

The study of Vorticity for Two-Layered Magnetohydrodynamic Flow Through Parallel Plates

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Article History:

Received:14-01-2025

Revised: 15-03-2025

Accepted:21-03-2025

Abstract

This paper investigates the vorticity characteristics within a two-layered Magnetohydrodynamic (MHD) model applied to a parallel plate hemodialyzer subjected to a uniform transverse magnetic field. Analytical expressions for the vorticity are derived for both the core fluid region and the peripheral plasma layer. These expressions are subsequently analyzed to obtain vorticity profiles as a function of the Hartmann number (M), with the thickness of the peripheral plasma layer (δ) held constant. The study reveals that, as the Hartmann number increases, the flow of the suspension exhibits a tendency to reduce the cell-free zone near the wall, while the plasma exposure area to the membrane increases. This results in a complex vorticity behavior where effective vorticity initially decreases and then increases with rising Hartmann number (M). The findings suggest that these MHD effects can be leveraged to optimize energy management and mitigate the risks of red blood cell injury, such as blood clotting, near the membrane wall during the dialysis process. This work provides a novel insight into the role of magnetic fields in enhancing the hemodialysis process through improved flow dynamics.

Keywords: Hemodialysis; Two Layered Model; Artificial Kidney; Magnetohydrodynamics; Hartmann Number.

1 Introduction

Fatal kidney malfunctions, though infrequent, predominantly affect individuals of a younger age group. The kidneys play a crucial role in eliminating metabolic by products and other undesirable substances from the bloodstream. When this function is compromised, it can lead to kidney failure. In instances where traditional medical interventions prove insufficient in addressing the malfunction, patients often require a series of artificial kidney treatments.

The artificial kidney, also known as a dialyzer, serves as a life support system engineered to extract waste products from the patient's body. Various types of dialyzers

exist, including flat plate, tubular, and helical designs. This paper primarily focuses on the fluid mechanics associated with flat plate dialyzers.

In the context of blood flow and magnetohydrodynamic (MHD) models, numerous researchers have explored the impact of magnetic fields on the dynamics of blood and other fluids.

It has been observed that the flow of blood tends to be two-layered particularly when the distance between the plates is very small [1–6]. Lightfoot described the flow of blood as a suspension of cells (red blood cells, white blood cells, and platelets, etc.) in plasma [7]. Additionally, it has been noted that red blood cell and clot accumulations in the dialyzer's membrane pores over time cause a reduction in the efficiency of the device. Red blood cells are furthermore damaged by membrane interaction. Studying a model that requires less time for dialysis and reduces red cell injury/harm is therefore interesting.

Woodcock demonstrated that red blood cells, which make up around 50% of the total blood volume, have a negative blood charge [8]. Therefore, blood can be seen as a fluid that conducts electricity. Numerous scholars have investigated one-layered magnetohydrodynamic (MHD) models of blood flow [9, 10]. It is well known that magnetic fields can be utilized to regulate the motion of charged particles because of MHD investigations. Because of this, it might be able to design a magnetic field configuration so that red cells are controlled by forced away from the walls, like the pinch effect.

This could ultimately lengthen the time of dialysis by increasing the amount of plasma exposed to the dialysate and preventing red blood cells from obstructing membrane pores. The red cell injury is lessened because red cells are separated from the membrane. To improve the effectiveness of the dialyzer, Chaturani and Saxena created the two-layered MHD model under the assumption that blood is a Newtonian fluid [11]. Mittal et al. also contributed to this field by investigating vorticity in hydrodynamic and MHD flows in ducts of various shapes and sizes, thus providing insights into energy losses and flow dynamics in such systems [12-14].

Aruna Sharma and Dubewar [17] studied MHD flow between parallel plates with an inclined magnetic field, employing finite difference methods. Their work is crucial for understanding blood flow dynamics in dialysis devices. Similarly, Singh and Wanja [18] analyzed steady MHD flow between parallel plates with an inclined magnetic field, offering solutions that contribute to optimizing blood flow configurations in such devices.

Further, recent studies by Mburu, Kwanza, and Onyango [19], as well as Naceur et al. [20], explored the effects of inclined magnetic fields and pressure gradients on MHD blood flow between parallel plates. Their findings offer valuable insights into the optimization of magnetic field configurations and the stability of MHD flow, which are directly applicable to improving blood dialysis systems by enhancing plasma exposure and minimizing red blood cell damage.

Further research in MHD models has explored various configurations of blood flow, including the effects of an inclined magnetic field and pressure gradients on flow characteristics between parallel plates [22–24]. For instance, Aydin and Oztop examined the effects of an inclined magnetic field on MHD flow between parallel plates, focusing on the velocity profiles and stability of the flow [23]. In a similar vein, Sundararajan and Sivaraj studied the instability of Hartmann layers in MHD flows between parallel plates, offering valuable insights into the stability of the flow under magnetic influence [25].

Moreover, studies on the heat transfer characteristics in MHD two-layered fluid flows, such as those by Zhang and Zhou, expanded the understanding of thermal and flow behaviors in systems exposed to magnetic fields [26]. These findings provide important implications for improving the thermal efficiency and stability of MHD flows in biomedical applications like blood dialysis. Other studies have delved into the influence of viscosity and heat generation on MHD flow in parallel plate configurations, exploring how these factors impact the overall performance of MHD systems in various engineering applications [28, 29].

To reduce red cell damage close to the membrane wall by reducing energy losses during dialysis, the current study's goal is to examine the blood flow vorticity in a two layered parallel plate hemodialyzer. The numerical solution for the vorticity profile has been shown graphically for various values of the Hartmann number, M , and thickness of the PPL, δ , and the analytical equation for velocity and vorticity has been produced in the core and peripheral plasma layer.

2 Mathematical model and solution of the problem

In order to construct the mathematical model of the problem, blood is assumed to flow between two stationary immovable plates in the +ve x -direction while being affected by a uniform magnetic field B_0 in the +ve z -direction. The fluid properties of blood are considered to be stable, viscous, incompressible, and Newtonian. According to theories, the plates are extremely long and wide in comparison to how far apart they are. Therefore, the issue can be roughly described as a 1-D plane Poiseuille flow between two infinite parallel plates at $z = \pm h$.

Red cells are suspended in plasma in the blood, and when the cells travel out from the walls, a cell-free zone known as the peripheral plasma layer (PPL) forms close to the walls and a core region made up of cells and plasma. The flow in PPL is unaffected by the magnetic field since plasma is an electrically non-conducting fluid; however, the flow in the core region is impacted by the magnetic field because of the electric charge on red cells. The following equations provide the governing equations for a two-layered MHD flow of blood between parallel plates:

$$\frac{d^2 u_c}{dz^2} - \frac{\sigma B_0^2}{\eta_c} u_c = -\frac{P_0}{\eta_c}, \quad -(h - \delta) \leq z \leq (h - \delta) \quad (1)$$

$$\frac{d^2 u_p}{dz^2} = -\frac{P_0}{\eta_p}, \quad -h \leq z \leq -(h - \delta) \text{ and } -(h - \delta) \leq z \leq (h - \delta) \quad (2)$$

Where:

u_c/u_p : Velocity of the fluid in the core / peripheral plasma region,

σ : Electrical conductivity of blood,

B_0 : Applied constant uniform magnetic field,

η_c/η_p : Viscosity of the fluid in the core/ peripheral plasma region,

P_0 ($= -\frac{\partial p}{\partial x}$): Pressure gradient in the x-direction, and

δ = Thickness of peripheral plasma layer.

Subjected to the following boundary conditions:

$$u_c = u_p \quad \text{at} \quad z = \pm(h - \delta) \quad \text{and} \quad u_p = 0 \quad \text{at} \quad z = \pm h \quad (3)$$

The solution of equation (1) and (2), subjected to the boundary condition (3) is given by

$$u_c = \frac{P_0}{\sigma B_0^2} \left[1 - \frac{\cos M \bar{z}}{\cos M \left(1 - \frac{\delta}{h}\right)} \right] - \frac{P_0 h^2}{2 \eta_p} \left(\frac{\delta}{h} \right) \left(\frac{\delta}{h} - 2 \right) \frac{\cos M \bar{z}}{\cos M \left(1 - \frac{\delta}{h}\right)} \quad (4)$$

Where: $\bar{z} = \frac{z}{h}$ and $M = B_0 h \sqrt{\frac{\sigma}{\eta_c}}$

M: Non-dimensional number, called Hartmann number. The velocity in peripheral plasma layer is given by

$$u_p = \frac{P_0 h^2}{2 \eta_p} - (\bar{z}^2 - 1) \quad (5)$$

The vorticity of fluid flow in the core/ peripheral plasma region is given by

$$\zeta_c = -\frac{P_0 h}{\eta_c M} \left(\frac{\sinh M \bar{z}}{\cosh M \left(1 - \frac{\delta}{h}\right)} \right) - \frac{P_0 h}{2 \eta_p} \left(\frac{\delta}{h} \right) \left(\frac{\delta}{h} - 2 \right) \frac{M \sinh M \bar{z}}{\cosh M \left(1 - \frac{\delta}{h}\right)} \quad (6)$$

$$\zeta_p = -\left(\frac{P_0 h}{\eta_c} \right) \bar{z} \quad (7)$$

Where $\frac{\zeta_c}{\zeta_p}$ Vorticity of the fluid in the core/ peripheral plasma region.

3 Numerical Results and Discussion

Numerical investigations were conducted to assess the quantitative effects of both the peripheral plasma layer and the Hartmann number (M) on the vorticity within the flow. These numerical studies were based on the analytical findings derived from equations (6 and 7). The following parameters were employed for the quantitative analysis: $h = 5 \times 10^{-4} m$, $\sigma = 1.4 mho/m$, $\eta_p = 1.2 \times 10^{-3} kg./m-sec$, $P_0 = 5 \times 10^4 N/m^3$. The values of η_c are listed in Table 1.

Table 1: Viscosity of blood in the core region for different PPL thickness

$\frac{\delta}{h}$	$\eta c (\times 10^{-3} kg / m - sec)$	$\frac{\delta}{h}$	$\eta c (\times 10^{-3} kg / m - sec)$
0.02	5.25	0.10	5.82
0.04	5.37	0.12	5.99
0.06	5.51	0.14	6.18
0.08	5.66	0.16	6.39

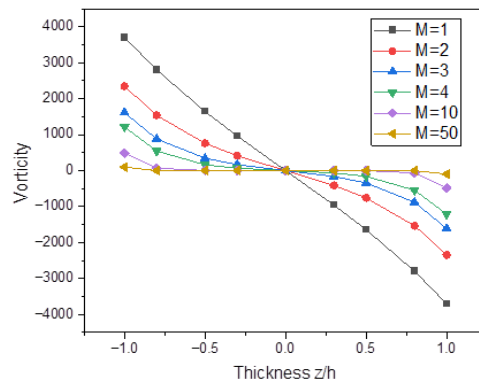


Fig. 1: Vorticity profile of one-layered MHD model for different Values of M and fixed values of $\frac{\delta}{h} = 0$

In order to gain a better physical understanding of the issue at hand, we conducted analytical studies, keeping the thickness, δ of the peripheral plasma layer constant while varying the Hartmann number, M. The vorticity profiles of one-layered and two-layered magnetohydrodynamic (MHD) model flows were plotted in Figures 1 and 2 across different values of the Hartmann number (M).

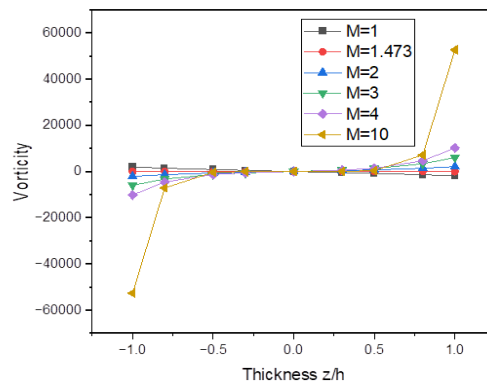


Fig. 2: Vorticity profile of one-layered MHD model for different Values of M and fixed values of $\frac{\delta}{h} = 0.10$

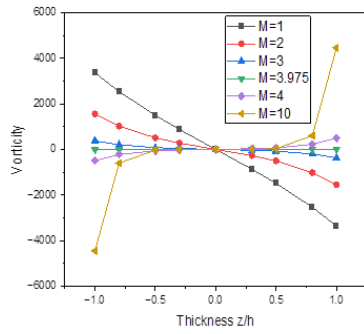
Notably, it's intriguing to observe that the vorticity profile for the two-layered MHD flow models in Figure 2 exhibits significant differences compared to the one-layered MHD flow models shown in Figure 1. The vorticity profile for the one-layered MHD flow model can be derived as a special case of the two-layered MHD flow model by substituting ($\delta = 0$) into equation (6).

To gain a better grasp of the issue at hand, we've conducted analytical computations for both one-layered and two-layered magnetohydrodynamic (MHD) model flows. These calculations are showcased in Figures 1 and 2, demonstrating how vorticity varies with different values of the Hartmann number (M) while keeping the thickness of the outer plasma layer constant.

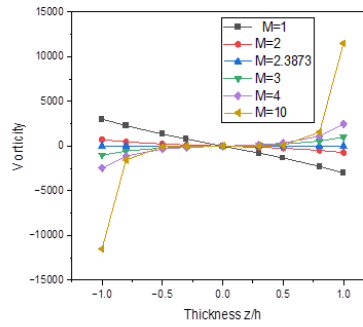
Notably, what piques our interest is the stark contrast in vorticity profiles between the two-layered MHD flow models depicted in Figure 2 and the one-layered MHD flow models presented in Figure 1. It's worth emphasizing that the vorticity profile for the one-layered MHD flow model can be derived as a specific instance of the two-layered MHD flow model by making certain substitutions, such as setting the outer plasma layer thickness (δ) to zero, as an illustrative example.

At magnetic fields of M greater than and equal to 50, the blood flow becomes nearly irrotational, leading to minimal energy losses within the bloodstream. However, this condition also raises the probability of red blood cells encountering the membrane walls, which, in turn, can decelerate the dialysis process, as demonstrated in Figure 1.

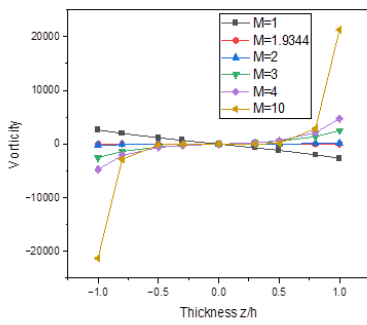
In the two-layered MHD model shown in Figure 2, as peripheral plasma layer (PPL) thickness is induced ($\delta/h = 0.10$), along with the application of a magnetic field, induces a complex behavior in the vorticity of the flow. Initially, the vorticity decreases and then subsequently increases. Consequently, a critical value of the Hartmann number (M) emerges, denoted as M_{CR} , which is a novel characteristic of this model (such an occurrence was not observed in the one-layered model). Notably, in the two-layered model, it has been observed that M_{CR} is dependent on the thickness of the PPL. Specifically, an increase in PPL thickness leads to a reduction in M_{CR} , as indicated in Table 2.



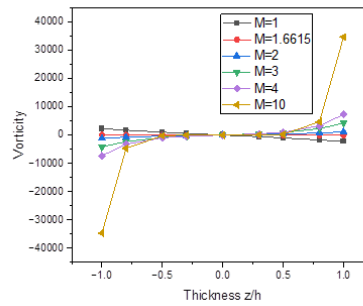
(a) $\frac{\delta}{h} = 0.02$



(b) $\frac{\delta}{h} = 0.04$



(c) $\frac{\delta}{h} = 0.06$



(d) $\frac{\delta}{h} = 0.08$

Fig. 3: Vorticity profile of two-layered MHD model showing variation of critical values of, M with PPL thickness of $\frac{\delta}{h}$

Furthermore, it is worth emphasizing that as the thickness of the peripheral plasma layer (PPL) increases (as shown in Figure 3), the region characterized by no vorticity or irrotationality diminishes, especially in the case of lower magnetic fields. Consequently, when the blood flow becomes irrotational, it leads to the elimination of energy losses. However, concurrently, this also raises the likelihood of blood coming into contact with a wall or clotting near a membrane wall, which can be adjusted according to needs.

Table 2: Critical values of Hartmann number for different PPL thickness

$\frac{\delta}{h}$	M_{CR}
0.02	3.975
0.04	2.3873
0.06	1.9344
0.08	1.6615
0.10	1.473

The current model suggests that due to the vortex motion induced by alterations in vorticity, the likelihood of red blood cells interacting with the membrane wall is diminished. It has been observed that as the thickness of the membrane increases, vorticity decreases but remains adequate to prevent contact between red blood cells and the membrane wall. This reduction in contact leads to decreased clotting and enhances the efficiency of the dialysis process.

4 Conclusion

This study investigates a two-layered Magnetohydrodynamic (MHD) model applied to a parallel plate hemodialyzer under the influence of a uniform transverse magnetic field. The focus is on analyzing the vorticity dynamics within the system, which plays a critical role in understanding the fluid behavior and the effectiveness of the hemodialysis process. The present analysis demonstrates that the operational efficiency of a parallel plate hemodialyzer can be optimized by controlling the magnetic field strength and maintaining a constant thickness of the peripheral plasma layer. This study provides valuable insights into how MHD effects can be utilized to enhance the performance of hemodialyzers by improving flow dynamics and enhancing mass transfer rates.

It is noteworthy that the two-layered magnetohydrodynamic (MHD) flow model offers an advantage over the conventional single-layered hemodialyzer by exhibiting variations in vorticity, wherein it decreases and then increases, particularly around the critical value of the applied magnetic field, M_{CR} . This effect is especially pronounced in regions where the thickness of the peripheral plasma layer (PPL) induces a zone characterized by no vorticity, and this occurs at lower magnetic field strengths, corresponding to lower critical values of the Hartmann number, M_{CR} .

Therefore, this characteristic of the current model can be employed to operate in a mode with minimal energy losses (when vorticity reaches zero) by applying a lower magnetic field. Simultaneously, it can be used to prevent the contact of blood particles with the membrane wall (when vorticity is not zero), thereby increasing the efficiency and speed of the dialysis process. Consequently, to enhance the efficiency of the hemodialyzer within clinical settings, one can leverage the impact of peripheral plasma layer thickness (δ) and the Hartmann number (M) on fluid dynamic parameters of physiological significance as well as vorticity.

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