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Solving Nonlinear Diffusion Equation of Porous Medium with linear pressure profile by Homotopy Perturbation Transform Method and He's polynomials.

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Abstract: Most physical phenomenon and processes in the field of Fluid Mechanics are governed by partial differential equations. Many nonlinear partial differential equations do not possess analytical solution, so numerical methods are commonly used to solve these equations. In this paper we have discussed an analytical method, which is a combination of Laplace Transforms and Homotopy Perturbation method called as homotopy perturbation transform method (HPTM). Our aim is to reduce the volume of computational work in finding the exact solution of nonlinear partial differential equation as compared to the classical methods while still maintaining the high accuracy of the numerical solution. One such nonlinear diffusion equation of porous medium with linear pressure profile is considered here with different initial conditions. Laplace Transform alone is incapable of handling nonlinear terms, so He's polynomials are used to simplify the nonlinear terms appearing in the equation. Many researchers have used Adomian Polynomials to decompose the nonlinear terms of the partial differential equations arising in Fluid Mechanics. But the complexity and calculation process are much easier in HPTM compared to Adomian polynomials.

Keywords: Computational Mathematics, Laplace Transform, Homotopy Perturbation Method, Porous Medium, Nonlinear Equations.

1. Introduction

The nonlinear diffusion equation is a prominent example of porous medium equation. Most phenomena described by nonlinear equations are still difficult to understand analytically hence approximate numerical method remains the only option. Analytic techniques like Adomian decomposition, the variational iteration methods etc. are approximate in nature. These methods have their limitations to calculate complex Adomian polynomials, Lagrange's multipliers etc. Simpler analytic method is discussed in the present paper.
Homotopy Perturbation Transform Method (HPTM):
It is good old homotopy perturbation method combined with Laplace Transform method named as Homotopy Perturbation Transform Method (HPTM). This method has proved to be quite promising in solving linear as well as nonlinear problems arising in different fields of science in general and Fluid Dynamics in particular. Initially J. H. He (1999) [1] described the fundamentals of Homotopy Perturbation method. He had showed by taking some examples that the homotopy perturbation technique does not depend upon a small parameter in the equation. In topology, a homotopy is constructed with an embedding parameter

$q \in [0,1]$ which is considered as a "small parameter". It was demonstrated through his method that the approximate solutions obtained by the proposed method are uniformly valid for both small and large parameters. It is considered as a promising and evolving method. Not only it has importance in solving mathematical equations, it can be applied to other branches of modern sciences. Lan Xu (2007) [2] had applied the Technique to find approximate solution of nonlinear boundary layer equations in unbounded domain. The results are compared with the results in available literature. It is verified that the method is simple and useful.

Ghorbani (2009) [3] used He's polynomials over Adomian polynomials, to obtain chaos solutions Fractals. Sari (2009) [4] demonstrated compact finite difference method in space and a low-storage total variation diminishing third-order Runge-Kutta scheme in time to find the solutions of the porous media equation in nonlinear problems of heat and mass transfer and in biological systems. In the calculation of the numerical derivatives, A tridiagonal band matrix algorithm was used in the calculation of the numerical derivative. However, the method used is not containing much numerical errors and storage capacity is also small. The solution obtained by this method is compared with the exact solutions to show the accuracy of the method. The approximate solutions of the equation have been computed. Numerical solution is compared with exact solution and it is found that they are almost same. So the method was proved to be a very good alternative method to some available techniques in various realistic problems.

In HPTM, the nonlinear terms can be easily simplified by the use of He's Polynomials is shown by Khan and Wu (2010) [5]. Gupta and Gupta (2011) [6] had showed applications of homotopy perturbation transform method for solving initial boundary value problem of variable coefficients. Madani et al (2011) [7] applied the technique in mathematical modelling. By using this new method, a combination of the Laplace transforms and Homotopy Perturbation Method (LHPM), all conditions could be satisfied. Also, very accurate results were obtained in a wide range via one or two iteration steps. Second order equation with respect to t is also giving exact solution with this method. It was proposed that Laplace transform Homotopy Perturbation Method (LHPM) to be used to handle the time dependent non-homogeneous partial differential equations. Furthermore, comparisons were made between the present method and the HPM, in order to verify the efficiency of the present method for its application on wider range. Majid Khan et al (2012) [8] had given the approximate solution of nonlinear Blasius flow equations by using homotopy analysis transform method (HATM) and HPTM. The nonlinear terms are simplified with the help of He's polynomials. The solution obtained with HATM is compared with HPTM and other iterative methods.

Jagdev Singh et al (2012) [9] have applied HPTM for linear and non-linear Klein-Gordon equations.

Devendra Kumar et al (2013) [10] have applied the homotopy perturbation transform method (HPTM) to obtain the solution of linear and nonlinear Schrodinger equations. The main feature of this method is to find the solution without any discretization or restrictive assumptions and avoids the round-off errors. The fact that this technique solves nonlinear problems without using Adomian polynomials can be considered as a clear advantage of this algorithm over the Adomian decomposition method (ADM).

U. Filobello-Nino et al (2013) [11] have proposed this method to solve nonlinear differential equation with Dirichlet, Neumann and mixed boundary conditions. Comparison is made between the figures of exact and approximate solutions, and it is seen that they are of high accuracy. Another advantage of this method is that, we need not have to solve several recurrence differential equations.

Saleem et al (2017) [12] demonstrated that, for solving nonlinear equations, a combination form of the Laplace transform method along with the Homotopy perturbation method was used. Proposed method solves nonlinear problems without using Adomian polynomials can be thought as a vibrant advantage of this algorithm over the Adomian decomposition method.
Swielam and Khadar (2018) [13] obtained exact solution of some coupled non-linear differential equations using homotopy perturbation method using Laplace transform and Pade approximation. The examples are considered on the coupled nonlinear system of Burger equations and the coupled nonlinear system in one dimensional thermoelasticity. The results obtained are verified with exact solutions. So the method is strongly applicable in solving large number of nonlinear differential equations.
Comparison of homotopy perturbation method (HPM) and homotopy perturbation transform method (HPTM) is effectively shown by Mohammad Elbadri (2018) [14]. Authors have claimed that homotopy perturbation transform method is very fast convergent to the solution of the partial differential equation. For illustration and more explanation of the idea, some examples are provided through which it is shown that most of the inhomogeneous problems give exact solution by HPTM whereas HPM gives an approximate solution to these problems.
Mohamed Jleli et al (2020) [15] have successfully applied HPTM for solving multidimensional time fractional partial differential equations involving the recent introduced Yang-Abdel-Aty-Cattani fractional derivative. The presented examples show that this approach is very powerful and efficient for finding approximate solutions for wide classes of problems arising in science and technology.

2. Objectives

Since last two decades, there is a rapid development in nonlinear Science. Many researchers have used numerical and analytical technique to solve the nonlinear differential equations. Diffusion equation is one of these differential equations in the areas of heat and mass transfer, biological systems and the processes involving fluid flow. To solve these differential equations analytically and to obtain formal and approximate solutions in the series form is the main objective here. In this paper, Homotopy Perturbation Transform Method is applied to solve these nonlinear diffusion equations with various boundary conditions.

3. Methods

The proposed algorithm provides the solution in a rapid convergent series which may lead to the solution in a closed form. The advantage of this method is its capability of combining two powerful methods for obtaining exact or approximate solution for non-linear equations.
To illustrate the basic idea for HPTM, we consider a general non-linear partial differential equation with the initial conditions of the form

$$D\theta(x,t) + R\theta(x,t) + N\theta(x,t) = f(x,t) \tag{1}$$

with initial condition

$\theta(x,0) = h(x)$ and $\theta_t(x,0) = g(x)$

where $D = \frac{\partial^2}{\partial t^2}$, R is the linear differential operator of less order than D, N represents the general non-linear differential operator and $f(x,t)$ is the source term.
Taking Laplace transform of both sides of Eq. (1)

$$L[D\theta(x,t)] + L[R\theta(x,t)] + L[N\theta(x,t)] = L[f(x,t)] \tag{2}$$

using differentiation property of Laplace transform

$$L[\theta(x,t)] = \frac{h(x)}{s} + \frac{f(x)}{s^2} - \frac{1}{s^2}L[R\theta(x,t)] + \frac{1}{s^2}L[f(x,t)] - \frac{1}{s^2}L[N\theta(x,t)] \quad (3)$$

Taking Laplace inverse on both sides of Eq. (3)

$$\theta(x,t) = F(x,t) - L^{-1}\left[\frac{1}{s^2}L(R\theta(x,t) + N\theta(x,t))\right] \quad (4)$$

where $F(x,t)$ represents the term arising from the source term with prescribed intial conditions
Now by applying homotopy perturbation method

$$\theta(x,t) = \sum_{n=0}^{\infty} q^n \theta_n(x,t) \quad (5)$$

where q is the embedding parameter & $q \in [0,1]$
the non-linear term can be decomposed as

$$N\theta(x,t) = \sum_{n=0}^{\infty} q^n H_n(\theta) \quad (6)$$

for some He's polynomials $H_n(\theta)$ that are given by

$$H_n(\theta_0, \theta_1, \theta_2 \dots \theta_n) = \frac{1}{n!}\frac{\partial^n}{\partial q^n}\left[N\sum_{i=0}^{\infty} q^i \theta_i\right]_{q=0} \quad n = 0,1,2,\dots$$

Substituting equations (5) and (6) in Eq. (4) we get

$$\sum_{n=0}^{\infty} q^n \theta_n(x,t) = F(x,t) - q\left\{L^{-1}\left[\frac{1}{s^2}L\left(R\ \sum_{n=0}^{\infty} q^n \theta_n(x,t) + \sum q^n H_n(\theta)\right)\right]\right\}$$

(7)

which is the coupling of the Laplace transform and the homotopy perturbation method using He's polynomials.
Comparing the co-efficient of like powers of q, the following approximations are obtained.

$q^0 : \theta_o(x,t) = F(x,t)$ $\quad$ $q^1 : \theta_1(x,t) = -\frac{1}{s^2}L[R\theta_0(x,t) + H_0(\theta)]$ $\quad$ $q^2 : \theta_2(x,t) = -\frac{1}{s^2}L[R\theta_1(x,t) + H_1(\theta)]$ $q^3 : \theta_3(x,t) = -\frac{1}{s^2}L[R\theta_2(x,t) + H_2(\theta)]$

The best approximation for the solution is

$$\theta = \theta_0 + \theta_1 + \theta_2 + \dots\dots$$

4. Discussion and Results

To evaluate the solution procedure of HPTM, we discuss the non-linear diffusion equation of porous medium with linear pressure profile as

Problem I

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[T\frac{\partial T}{\partial x}\right] + \frac{1}{\rho}\frac{\partial p}{\partial x} \quad (1)$$

where p = linear pressure = $ax + b$, with $a < 0$ and ρ = constant density with $T(x,0) = e^x$
So, Eq. (1) takes the form

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[T\frac{\partial T}{\partial x}\right] + \frac{1}{\rho}(a) \tag{2}$$

Consider Laplace transform of Eq. (2)

$$L\left[\frac{\partial T}{\partial t}\right] = L\left[\left(\frac{\partial T}{\partial x}\right)^2\right] + L\left[T\frac{\partial^2 T}{\partial x^2}\right] + L\left[\frac{a}{\rho}\right]$$

Using the initial condition and Laplace transform of derivative

$$T(x,s) = \frac{e^x}{s} + \frac{1}{s}L\left[\left(\frac{\partial T}{\partial x}\right)^2\right] + \frac{1}{s}L\left[T\frac{\partial^2 T}{\partial x^2}\right] \tag{3}$$

Taking inverse transform of both sides of Eq. (3)

$$T(x,t) = e^x + \frac{a}{\rho}t + L^{-1}\left\{\frac{1}{s}L\left[\left(\frac{\partial T}{\partial x}\right)^2 + T\frac{\partial^2 T}{\partial x^2}\right]\right\} \tag{4}$$

by Homotopy perturbation method

$$T(x,t) = \sum_{n=0}^{\infty} q^n T_n(x,t) \tag{5}$$

Substituting Eq. (5) in Eq. (4) and using He's polynomials

$$\sum_{n=0}^{\infty} q^n T_n(x,t) = e^x + \frac{a}{\rho}t + q\,L^{-1}\left\{\frac{1}{s}L\left(\sum q^n H_n(T)\right)\right\}$$

Where

$$H_0(T) = \left(\frac{\partial T_0}{\partial x}\right)^2 + T_o\frac{\partial^2 T_0}{\partial x^2}$$

$$H_1(T) = 2\left(\frac{\partial T_0}{\partial x}\right)\left(\frac{\partial T_1}{\partial x}\right) + T_0\frac{\partial^2 T_1}{\partial x^2} + T_1\frac{\partial^2 T_0}{\partial x^2}$$

Comparing the co-efficient of like powers of q, the following approximations are obtained

$$q^0 : T_0(x,t) = e^x + \frac{a}{\rho}t$$

$$q^1 : T_1(x,t) = L^{-1}\left[\frac{1}{s}L\left[\left(\frac{\partial T_0}{\partial x}\right)^2 + T_0\frac{\partial^2 T_0}{\partial x^2}\right]\right] = L^{-1}\left[\frac{1}{s}\left(\frac{2e^{2x}}{s} + \frac{a}{\rho}\frac{e^x}{s^2}\right)\right]$$

$$q^1 : T_1(x,t) = 2e^{2x}t + \frac{a}{\rho}e^x t$$

$$q^2 : T_2(x,t) = L^{-1}\left[\frac{1}{s}L\left\{2\left(\frac{\partial T_0}{\partial x^2}\right)\left(\frac{\partial T_1}{\partial x}\right) + \left(T_0\frac{\partial^2 T_1}{\partial x^2} + T_1\frac{\partial^2 T_0}{\partial x^2}\right)\right\}\right]$$

$$= t^2\left[9e^{3x} + \frac{2a}{\rho}e^{2x}\right] + t^3\left[\frac{8ae^{2x}}{3\rho} + \frac{a^2e^x}{3\rho^2}\right]$$

………..

$$\therefore T(x,t) = T_0 + T_1 + T_2 + \cdots$$

$$= e^x + t\left[\frac{a}{\rho} + 2e^{2x} + \frac{a}{\rho}e^x\right] + t^2\left[9e^{3x} + \frac{2a}{\rho}e^{2x}\right] + t^3\left[\frac{8a}{3\rho}e^{2x} + \frac{a^2}{3\rho^2}e^x\right] + \cdots$$

as an approximate solution converges for $t < 1$.

Problem II:

Consider non-linear diffusion equation of porous medium with pressure profile is

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[T\frac{\partial T}{\partial x}\right] + \frac{1}{p}\frac{\partial p}{\partial x}$$

With initial condition $T(x,0) = -x \; and \; p(x) = ax + b, with \; a < 0$

Applying Laplace transform on both sides

$$L\left[\frac{\partial T}{\partial x}\right] = L\left[\left(\frac{\partial T}{\partial x}\right)^2\right] + L\left[T\frac{\partial^2 T}{\partial x^2}\right] + L\left[\frac{1}{\rho}\frac{\partial p}{\partial x}\right] \tag{1}$$

Using initial condition

$$\Rightarrow T(x,s) = \frac{-x}{s} + \frac{1}{s}L\left[\left(\frac{\partial T}{\partial x}\right)^2 + T\frac{\partial^2 T}{\partial x^2}\right] + \frac{a}{\rho s^2} \tag{2}$$

Taking inverse transform of Eq. (2)

$$\mathrm{T}(x,t) = -x + \frac{a}{\rho}t + L^{-1}\left[\frac{1}{s}L\left(\left(\frac{\partial T}{\partial x}\right)^2 + T\frac{\partial^2 T}{\partial x^2}\right)\right] \tag{3}$$

by *HPM*

$$T(x,t) = \sum_{n=0}^{\infty} q^n T_n(x,t) \tag{4}$$

and using He's polynomials, Substituting Eq. (4) in Eq. (3)

$$\Rightarrow \sum_{n=0}^{\infty} q^n T_n(x,t) = -x + \frac{a}{\rho}t + q\,L^{-1}\left\{\frac{1}{s}L\left(\sum q^n H_n(T)\right)\right\}$$

Where

$$H_0(T) = \left(\frac{\partial T_0}{\partial x}\right)^2 + T_0\frac{\partial^2 T_0}{\partial x^2}$$

$$H_1(T) = 2\left(\frac{\partial T_0}{\partial x}\right)\left(\frac{\partial T_1}{\partial x}\right) + T_0\frac{\partial^2 T_1}{\partial x^2} + T_1\frac{\partial^2 T_0}{\partial x^2}$$

Comparing the co-eff. of like powers of q we get

$$q^0 : T_0(x,t) = -x + \frac{a}{\rho}t$$

$$q^1 : T_1(x,t) = L^{-1}\left\{\frac{1}{s}L\left[\left(\frac{\partial T_0}{\partial x}\right)^2 + T_0\frac{\partial^2 T_0}{\partial x^2}\right]\right\}$$

$$T_1(x,t) = L^{-1}\left[\frac{1}{s^2}\right] = t$$

$$q^2 : T_2(x,t) = L^{-1}\left\{\frac{1}{s}L\left[2\left(\frac{\partial T_0}{\partial x}\right)\left(\frac{\partial T_1}{\partial x}\right) + \left(T_0\frac{\partial^2 T_1}{\partial x^2} + T_1\frac{\partial^2 T_0}{\partial x^2}\right)\right]\right\}$$

$$T_2(x,t) \;=\; 0$$

$$\therefore T(x,t) \;=\; T_0 + \; T_1 + \; T_2 + \cdots$$

$$= \; -x\left(\frac{a}{\rho}+1\right)t$$

is an exact solution.

Problem III

Consider the diffusion equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[T\frac{\partial T}{\partial x}\right] + \frac{1}{\rho}\frac{\partial p}{\partial x}$$

with linear pressure

$p(x) = ax + b, with\ a < 0\ and$ initial condition $T(x,0) = sinx$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[T\frac{\partial T}{\partial x}\right] + \frac{1}{\rho}\;(a) \tag{1}$$

Consider Laplace transform of Eq. (1)

$$L\left[\frac{\partial T}{\partial t}\right] = L\left[\left(\frac{\partial T}{\partial x}\right)^2\right] + L\left[T\frac{\partial^2 T}{\partial x^2}\right] + L\left[\frac{a}{\rho}\right]$$

Using the initial condition and Laplace transform of derivative

$$T(x,s) = \frac{sinx}{s} + \frac{1}{s}\;L\left[\left(\frac{\partial T}{\partial x}\right)^2 + T\frac{\partial^2 T}{\partial x^2}\right] + \frac{a}{\rho s^2} \tag{2}$$

Taking inverse transform of both sides of Eq. (2)

$$T(x,t) = \; sinx + \frac{a}{\rho}\;t\; + L^{-1}\left[\frac{1}{s}\;L\left(\left(\frac{\partial T}{\partial x}\right)^2 \; + T\frac{\partial^2 T}{\partial x^2}\right)\right] \tag{3}$$

by Homotopy perturbation method

$$T(x,t) = \sum_{n=0}^{\infty} q^n T_n(x,t) \tag{4}$$

Substituting Eq. (4) in Eq. (3) and using He's polynomials

$$\Rightarrow \sum_{n=0}^{\infty} q^n T_n(x,t) = \; sinx + \frac{a}{\rho}t + q\;L^{-1}\left\{\frac{1}{s}\;L\;\left(\sum q^n H_n\;(T)\right)\right\}$$

Where

$$H_0(T) \quad = \left(\frac{\partial T_0}{\partial x}\right)^2 + \; T_0\frac{\partial^2 T_0}{\partial x^2}$$

$$H_1(T) = 2\left(\frac{\partial T_0}{\partial x}\right)\left(\frac{\partial T_1}{\partial x}\right) + T_0\frac{\partial^2 T_1}{\partial x^2} + T_1\frac{\partial^2 T_0}{\partial x^2}$$

Comparing the co-eff. of like powers of q we get

$$q^0 : T_0(x,t) = sinx + \frac{a}{\rho}t$$

$$q^1 : T_1(x,t) = L^{-1}\left\{\frac{1}{s} L\left[\left(\frac{\partial T_0}{\partial x}\right)^2 + T_0\frac{\partial^2 T_0}{\partial x^2}\right]\right\}$$

$$= L^{-1}\left[\frac{1}{s}\left(\frac{cos^2x}{s} + \frac{sinx}{s} - \frac{a}{\rho}\frac{sinx}{s^2}\right)\right]$$

$$q^1 : T_1(x,t) = tcos^2x + tsinx - \frac{a}{2\rho}t^2 sinx$$

$$q^2 : T_2(x,t) = L^{-1}\left\{\frac{1}{s} L\left[2\left(\frac{\partial T_0}{\partial x}\right)\left(\frac{\partial T_1}{\partial x}\right) + \left(T_0\frac{\partial^2 T_1}{\partial x^2} + T_1\frac{\partial^2 T_0}{\partial x^2}\right)\right]\right\}$$

$$= t^4\frac{a^2 sinx}{8\rho} - t^3\frac{a(3cos2x + sinx)}{3\rho} + t^2\frac{(2cos2x - 2sinxcos2x - 5sinxcos^2x}{2}$$

$$\therefore T(x,t) = T_0 + T_1 + T_2 + \cdots$$

$$= sinx + t\left[\frac{a}{\rho} + cos^2x + sinx\right] - t^2\left[\frac{asinx}{2\rho} + \frac{(2cos2x - 2sinxcos2x - 5sinxcos^2x)}{2}\right] + \cdots.$$

as an approximate solution, converges for $t < 1$.

5. Conclusion

In this paper, a homotopy perturbation transform method (HPTM) is discussed for solving non-linear diffusion equation of porous medium with linear pressure gradient under different initial conditions. From this method we can conclude that the non-linear problems possess approximate or exact solution by analytical method.
This method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result. As in problem I, with initial condition as an exponential term it gives approximate solution with the condition $t < 1$. Where as in problem II, with initial condition as a linear term gives exact solution for any value of t. In problem III, initial condition is the sinusoidal function, which gives approximate solution for $t < 1$.

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NOMENCLATURE				
T	Temperature variable in problems		x	Distance variable
θ	Temperature variable in method		t	Time variable
p	Pressure		q	Embedding parameter
ρ	Density		H	He's Polynomial term
L	Laplace Operator		θ_t	Velocity
L^{-1}	Inverse Laplace operator		a	Constant in pressure term