. Calculate the neutrosophic normalized

decision matrix using the formula

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^m d_{ij}^2}}$$

Table 4.3: TRUTH VALUES

Alternative	Environmental Impact (E)	Cost (C)	Efficiency	Community Acceptance
Recycling System (R)	0.5657	0.5774	0.6782	0.4619
Advanced Treatment Plant (T)	0.6364	0.4122	0.6027	0.5385
Waste Reduction Program (W)	0.4944	0.7416	0.4520	0.6154

Table 4.4: INDETERMINACY VALUES

Alternative	Environmental Impact (E)	Cost (C)	Efficiency	Community Acceptance
Recycling System (R)	0.4082	0.5164	0.1526	0.8018
Advanced Treatment Plant (T)	0.2041	0.7745	0.3052	0.5345
Waste Reduction Program (W)	0.8165	0.1291	0.9158	0.2673

Table 4.5: FALSITY VALUES

Alternative	Environmental Impact (E)	Cost (C)	Efficiency	Community Acceptance
Recycling System (R)	0.6667	0.4082	0.3333	0.5774
Advanced Treatment Plant (T)	0.3333	0.8165	0.6667	0.5774
Waste Reduction Program (W)	0.6667	0.2041	0.6667	0.5774

The weighted normalized decision matrix using the formula,

$$v_{ij} = w_j \times r_{ij}$$

Alternative	Environmental Impact (E)	Cost (C)	Efficiency	Community Acceptance
Recycling System (R)	T-0.2263 I-0.1632 F-0.2666	T-0.1155 I-0.1032 F-0.0816	T-0.2035 I-0.0457 F-0.0999	T-0.0462 I-0.0801 F-0.0577
Advanced Treatment Plant (T)	T-0.2546 I-0.0816 F-0.1333	T-0.0824 I-0.1549 F-0.1633	T-0.1808 I-0.0915 F-0.2000	T-0.0538 I-0.0534 F-0.0577
Waste Reduction Program (W)	T-0.1977 I-0.3266 F-0.2666	T-0.1483 I-0.0258 F-0.0408	T-0.1356 I-0.2747 F-0.2000	T-0.0615 I-0.0267 F-0.0577

Step 5: Calculate the ideal (A*) and negative-ideal (A^-) solutions for the Truth(T), Indeterminacy(I) and Falsity(F) values using the formula

$$A^+ = \{(\max v_{ij} | j \in J), (\min v_{ij} | j \in J')\}$$

$$A^- = \{(\min v_{ij} | j \in J), (\max v_{ij} | j \in J')\}$$

Table 4.7

Recycling System(R)				
A*	0.2546	0.1483	0.2035	0.0615
A-	0.1978	0.0824	0.1356	0.0462
Advanced Treatment Plant(T)				
A*	0.3266	0.1549	0.2747	0.0801
A-	0.0816	0.0258	0.0457	0.0262
Waste Reduction Program(W)				
A*	0.2666	0.1633	0.2000	0.0577
A-	0.1333	0.0408	0.0999	0.0577

Step 6: Calculate the distance of each alternative from D* and D^- using Euclidean distance for each alternative from the positive and negative ideal solutions for Truth(T), Indeterminacy(I) and Falsity(F) using the formula,

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}$$

Table 4.8

Alternatives	Truth	Indeterminacy	Falsity
d_R^*	0.02960	0.21851	0.09172

d_R^-	0.03420	0.09049	0.13496
d_T^*	0.06647	0.27927	0.13333
d_T^-	0.05890	0.13193	0.13252
d_W^*	0.06141	0.13195	0.12251
d_W^-	0.06595	0.29744	0.14332

Step 7: Calculate the Relative closeness for Truth(T), Indeterminacy(I) and Falsity(F) values as follows,

Truth(T)

$$C_R^* = \frac{d_R^-}{d_R^- + d_R^*} = \frac{0.03420}{0.03420 + 0.02960} = 0.5360$$

$$C_T^* = \frac{d_T^-}{d_T^- + d_T^*} = \frac{0.05890}{0.05890 + 0.06647} = 0.4698$$

$$C_W^* = \frac{d_W^-}{d_W^- + d_W^*} = \frac{0.06595}{0.06595 + 0.06141} = 0.5178$$

Indeterminacy(I)

$$C_R^* = \frac{d_R^-}{d_R^- + d_R^*} = \frac{0.29744}{0.29744 + 0.13195} = 0.2928$$

$$C_T^* = \frac{d_T^-}{d_T^- + d_T^*} = \frac{0.13193}{0.13193 + 0.27927} = 0.3208$$

$$C_W^* = \frac{d_W^-}{d_W^- + d_W^*} = \frac{0.09049}{0.09049 + 0.21851} = 0.2927$$

Falsity(F)

$$C_R^* = \frac{d_R^-}{d_R^- + d_R^*} = \frac{0.13496}{0.13496 + 0.09172} = 0.3984$$

$$C_T^* = \frac{d_T^-}{d_T^- + d_T^*} = \frac{0.13252}{0.13252 + 0.13333} = 0.5953$$

$$C_W^* = \frac{d_W^-}{d_W^- + d_W^*} = \frac{0.14332}{0.14322 + 0.12251} = 0.5391$$

Step 8: Ranking the Alternatives:

Based on the relative closeness values for Truth(T), Indeterminacy(I) and Falsity(F)

- For Truth(T) values, the **Recycling System (R)** is the best alternative with a relative closeness of **0.5360**
- For Indeterminacy(I) and Falsity(F) values, the **Recycling System (R)** are the least alternative with a relative closeness of **0.2928 and 0.3984**

In the context of neutrosophic hypersoft sets, the **Recycling System(R)** is the best alternative for the water waste management system when considering the given criteria and weights. This conclusion is derived from its highest relative closeness values in terms of truth, indeterminacy and falsity indicating its overall superior performance and reliability in managing water waste effectively.

5. Conclusion

The **Recycling System (R)** emerged as the best alternative for managing waste water, scoring the highest relative closeness to the ideal solution. This indicates that, given the criteria and their respective weights, the Recycling System (R) provides the most balanced and optimal solution, effectively minimizing environmental impact while maintaining cost-efficiency, operational efficiency, and community acceptance. This decision supports the implementation of the Recycling System (R) as the preferred choice for a sustainable waste water management system.

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