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Decomposition of $(\Psi_\alpha h_b, \Delta)$-Continuity

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Abstract: In this article, we explore and introduce the variety of open sets in H-GTS. Additionally, we get the decomposition of $(\Psi_\alpha H_b, \delta)$-C.

Keywords: hereditary generalized topology, α-H_b-open, π-H_b-open and π-H_b-open sets.

1. Introduction

Let Z be a nonempty set and ϑ be a collection from the subsets of Z. Then ϑ is called a generalized topology (briefly GT) [1], iff $\emptyset \in \vartheta$ and union of open sets of ϑ is open in ϑ. The closure of a subset B of Z, denoted by $c_\vartheta(B)$ is the smallest ϑ-closed sets containing B and the interior (resp. ϑ-σ-interior) of B, denoted by $i_\vartheta(B)$ (resp. $i_\sigma(B)$) is the largest ϑ-open (resp. ϑ-σ-open) sets contained in B.

Definition 1.1. A subset $B \subset Z$ is called

1. ϑ-α-open [2], if $B \subset i_\vartheta c_\vartheta i_\vartheta(B)$.
2. ϑ-σ-open [2], if $B \subset c_\vartheta i_\vartheta(B)$.
3. ϑ-π-open [2], if $B \subset i_\vartheta c_\vartheta(B)$.
4. ϑ-β-open [2], if $B \subset c_\vartheta i_\vartheta c_\vartheta(B)$.
5. ϑ-b-open [14], if $B \subset c_\vartheta i_\vartheta(B) \cup i_\vartheta c_\vartheta(B)$.

Definition 1.2. A collection H of subsets of Z is called as a hereditary class [3], if $B \in H$ and $V \subset B$, then $V \in H$.

Definition 1.3. For a hereditary class H on Band $B \subset Z$, we define $B^*(H, \vartheta) = \{a \in Z: B \cap V \notin H$ for all $V \in \vartheta$ such that $a \in V\}$ [3].

Definition 1.4. A subset $B \subset Z$ is called

1. α-H-open [3], if $B \subseteq i_\vartheta c_\vartheta^* i_\vartheta(B)$,
2. σ-H-open [3], if $B \subseteq c_\vartheta^* i_\vartheta(B)$,
3. π-H-open [3], if $B \subseteq i_\vartheta c_\vartheta^*(B)$,
4. β-H-open [3], if $B \subseteq c_\vartheta i_\vartheta c_\vartheta^*(B)$,
5. ϑ^*-closed [3], if $c_\vartheta^*(B) \subset B$.
6. b-H-open [8], if $B \subseteq i_\vartheta c_\vartheta^*(B) \cup c_\vartheta^* i_\vartheta(B)$
7. $\sigma\vartheta^*$-closed [5], if $B_\sigma^* \subseteq B$

Definition 1.5. A subset $B \subset Z$ is said to be

1. α-H_σ-open [11], if $B \subseteq i_\vartheta c_\sigma^* i_\vartheta(B)$,
2. σ-H_σ-open [11], if $B \subseteq c_\sigma^* i_\vartheta(B)$,
3. π-H_σ-open [11], if $B \subseteq i_\vartheta c_\sigma^*(B)$,
4. β-H_σ-open [11], if $B \subseteq c_\vartheta i_\vartheta c_\sigma^*(B)$.
5. b-H_σ-open [10], if $B \subseteq i_\vartheta c_\sigma^*(B) \cup c_\sigma^* i_\vartheta(B)$.

Definition 1.6. Consider B be a subset of H-GTS (Z, ϑ, H). Then $B_b^*(H, \vartheta) = \{z \in Z: B \cap V \notin H$ for All $V \in \vartheta$-b-open such that $z \in V\}$.

Consider (Z, ϑ, H) be a hereditary generalized topological space. For $B \subset Z$, define $c_b^*(B) = B \cup B_b^*(H, \vartheta)$ and $c_b^*(B)$ is enlarging, monotone and idempotent.

Definition 1.7. [12] A subset $B \subset Z$ is called

1. Δ-H_ψ-open, if $B \subseteq i_\sigma c_\sigma{}^* i_\sigma(B)$.
2. Σ-H_ψ-open, if $B \subseteq c_\sigma{}^* i_\sigma(B)$.
3. Φ-H_ψ-open, if $B \subseteq i_\sigma c_\sigma{}^*(B)$.
4. Ω-H_ψ-open, if $B \subseteq c_\vartheta i_\sigma c_\sigma{}^*(B)$.
5. B-H_ψ-open, if $B \subseteq c_\sigma{}^* i_\sigma \cup i_\sigma c_\sigma{}^*(B)$.

Definition 1.8. [13] A subset B of a H-GTS (Z, ϑ, H) is called as

1. α-H_b-open, if $B \subseteq i_\vartheta c_b * i_\vartheta(B)$
2. σ-H_b-open, if $B \subseteq c_b * i_\vartheta(B)$
3. π-H_b-open, if $B \subseteq i_\vartheta c_b *(B)$
4. β-H_b-open, if $B \subseteq c_\vartheta i_\vartheta c_b *(B)$
5. b-H_b-open, if $A \subseteq i_\vartheta c_b *(A) \cup c_b * i_\vartheta(A)$.

2. Generalized ΨH_b-open sets

Definition 2.1. A subset B of a H-GTS (Z, ϑ, H) is called as

1. Ψ_α-H_b -open, if $B \subseteq i_\sigma c_b{}^* i_\sigma(B)$
2. Ψ_σ-H_b -open, if $B \subseteq c_b{}^* i_\sigma(B)$
3. Ψ_π-H_b -open, if $B \subseteq i_\sigma c_b{}^*(B)$
4. Ψ_β-H_b -open, if $B \subseteq c_\vartheta i_\sigma c_b{}^*(B)$
5. Ψ_b-H_b -open, if $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$.

The sets Ψ_α-H_b-open (resp. Ψ_σ-H_b-open, Ψ_π-H_b-open, Ψ_β-H_b-open, Ψ_b-H_b-open, ϑ-pen) denoted by Ψ_α-$H_bO(Z)$ (resp. Ψ_σ-$H_bO(Z)$, Ψ_π-$H_bO(Z)$, Ψ_β-$H_bO(Z)$, Ψ_b-$H_bO(Z)$, $\vartheta O(Z)$).

Theorem 2.2. In H - GT S (Z, ϑ, H) :

1. Any $\vartheta O(Z)$ set is $\Psi\alpha$-HbO(Z).
2. Any $\vartheta O(Z)$ set is $\Psi\sigma$-HbO(Z).
3. Any $\vartheta O(Z)$ set is $\Psi\pi$-HbO(Z).
4. Any $\vartheta O(Z)$ set is $\Psi\beta$-HbO(Z).
5. Any $\vartheta O(Z)$ set is Ψb-HbO(Z).

Proof. (1). Consider a subset B of H-GTS (Z, ϑ, H) is $\vartheta O(Z)$. Then $B \subseteq i_\vartheta(B) \subseteq i_\vartheta{}^*(B) \subseteq i_\vartheta c_b{}^* i_\vartheta(B) \subseteq i_\sigma c_b{}^* i_\sigma(B)$. Hence B is Ψ_α-$H_bO(Z)$.
(2). Consider a subset B of H-GTS (Z, ϑ, H) is $\vartheta O(Z)$. Then, $B \subseteq i_\vartheta(B) \subseteq c_b{}^* i_\vartheta(B) \subseteq c_b{}^* i_\sigma(B)$. Hence B is Ψ_σ-$H_bO(Z)$.
(3). Consider a subset B of H-GTS (Z, ϑ, H) is $\vartheta O(Z)$. Then, $B \subseteq i_\vartheta(B) \subseteq i_\vartheta\, c_b{}^*(B) \subseteq i_\sigma\, c_b{}^*(B)$. Hence B is Ψ_π-$H_bO(Z)$.
(4). Consider a subset B of H-GTS (Z, ϑ, H) is $\vartheta O(Z)$. Then, $B \subseteq i_\vartheta(B) \subseteq c_\vartheta i_\vartheta\, c_b{}^*(B) \subseteq c_\vartheta i_\sigma c_b{}^*(B)$. Hence B is Ψ_β-$H_bO(Z)$.
(5). Consider a subset B of H-GTS (Z, ϑ, H) is $\vartheta O(Z)$. Then, $B \subseteq i_\vartheta(B) \subseteq i_\vartheta c^*(B) \subseteq i_\vartheta\, c_b{}^*(B) \cup c_b{}^* i_\vartheta(B) \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$. Hence, B is Ψ_b-$H_bO(Z)$.

Theorem 2.3. In H-GTS (Z, ϑ, H):

1. Any α-HbO(Z) is $\Psi\alpha$-HbO(Z).
2. Any σ-HbO(Z) is $\Psi\sigma$-HbO(Z).
3. Any π-HbO(Z) is $\Psi\pi$-HbO(Z).
4. Any β-HbO(Z) is $\Psi\beta$-HbO(Z).
5. Any b-HbO(Z) is Ψb-HbO(Z).

Proof. 1. Consider B be α-$H_bO(Z)$. Then we have, $B \subseteq i_\vartheta c_b{}^* i_\vartheta(B) \subseteq i_\sigma c_b{}^* i_\sigma(B)$. Hence Ψ_α-$H_bO(Z)$.

2. Consider B be σ-$H_bO(Z)$. Then we have, $B \subseteq c_b{}^* i_\vartheta(B) \subseteq c_b{}^* i_\sigma(B)$. Hence Ψ_σ-$H_bO(Z)$.
3. Consider B be π-$H_bO(Z)$. Then we have, $B \subseteq i_\vartheta c_b{}^*(B) \subseteq i_\sigma c_b{}^*(B)$. Hence Ψ_π-$H_bO(Z)$.
4. Consider B be β-$H_bO(Z)$. Then we have, $B \subseteq c_\vartheta i_\vartheta c_b{}^*(B) \subseteq c_\vartheta i_\sigma c_b{}^*(B)$. Hence Ψ_β-$H_bO(Z)$.
5. Consider B be b-$H_bO(Z)$. Then we have, $B \subseteq i_\vartheta c_b{}^*(B) \cup c_b{}^* i_\vartheta(B) \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$. Hence Ψ_b-$H_bO(Z)$.

Theorem 2.4. In H-GTS (Z, ϑ, H):

1. Any Ψ_α-$H_bO(Z)$ is Δ-HO(Z).
2. Any Ψ_σ-$H_bO(Z)$ is Σ-HO(Z).
3. Any Ψ_π-$H_bO(Z)$ is Φ-HO(Z).

4. Any Ψ_β-$H_bO(Z)$ is Ω - bO(Z).
5. Any Ψb - HbO(Z) is B -$H_bO(Z)$.

Proof. 1. Consider B be Ψ_α-$H_bO(Z)$. Then we have, $B\subseteq i_\sigma c_b{}^* i_\sigma(B) \subseteq i_\sigma c_\sigma{}^* i_\sigma(B)$. Hence Δ-HO(Z).

2. Consider B be Ψ_σ-$H_bO(Z)$. Then we have, $B\subseteq c_b{}^* i_\sigma(B)\subseteq c_\sigma{}^* i_\sigma(B)$. Hence Σ-HO(Z).

3. Consider B be Ψ_π-$H_bO(Z)$. Then we have, $B\subseteq i_\sigma c_b{}^*(B)\subseteq i_\sigma c_\sigma{}^*(B)$. Hence B is Φ-HO(Z).

4. Consider B be Ψ_β-$H_bO(Z)$. Then we have, $B\subseteq c_\sigma i_\sigma c_b{}^*(B)\subseteq c_\sigma i_\sigma c_\sigma{}^*(B)$. Hence B is Ω - $H_bO(Z)$.

5. Consider B be Ψ_b - $H_bO(Z)$. Then we have, $B \subseteq i_\sigma c^*(B) \cup c_b{}^* i_\sigma(B)\subseteq i_\sigma c_\sigma{}^*(B) \cup c_\sigma{}^* i_\sigma(B)$. Hence B is B-$H_bO(Z)$.

Theorem 2.5. If B is Ψ_α - $H_bO(Z)$, then it is $\alpha(\sigma)$ -open.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_α-$H_bO(Z)$. Then, $B\subseteq i_\sigma c_b{}^* i_\sigma(B)\subseteq i_\sigma c_\sigma{}^* i_\sigma(B) \subseteq i_\sigma c_\sigma i_\sigma(B)$. Hence B is $\alpha(\sigma)$ -open.

Theorem 2.6. If B is Ψ_σ-$H_bO(Z)$, then it is $\sigma(\sigma)$ -open.
Proof. Consider a subset B of H-GT S (Z, ϑ, H) is Ψ_σ-$H_bO(Z)$. Then, $B\subseteq c_b{}^* i_\sigma(B)\subseteq c_\sigma{}^* i_\sigma(B) \subseteq c_\sigma i_\sigma(B)$. Hence B is $\sigma(\sigma)$ -open.

Theorem 2.7. If B is Ψ_π-$H_bO(Z)$, then it is $\pi(\sigma)$ -open.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_π-$H_bO(Z)$. Then, $B\subseteq i_\sigma c_b{}^*(B) \subseteq i_\sigma c_\sigma{}^*(B) \subseteq i_\sigma c_\sigma(B)$. Hence B is $\pi(\sigma)$ -open.

Theorem 2.8. In H-GT S (Z,ϑ,H), any Ψ_α-$H_bO(Z)$ is Ψ_σ-$H_bO(Z)$.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_α-$H_bO(Z)$. Then, $B\subseteq i_\sigma c_b{}^* i_\sigma(B)\subseteq c_b{}^* i_\sigma(B)$. Hence Ψ_σ - $H_bO(Z)$.

Theorem 2.9. In H-GTS (Z,ϑ,H), any Ψ_α - $H_bO(Z)$ is Ψ_π - $H_bO(Z)$.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_α-$H_bO(Z)$. Then, $B\subseteq i_\sigma c_b{}^* i_\sigma(B)\subseteq i_\sigma c^*(B)$. Hence Ψ_π-$H_bO(Z)$.

Theorem 2.10. A subset B of a H-GT S (Z,ϑ,H), the following results are equivalent.

1. Ψ_α-$H_bO(Z)$
2. Ψ_σ- $H_bO(Z)$ and Ψ_π-$H_bO(Z)$.

Proof. (1)⇒(2). Consider B is Ψ_α-$H_bO(Z)$. Then by theorem 2.8 and 2.9, Ψ_σ-$H_bO(Z)$ and Ψ_π-$H_bO(Z)$.
(2) ⇒ (1). Consider B is both Ψ_σ-$H_bO(Z)$ and Ψ_π-$H_bO(Z)$. Then $B\subseteq i_\sigma c_b{}^*(B) \subseteq i_\sigma c_b{}^* c_b{}^* i_\sigma(B) \subseteq i_\sigma c_b{}^* i_\sigma(B)$. Hence Ψ_α-$H_bO(Z)$.

Example 2.11. Consider $Z=\{z_1,z_2,z_3,z_4,z_5\}$, $\vartheta=\{\emptyset, \{z_1\}, \{z_2\}, \{z_3\}, \{z_1,z_2\}, \{z_1,z_3\}, \{z_2,z_3\}, \{z_1,z_2,z_3\}, \{z_1,z_3,z_4\}, \{z_1, z_2,z_3\}, \{z_1,z_2,z_3,z_4\}\}$, $H=\{\emptyset, \{z_1\}\}$. Then $B = \{z_1\}, \{z_5\}\}$ is Ψ_σ-$H_bO(Z)$ but not Ψ_α-$H_bO(Z)$.

Example 2.12. Consider $Z=\{z_1,z_2,z_3,z_4\}$, $\vartheta=\{\emptyset, \{z_1,z_3\}, \{z_4\}, \{z_1,z_3,z_4\}, Z\}$ $H=\{\emptyset, \{z_3\}\}$. Then $B=\{z_1,z_2,z_4\}$ is Ψ_π-$H_bO(Z)$ but not Ψ_α-$H_bO(Z)$.

Remark 2.13. The notions of Ψ_σ - $H_bO(Z)$ and Ψ_π-$H_bO(Z)$ are independent.

Example 2.14. Consider $Z = \{z_1,z_2,z_3,z_4,z_5\}$, $\vartheta = \{\emptyset, \{z_1\}, \{z_2\}, \{z_3\}, \{z_1,z_2\}, \{z_1,z_3\}, \{z_2,z_3\}, \{z_1,z_2,z_3\}, \{z_1,z_3,z_4\}, \{z_2,z_3,z_4\}, \{z_1,z_2,z_3,z_4\}\}$, $H= \{\emptyset, \{z_1\}\}$. Then $B = \{z_1,z_3\}$ is Ψ_σ-$H_bO(Z)$ but not Ψ_π-$H_bO(Z)$.

Example 2.15. Consider $Z = \{z_1,z_2,z_3,z_4\}$ $\vartheta = \{\emptyset, \{z_1,z_3\}, \{z_4\}, \{z_1,z_3,z_4\}, Z\}, H = \{\emptyset, \{z_3\}\}$. Then $B = \{z_1,z_3,z_4\}$ is Ψ_π-$H_bO(Z)$ but not Ψ_σ-$H_bO(Z)$.

Theorem 2.16. Any Ψ_π-$H_bO(Z)$ set is Ψ_β-$H_bO(Z)$ set.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_π-$H_bO(Z)$. Then $B\subseteq i_\sigma c_b{}^*(B)\subseteq c_\vartheta i_\sigma c_b{}^*(B)$. Hence B is Ψ_β-$H_bO(Z)$.

Theorem 2.17. Any Ψ_α-$H_bO(Z)$ set is Ψ_β-$H_bO(Z)$ set.

Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_α-$H_bO(Z)$. Then $B \subseteq i_\sigma c_b{}^* i_\sigma(B) \subseteq i_\sigma c^*(B) \subseteq c_\vartheta i_\sigma c_b{}^*(B)$. Hence B is Ψ_β-$H_bO(Z)$.

Theorem 2.18. Any Ψ_σ-$H_bO(Z)$ set is Ψ_β-$H_bO(Z)$ set.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_σ-$H_bO(Z)$. Then $B \subseteq c_b{}^* i_\sigma(B) \subseteq c_\vartheta{}^* i_\sigma(B) \subseteq c_\vartheta i_\sigma(B) \subseteq c_\vartheta i_\sigma c_b{}^*(B)$. Hence B is Ψ_β-$H_bO(Z)$.

Theorem 2.19. Any Ψ_σ - $H_bO(Z)$ set is Ψ_b - $H_bO(Z)$ set.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_σ-$H_bO(Z)$. Then $B \subseteq c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \cup i_\sigma c_b{}^*(B)$. Hence B is Ψ_b-$H_bO(Z)$.

Theorem 2.20. Any Ψ_π-$H_bO(Z)$ set is Ψ_b-$H_bO(Z)$ set.
Proof. Consider a subset B of H-GTS (Z, ϑ, H) is Ψ_π-$H_bO(Z)$. Then $B \subseteq i_\sigma c_b{}^*(B) \subseteq c_b{}^* i_\sigma(B) \cup i_\sigma c_b{}^*(B)$. Hence B is Ψ_b-$H_bO(Z)$.

Theorem 2.21. If $B \subset Z$ is both Ψ_b-$H_bO(Z)$ and ϑ-σ-open, then it is Ψ_β-$H_bO(Z)$.
Proof. Let B is both Ψ_b-$H_bO(Z)$ and ϑ-σ-open. Now $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B) \subseteq c_b{}^*(B)$ and $c_\vartheta i_\vartheta(B) \subseteq c_\vartheta i_\sigma(B) \subseteq c_\vartheta i_\sigma c_b{}^*(B)$. Hence B is Ψ_β-$H_bO(Z)$.

Theorem 2.22. If $B \subset Z$ is both Ψ_b-$H_bO(Z)$ and ϑ-σ-open, then it is ϑ-β-open.
Proof. Let B is both Ψ_b-$H_bO(Z)$ and ϑ-σ-open. Then $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$ and $B \subseteq c_\vartheta i_\vartheta(B)$. Now $B \subseteq i_\vartheta c_b{}^*(B) \cup c_b{}^* i_\vartheta(B) \subseteq c_b{}^*(B)$, which implies $c_\vartheta i_\vartheta(B) \subseteq c_\vartheta i_\vartheta c_b{}^*(B) \subseteq c_\vartheta i_\vartheta c_b{}^*(B) \subseteq c_\vartheta i_\vartheta c_\vartheta(B)$ So $B \subseteq c_\vartheta i_\vartheta(B) \subseteq c_\vartheta i_\vartheta c_\vartheta(B)$. Hence B is ϑ - β -open.

Theorem 2.23. If $B \subset Z$ is both Ψ_b-$H_bO(Z)$ and ϑ^*-closed, then it is Ψ_σ-$H_bO(Z)$.
Proof. Let B is both Ψ_b-$H_bO(Z)$ and ϑ^*-closed. Then $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$ and $c^*(B) \subseteq B$. Now $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \cup i_\sigma(B) = c_b{}^* i_\sigma(B)$. Hence B is Ψ_σ-$H_bO(Z)$.

Theorem 2.24. If $B \subset Z$ is both Ψ_b-$H_bO(Z)$ and $\vartheta*$-closed, then it is σ(σ)-open.
Proof. Let B is both Ψ_b-$H_bO(Z)$ and ϑ^*-closed. Then $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$ and $c_b{}^*(B) \subseteq B$. Now $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \cup i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \subseteq c_\sigma i_\sigma(B)$. Hence B is σ(σ)-open.

Theorem 2.25. If $B \subset Z$ is both Ψ_b-$H_bO(Z)$ and $b\vartheta^*$-closed, then it is Ψ_σ-$H_bO(Z)$.
Proof. Let B is both Ψ_b-$H_bO(Z)$ and $b\vartheta^*$-closed. Then $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$ and $c_b{}^*(B) \subseteq B$. Now $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \cup i_\sigma(B) = c_b{}^* i_\sigma(B)$. Hence B is Ψ_σ-$H_bO(Z)$.

Theorem 2.26. If $B \subset Z$ is both Ψ_b-$H_bO(Z)$ and $b\vartheta^*$-closed, then it is σ(σ)-open.
Proof. Let B is both Ψ_b-$H_bO(Z)$ and $b\vartheta^*$-closed. Then $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B)$ and $c_b{}^*(B) \subseteq B$. Now $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \cup i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \subseteq c_b{}^* i_\sigma(B) \subseteq c_\sigma i_\sigma(B)$. Hence B is σ(σ)-open.

Theorem 2.27. If $B \subset Z$ is Ψ_b-$H_bO(Z)$ such that $i_\sigma(B) = \emptyset$, then it is Ψ_π-$H_bO(Z)$.
Proof. Let B be a Ψ_b-$H_bO(Z)$ and $i_\sigma(B) = \emptyset$. Then $B \subseteq i_\sigma c_b{}^*(B) \cup c_b{}^* i_\sigma(B) = i_\sigma c_b{}^*(B)$. Hence B is Ψ_π-$H_bO(Z)$.

Theorem 2.28. If $B \subset Z$ is both Ψ_π-$H_bO(Z)$ and ϑ^*-closed, then it is σ(σ) - open.
Proof. Let B is both Ψ_π-$H_bO(Z)$ and ϑ^*-closed. Then $B \subseteq i_\sigma c_b{}^*(B)$ and $c_b{}^*(B) \subseteq B$. Now $B \subseteq i_\sigma c_b{}^*(B) \subseteq i_\sigma(B)$. Hence B is ϑ-σ-open.

Theorem 2.29. If $B \subset Z$ is both Ψ_π-$H_bO(Z)$ and $b\vartheta^*$-closed, then it is σ(σ) - open.
Proof. Let B is both Ψ_π-$H_bO(Z)$ and $b\vartheta^*$-closed. Then $B \subseteq i_\sigma c_b{}^*(B)$ and $c_b{}^*(B) \subseteq B$. Now $B \subseteq i_\sigma c_b{}^*(B) \subseteq i_\sigma(B)$. Hence B is ϑ-σ-open.

3. Decomposition of $(\Psi_\alpha H_b, \delta)$-Continuity

Definition 3.1. A map $j:(Z, \vartheta, H) \to (W, \delta)$ is $(\Psi_\alpha H_b, \delta)$ -continuous ($(\Psi_\alpha H_b, \vartheta)$-C), if $j^{-1}(V)$ is Ψ_α-$H_bO(Z)$ for each $\delta O(Z)$ set V in (W, δ).

Definition 3.2. A map $j: (Z, \vartheta, H) \rightarrow (W, \delta)$ is $(\Psi_\sigma H_b, \delta)$ -continuous ($(\Psi_\sigma H_b, \vartheta)$ -C), if $j^{-1}(V)$ is Ψ_σ-$H_bO(Z)$ for each $\delta O(Z)$ set V in (W, δ).

Definition 3.3. A map $j: (Z, \vartheta, H) \rightarrow (W, \delta)$ is $(\Psi_\pi H_b, \delta)$-continuous ($(\Psi_\pi H_b, \vartheta)$ -C), if $j^{-1}(V)$ is Ψ_π-$H_bO(Z)$ for each $\delta O(Z)$ set V in (W, δ).

Theorem 3.4. For a map $j : (Z, \vartheta, H) \rightarrow (W, \delta$, the following results are equivalent.

1. j is $(\Psi_\alpha H_b, \vartheta)$ - C.
2. j is $(\Psi_\sigma H_b, \vartheta)$ - C and $(\Psi_\pi H_b, \vartheta)$ - C.

Proof. Proof is trivial from Theorem 2.14.

4. Conclusion

In this paper, we introduced Ψ_α - $H_bO(Z)$, Ψ_σ - $H_bO(Z)$, Ψ_π - $H_bO(Z)$, Ψ_β - $H_bO(Z)$, and Ψ_b - $H_bO(Z)$ sets and obtained decomposition of $(\Psi_\alpha H_b, \vartheta)$ - C. In future work we will introduce new types of generalized open sets related to these sets and obtain new decomposition of (ϑ, δ) - C.

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