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Edge binary coding of Cayley network

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Abstract:
Let $\mathcal{G}(V, L)$ = Cay(Γ, Ω) is an algebraic network model of the Cayley graph. A binary coding edge function $\mathcal{F}$: $L(\mathcal{G}) \to \{0,1\}$, it induces $V(\mathcal{G})$ as $\mathcal{F}(v) = \Sigma_{uv\epsilon L(\mathcal{G})} \mathcal{F}(uv)(mod\ 2)$. The function $\mathcal{F}$ is said to be an edge cordial function of $\mathcal{G}$, if the difference between the number of vertices (edges) labeled by zero and the number of vertices (edges) labeled by one is at most one. In this paper, we show the edge binary coding of the Cayley graph network model which satisfies the edge cordial constraint.

Keywords and Phrases: Cayley network, Binary coding, edge cordiality, labeling event, congruence classes.
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1. Introduction and Notations

In mathematical modelling, algebraic graphs are the evergreen trending solution domain for many practical problems. It describes the concept and make clarity on a concrete solution to the lot of abstract problems through graphically, that's the reason new graphical techniques and terminologies are emerging in inter and under disciplined with the algebraic base. Labelling is one of such encoding technique, contributed by Alex Rosa[4]in 1967. Nowadays different types of labelling techniques are utilized in various fields of sciences such as coding theory, X-ray diffraction, crystallography, missile guidance, astronomy, circuit designing, communication network addressing[3] etc. Various labelling terminologies are proven on the Cayley graphs however we mainly focus the classical binary encoding technique, which is easy to decode than the other. We may say, binary encoding is a labelling where as all the labelling may not be a binary coding because every labelling has its own well defined terminology. When a binary encoding inherits the (vertex or edge or some) cordial labelling terminology is known as a cordial labelling, therefore cordial functions are the massive operator in logical and decision making algorithms. This NP-complete problems can be executed in polynomial time. "Cordial labelling may be considered as a weakened version of Harmonious and Graceful labelling" is said by I. Cahit[1] and introduced the concept of cordial labelling in 1987 with the necessary and sufficient condition for that and R. Yilmaz and I. Cahit[7] investigated the edge cordiality of some special featured graphs such as complete bipartite, wheel, cycles etc., in 1997. Any graph of order is congruent to 2(mod 4) will not admit binary edge labelling and suppose a graph satisfies the binary coding under any cordial constraints, from that expected optimal solution derived by decoding.

Given network model is the Cayley graph, $\mathcal{G}$ = Cay(Γ, Ω) whose vertices are the elements of group Γ and the adjacency relation corresponds to the operation on every element of Γ with generators such that $L(\mathcal{G}) = \{(\gamma, \gamma\omega): \gamma\epsilon\ \Gamma,\ \omega\epsilon\ \Omega\}$. Identity free set Ω which is closed under inverse, leads to the loop free and symmetric structure of the graph. Its vertex and edge transitivity feature will facilitate the lot of applications in the network models with fault handling sensor, in human resource development as a sociometry diagrams and fixation of radio frequencies etc. Let $\mathcal{F}$ be a binary function is defined from the edge set or the vertex set of a graph $\mathcal{G}$ to a set $\{0, 1\}$ said to be a cordial function if $\mathcal{F}$ satisfies any one of cordial constrain. Suppose $\mathcal{F}$ is said to be an edge cordial function, it is defined from the edge set of a graph to a cordial set $\{0, 1\}$ and it will satisfy the difference between the number of edges labelled by zero and number of edges labelled by one is at most one, at the same time it will induce the vertex labels (sum of incident edge labels) and will meet the similar difference as we discussed for edges of $\mathcal{G}$. It is denoted $|v_{\mathcal{F}}(0) - v_{\mathcal{F}}(1)| \leq 1$ and $|e_{\mathcal{F}}(0) - e_{\mathcal{F}}(1)| \leq 1$. In this paper, highly symmetric network model Cayley graph is binary coded through the edge cordial constraint, is said to be an edge cordiality of the given network. The notations used in the manuscript are given below.

$\mathcal{G}$	Cayley graph	$O(a)$	order of an element a
$V(\mathcal{G})$	vertex set of $\mathcal{G}$	η	n^{th} non self inverse element
$L(\mathcal{G})$	edge set of $\mathcal{G}$	δ	n^{th} self inverse element
$\mathcal{F}$	cordial function	ϕ	number of cycles produced by $\mathcal{N}$
Γ	finite group	ℓ	length of the cycle
Ω	generating subset of Γ	ρ	order of a finite group
ξ	a matching	$\mathcal{N}$	non-self inverse element of Γ
ζ	a cycle	$\mathcal{S}$	self inverse element of Γ
μ, i, j, z	variables	θ	cycle producing generators
v and e	vertex and edge	α	matching producing generators

2. Edge binary coding on Cayley graph

For convenience, the vertex set $V(\mathcal{G})$ contains the elements of a finite group Γ and the edge set $L(\mathcal{G})$ can be split into the sets $\mathcal{N}$ and S whose edges are generated by the non-self inverse elements and self inverse elements of Γ respectively. Suppose, the set of $L(\mathcal{G}) = \{\mathcal{N}1, \mathcal{N}2,...,\mathcal{N}\eta, S_1, S_2,... , S_\delta\}$. Assume that $\Omega = \{\theta_1, \theta_2, \ldots, \theta_{2\eta}, \alpha_1, \alpha_2, \ldots, \alpha_\delta\}$, where $\theta_i, 1 \leq i \leq 2\eta$ be the generators which produces a cycles and $\alpha_j, 1 \leq j \leq \delta$ be the generator which produces matchings in $\mathcal{G}$. Let φ be the number of cycles produced by a generator of the set $\mathcal{N}$ which is a fraction of the group order (ρ) and the order of corresponding generator (ℓ), where ℓ is known as a length of that cycle. As per the requirement, algebraic equations and congruence classes are defined and executed in every labeling event. Let us consider the sets C_1, C_2, C_3 and C_4 consisting the elements of congruence classes of $[1]_4 \cup [3]_4$, $[0]_4 \cup [2]_4$, $[1]_4 \cup [2]_4$ and $[0]_4 \cup [3]_4$ respectively. Here, $[a]_n$ is the notation for the congruence classes of a modulo n such that for a fixed non zero integer $n, [a]_n = \{z \in \mathbf{Z} / z \equiv a (mod\, n)\}$.

Proposition 2.1. For a group Γ of order $\rho = [0]_4$ and $\Omega \subseteq \Gamma$, where Ω is free from non-self inverse elements of Γ. Then the Cayley graph $\mathcal{G}$ admits edge cordiality.

Proof. If $\Omega = \{\alpha_1, \alpha_2, \ldots, \alpha_\delta\}$. Then the $L(\mathcal{G}) = S_1 \cup S_2 \cup \cdots \cup S_\delta$ and without loss of generality, $\alpha_1 \in \Omega, O(\alpha_1) = 2$ which generates the $\frac{\rho}{2}$ matchings in S_1. i.e, $S_1 = S_{\xi_1}1 \cup S_{\xi_2} \cup \ldots \cup S_{\xi_{\frac{\rho}{2}}}$ where $S_{\xi_i} = v_i \,.\, \alpha_1$ with each matching of length exactly 2.

Event 1. If $|\Omega|$ is odd, then δ must be odd. Executing a cordial function $\mathcal{F}$ from $L(\mathcal{G})$ to the set $\{0,1\}$ such that

$$\mathcal{F}(e)=\begin{cases}0 & \text{if } e\in\mathcal{S}_1 \ \&\ e\in\mathcal{S}_{\xi_\mu},\ 1\le\mu\le\frac{\rho}{2},\ \mu \text{ odd},\\ 1 & \text{if } e\in\mathcal{S}_1 \ \&\ e\in\mathcal{S}_{\xi_\mu},\ 1\le\mu\le\frac{\rho}{2},\ \mu \text{ even},\\ [j]_2 & \text{if } e\in\mathcal{S}_j,\ 2\le j\le\delta.\end{cases}$$

The above labelled assignment persuading that, the sum of the labels of incident edges on every vertex such that for all $1 \le i \le \rho$,

$$\mathcal{F}(v_i)=\sum_{e\in L(v)}\mathcal{F}(e)=\begin{cases}0 & \text{if } \rho \text{ is odd},\\ 1 & \text{if } \rho \text{ is even}.\end{cases}$$

Thus, $v_{\mathcal{F}}(0)=v_{\mathcal{F}}(1)=\frac{\rho}{2}$ and $e_{\mathcal{F}}(0)=e_{\mathcal{F}}(1)=\frac{\rho|\Omega|}{4}$.

Event 2. If $|\Omega|$ is even, then δ must be even. Executing a cordial function $\mathcal{F}$ on the edge set of $(\mathcal{G})$ such that,

$$\mathcal{F}(e)=\begin{cases}0 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ odd} \ \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_1,\\ 1 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ odd} \ \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_2,\\ 0 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ even} \ \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_1,\\ 1 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ even} \ \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_2.\end{cases}$$

The above labeled assignment persuading that, the sum of the labels of incident edges on every vertex for all $1 \le i \le \rho$,

$$\mathcal{F}(v_i)=\sum_{e\in L(v)}f(e)=\begin{cases}0 & \text{if } 1\le i\le\frac{\rho}{2},\\ 1 & \text{if } \frac{\rho}{2}+1\le i\le\rho.\end{cases}$$

Thus, $v_{\mathcal{F}}(0)=v_{\mathcal{F}}(1)=\frac{\rho}{2}$ and $e_{\mathcal{F}}(0)=e_{\mathcal{F}}(1)=\frac{\rho|\Omega|}{4}$.

Theorem 2.2. Let $\Omega \subseteq \Gamma$ be a generating set, of a group Γ whose order is $\rho = [0]_4$. If $|\Omega|$ is odd and Ω contains an element of order at least $4\ell, \ell \neq 0$. Then the Cayley graph G admits edge binary coding.

Proof. Suppose, $\Omega = \{\theta_1,\theta_2,\ldots,\theta_{2\eta},\alpha_1, \alpha_2,\ldots,\alpha_\delta\}$. Note that $|\Omega|$ is odd which implies δ must be odd. Let the edge set $L(\mathcal{G})$ has a partition $\{\mathcal{N}1, \mathcal{N}2,\ldots,\mathcal{N}\eta, S_1, S_2,\ldots, S_\delta\}$. Arbitrarily, we assume that Ω is arranged so that θ_η is an element, whose order is at least 4ℓ, $\ell \neq 0$. If $\mathcal{N}\eta$ contains

φ cycles and the edges of φ cycles are generated by η, can be represented by a set $\mathcal{N}_\eta = \mathcal{N}_{\zeta_1} \cup \mathcal{N}_{\zeta_2} \cup \ldots \cup \mathcal{N}_{\zeta_\varphi}$ with each cycle of length ℓ.

Event 1. If $\eta \geq 1$, η is even and δ is odd. Executing a cordial function $\mathcal{F}$ from $L(\mathcal{G})$ to the set $\{0, 1\}$ such that,

$$\mathcal{F}(e) = \begin{cases} [i]_2 & \text{if } e \in \mathcal{N}_i,\ 1 \leq i \leq \eta, \\ 0 & \text{if } e \in \mathcal{S}_1 \text{ and } e \in \mathcal{S}_{\xi_\mu},\ 1 \leq \mu \leq \frac{\varrho}{2},\ \mu \text{ odd}, \\ 1 & \text{if } e \in \mathcal{S}_1 \text{ and } e \in \mathcal{S}_{\xi_\mu},\ 1 \leq \mu \leq \frac{\varrho}{2},\ \mu \text{ even}, \\ [j]_2 & \text{if } e \in \mathcal{S}_j,\ 2 \leq j \leq \delta. \end{cases}$$

Event 2. If $\eta \geq 1$, η is odd which implies δ is odd. Executing a be a cordial function $\mathcal{F}$ from $L(\mathcal{G})$ to the set $\{0, 1\}$ such that

$$\mathcal{F}(e) = \begin{cases} [i]_2 & \text{if } e \in \mathcal{N}_i,\ 1 \leq i \leq \eta - 1, \\ 0 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \leq z \leq \phi,\ 1 \leq j \leq \ell \ \&\ j \text{ odd}, \\ 1 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \leq z \leq \phi,\ 1 \leq j \leq \ell \ \&\ j \text{ even}, \\ 0 & \text{if } e \in \mathcal{S}_1 \ \&\ e \in \mathcal{S}_{\xi_\mu},\ 1 \leq \mu \leq \frac{\varrho}{2},\ \mu \text{ odd}, \\ 1 & \text{if } e \in \mathcal{S}_1 \ \&\ e \in \mathcal{S}_{\xi_\mu},\ 1 \leq \mu \leq \frac{\varrho}{2},\ \mu \text{ even}, \\ [j]_2 & \text{if } e \in \mathcal{S}_j,\ 2 \leq j \leq \delta. \end{cases}$$

The above two events will ensure that the persuaded vertex sum of $V(\mathcal{G})$ is, for all $1 \leq i \leq \rho$.

$$\mathcal{F}(v_i) = \sum_{e \in L(v)} \mathcal{F}(e) = \begin{cases} 0 & \text{if } \rho \text{ is odd}, \\ 1 & \text{if } \rho \text{ is even}. \end{cases}$$

As a result, $v_{\mathcal{F}}(0) = v_{\mathcal{F}}(1) = \frac{\varrho}{2}$ and $e_{\mathcal{F}}(0) = e_{\mathcal{F}}(1) = \frac{\rho|\Omega|}{4}$.

Event 3. Suppose $\eta = 0$ and odd δ. At this instance the generating subset Ω contains only the self inverse(order two) elements of Γ. In this case, by Proposition 2.1 the proof is immediate. Thus the network model($\mathcal{G}$) can be encoded with the binary labels.

Example 2.3. The following figure shows the E-cordiality of the Cayley graph $\mathcal{G}$ with respect to the group S_4 whose order is 24 ≡ 0 (mod 4). This network is binary encoded by applying the binary encoding function of Theorem 2.2. The dotted lines and non-darkened nodes indicating the code zero and the non-dotted lines and darkened nodes indicating code is one.

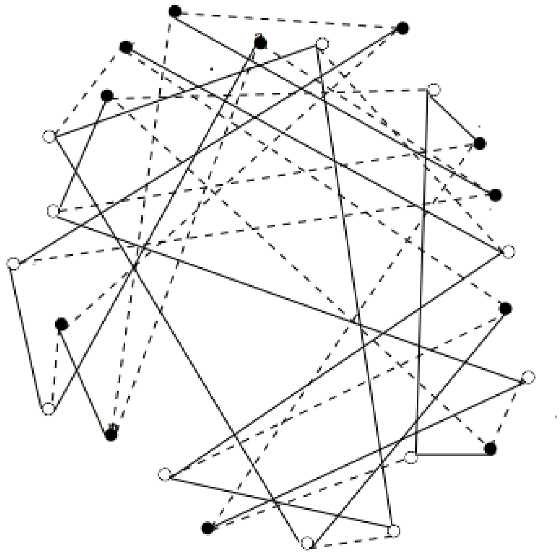

Figure 1: $\mathrm{Cay}(S_4, \Omega)$ where $\Omega = \{(24), (12)(34), (23)(14)\}$

Theorem 2.4. Let $\Omega \subseteq \Gamma$ be a generating subset of a group Γ whose order is $\rho = [0]_4$. If $|\Omega|$ is even and Ω contains an element of order at least $4\ell, \ell \neq 0$. Then the network model Cayley graph $\mathcal{G}$ is binary codable.

Proof. Suppose, $\Omega = \{\theta_1, \theta_2, \ldots, \theta_{2\eta}, \alpha_1, \alpha_2, \ldots, \alpha_\delta\}$ and $|\Omega|$ is even. Then δ must be even. We know that $L(\mathcal{G})$ has a partition such that $\{\mathcal{N}1, \mathcal{N}2, \ldots, \mathcal{N}\eta, S_1, S_2, \ldots, S_\delta\}$. With no loss of generality, we assume that $\theta_\eta \in \Omega$ is an element of order $4\ell, \ell \neq 1$. Let φ be the number of cycles produced by $\mathcal{N}_\eta$, the set of all edges of those cycles are generated by η where $\mathcal{N}_\eta = \mathcal{N}_{\theta_1} \cup \mathcal{N}_{\theta_2} \cup \cdots \cup \mathcal{N}_{\theta_\varphi}$ with each cycle of length 4ℓ.

Event 1. If $\eta \geq 1$ and $\delta = 0$.

Instance 1.1. For a odd η, executing a cordial function F from $L(\mathcal{G})$ to the set $\{0, 1\}$ such that

$$\mathcal{F}(e) = \begin{cases} [i]_2 & \text{if } e \in \mathcal{N}_i,\ 1 \le i \le \eta - 1, \\ 0 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ odd } \&\ j \in C_3, \\ 1 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ odd } \&\ j \in C_4, \\ 0 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ even } \&\ j \in C_3, \\ 1 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ even } \&\ j \in C_4. \end{cases}$$

Instance 1.2. For a even η, executing a cordial function $\mathcal{F}$ from $L(\mathcal{G})$ to the set $\{0, 1\}$ such that

$$\mathcal{F}(e)=\begin{cases}0 & \text{if } e=(\zeta_1\theta_\eta^j,\zeta_1\theta_\eta^{j+1})\in\mathcal{N}_1,\ 1\le j\le\rho\ \&\ j\in C_3,\\ 1 & \text{if } e=(\zeta_1\theta_\eta^j,\zeta_1\theta_\eta^{j+1})\in\mathcal{N}_1,\ 1\le j\le\rho\ \&\ j\in C_4,\\ [i]_2 & \text{if } e\in\mathcal{N}_i,\ 2\le i\le\eta-1,\\ 0 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ odd } \&\ j\in C_3,\\ 1 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ odd } \&\ j\in C_4,\\ 0 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ even } \&\ j\in C_3,\\ 1 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ even } \&\ j\in C_4.\end{cases}$$

In the above two instance of this event, sum of the labels of incident edges in every vertices as said to be a induced vertex sum of $V(\mathcal{G})$ for all $1 \le i \le \rho$,

$$\mathcal{F}(v_i)=\sum_{e\in L(v)}\mathcal{F}(e)=\begin{cases}0 & \text{if } \rho \text{ is odd},\\ 1 & \text{if } \rho \text{ is even}.\end{cases}$$

$$\text{Thus, } v_{\mathcal{F}}(0)=v_{\mathcal{F}}(1)=\tfrac{\varrho}{2} \text{ and } e_{\mathcal{F}}(0)=e_{\mathcal{F}}(1)=\tfrac{\rho|\Omega|}{4}.$$

Event 2. If $\eta \ge 1$ and $\delta \ne 0$.

Instance 2.1. For a odd η, executing a cordial function $\mathcal{F}$ from $L(\mathcal{G})$ to the set $\{0,1\}$ such that

$$\mathcal{F}(e)=\begin{cases}[i]_2 & \text{if } e\in\mathcal{N}_i,\ 1\le i\le\eta-1,\\ 0 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ odd } \&\ j\in C_3,\\ 1 & \text{if } e=(\zeta_z\theta_s^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ odd } \&\ j\in C_4,\\ 0 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ even } \&\ j\in C_3,\\ 1 & \text{if } e=(\zeta_z\theta_\eta^j,\zeta_z\theta_\eta^{j+1})\in\mathcal{N}_\eta,\ 1\le z\le\phi,\ 1\le j\le\ell,\ j \text{ even } \&\ j\in C_4,\\ 0 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ odd } \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_1,\\ 1 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ odd } \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_2,\\ 0 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ even } \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_1,\\ 1 & \text{if } e\in\mathcal{S}_j,\ 1\le j\le\delta,\ j \text{ even } \&\ e\in\mathcal{S}_{\xi_\mu},\ \mu\in C_2.\end{cases}$$

Instance 2.2. For a even η, executing a cordial function $\mathcal{F}$ from L($\mathcal{G}$) to the set $\{0, 1\}$ such that,

$$\mathcal{F}(e) = \begin{cases} 0 & \text{if } e = (\zeta_1\theta_\eta^j, \zeta_1\theta_\eta^{j+1}) \in \mathcal{N}_1,\ 1 \le j \le \rho\ \&\ j \in C_3\ , \\ 1 & \text{if } e = (\zeta_1\theta_\eta^j, \zeta_1\theta_\eta^{j+1}) \in \mathcal{N}_1,\ 1 \le j \le \rho\ \&\ j \in C_4\ , \\ [i]_2 & \text{if } e \in \mathcal{N}_i,\ 2 \le i \le \eta - 1, \\ 0 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ odd } \ \&\ j \in C_3, \\ 1 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ odd } \ \&\ j \in C_4, \\ 0 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ even } \&\ j \in C_3, \\ 1 & \text{if } e = (\zeta_z\theta_\eta^j, \zeta_z\theta_\eta^{j+1}) \in \mathcal{N}_\eta,\ 1 \le z \le \phi,\ 1 \le j \le \ell,\ j \text{ even and } j \in C_4 \\ 0 & \text{if } e \in \mathcal{S}_j,\ 1 \le j \le \delta,\ j \text{ odd } \ \&\ e \in \mathcal{S}_{\xi_\mu},\ \mu \in C_1, \\ 1 & \text{if } e \in \mathcal{S}_j,\ 1 \le j \le \delta,\ j \text{ odd } \ \&\ e \in \mathcal{S}_{\xi_\mu},\ \mu \in C_2, \\ 0 & \text{if } e \in \mathcal{S}_j,\ 1 \le j \le \delta,\ j \text{ even } \&\ e \in \mathcal{S}_{\xi_\mu},\ \mu \in C_1, \\ 1 & \text{if } e \in \mathcal{S}_j,\ 1 \le j \le \delta,\ j \text{ even } \&\ e \in \mathcal{S}_{\xi_\mu},\ \mu \in C_2. \end{cases}$$

At this event, by the above labelled assignments of both instance we will obtained the result, which will meet the edge cordiality such that, $v_{\mathcal{F}}(0) = v_{\mathcal{F}}(1) = \frac{\rho}{2}$ and $e_{\mathcal{F}}(0) = e_{\mathcal{F}}(1) = \frac{\rho|\Omega|}{4}$.

Event 3. Suppose $\eta = 0$ and even δ. At this instance the generating subset Ω contains only the self inverse(order two) elements of Γ. In this case by Proposition 2.1, the proof is immediate. Hence by the above three labeling events, it is clear that the Cayley graph($\mathcal{G}$) is binary coded.

Example 2.5. Consider the Cayley graph corresponding to the Dihedral group D_{16} which has more than one self inverse element as well as the non self inverse element. By applying the binary encoding labeling function on it through the Theorem 2.4. The dotted edges and non-darkened vertices indicates the holding weight is zero and the non-dotted lines and darkened nodes indicates the holding weight is one.

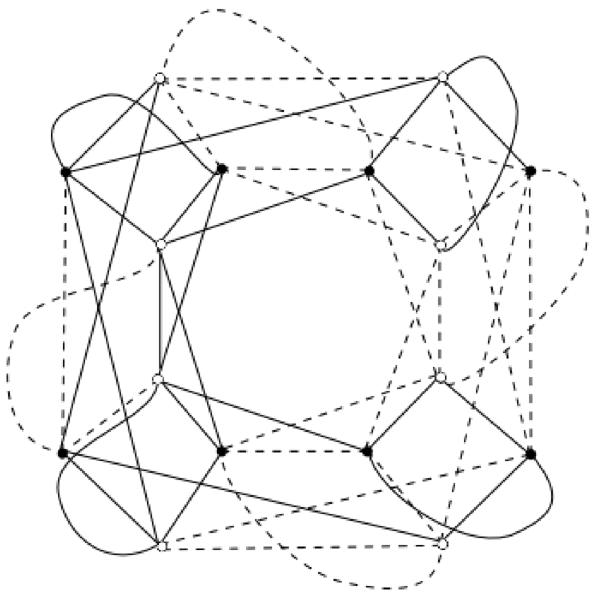

Figure 2: $\text{Cay}(D_{16}, \Omega)$ where $\Omega = \{r, r^2, s, rs\}$

Theorem 2.6. Let Ω be a generating subset of a group Γ of order ρ. Then the Cayley network $\mathcal{G}$ is binary codable, if $\rho \in [1]_4$ and $\rho \in [3]_4$.
Proof. Since $|\Omega|$ is even and $\Omega = \{\theta_1, \theta_2, \ldots, \theta_\eta, \theta_{\eta+1}, \theta_{\eta+2}, \ldots, \theta_{2\eta}\}, \theta_{i-1} = \theta_{i+\eta}, 1 \leq i \leq \eta$.
Event 1. If η is odd. Executing a cordial function $\mathcal{F}$ from $L(\mathcal{G})$ to the set $\{0, 1\}$ such that

$$\mathcal{F}(e) = \begin{cases} 0 & \text{if } e = \{\zeta_1\theta_\eta^j, \zeta_1\theta_\eta^{j+1}\} \in \mathcal{N}_1,\ 1 \leq j \leq \rho \ \&\ j \in C_3\ , \\ 1 & \text{if } e = \{\zeta_1\theta_\eta^j, \zeta_1\theta_\eta^{j+1}\} \in \mathcal{N}_1,\ 1 \leq j \leq \rho \ \&\ j \in C_4\ , \\ [i]_2 & \text{if } e \in \mathcal{N}_i,\ 2 \leq i \leq \eta - 1. \end{cases}$$

Event 2. If η is even. Executing a cordial function $\mathcal{F}$, from L($\mathcal{G}$) to the set {0, 1} such as

$$\mathcal{F}(e) = \begin{cases} 0 & \text{if } e = (\zeta_i\theta_\eta^j, \zeta_i\theta_\eta^{j+1}) \in \mathcal{N}_i, i \text{ is odd},\ 1 \leq j \leq \rho \ \&\ j \in C_3\ , \\ 1 & \text{if } e = (\zeta_i\theta_\eta^j, \zeta_i\theta_\eta^{j+1}) \in \mathcal{N}_i, i \text{ is odd},\ 1 \leq j \leq \rho \ \&\ j \in C_4\ , \\ 1 & \text{if } e = (\zeta_i\theta_\eta^j, \zeta_i\theta_\eta^{j+1}) \in \mathcal{N}_i, i \text{ is even},\ 1 \leq j \leq \rho \ \&\ j \in C_3\ , \\ 0 & \text{if } e = (\zeta_i\theta_\eta^j, \zeta_i\theta_\eta^{j+1}) \in \mathcal{N}_i, i \text{ is even},\ 1 \leq j \leq \rho \ \&\ j \in C_4. \end{cases}$$

By the above labeled events shows that for all $1 \leq i \leq \rho$,

$$\mathcal{F}(v_i) = \sum_{e \in L(v)} \mathcal{F}(e) = \begin{cases} 0 & \text{if } i \in C_3 \ \&\ \rho = [1]_4, \\ 1 & \text{if } i \in C_4 \ \&\ \rho = [1]_4, \\ 1 & \text{if } i \in C_3 \ \&\ \rho = [3]_4, \\ 0 & \text{if } i \in C_4 \ \&\ \rho = [3]_4. \end{cases}$$

Therefore $v_{\mathcal{F}}(0) = \lfloor \frac{\rho}{2} \rfloor + 1, v_{\mathcal{F}}(1) = \lfloor \frac{\rho}{2} \rfloor$ and $e_{\mathcal{F}}(0) = \lceil \frac{\rho|\Omega|}{4} \rceil + 1, e_{\mathcal{F}}(1) = \lceil \frac{\rho|\Omega|}{4} \rceil$ when $\rho = [1]_4$ and $v_{\mathcal{F}}(0) = \lfloor \frac{\rho}{2} \rfloor$, $v_{\mathcal{F}}(1) = \lfloor \frac{\rho}{2} \rfloor + 1$ and $e_{\mathcal{F}}(0) = \lceil \frac{\rho|\Omega|}{4} \rceil + 1$ and $e_{\mathcal{F}}(1) = \lceil \frac{\rho|\Omega|}{4} \rceil$ when $\rho = [3]_4$. □

3. **Conclusion**

Basically, Cayley graphs are the good network model, by implementing this edge binary encoding analogue on any practical problem, whose configuration resembles the Cayley graph at that instance, multiple non-binary output can be gained by the polynomial time algorithm as a decoder. Further, this research can be extended to encode the various families of Cayley graphs such as unitary Cayley, unitary addition Cayley and Euler totient Cayley graphs, etc., with the different algebraic structure as well as the different adjacency constraints.

References

[1] I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987), 201–207.
[2] J. A. Gallian, A dynamic survey of graph labeling, Electron. J. Combin., 18 (2011), #DS6.
[3] M. Heydemann, Cayley graphs and interconnection networks. In: G. Hahn and G. Sabidussi (Eds.), Graph Symmetry: Algebraic Methods and Applications, (1997), 167–224.
[4] A. Rosa, On certain valuation of the vertices of a graph, Theory of Graphs(International. Symposium, Rome, july 1966), Gordon and Breach, N. Y. and Dunodparis, The Electronic journal of combinatorics, 16 (1967), 349–355, #DS6.
[5] T. Tamizh Chelvam, N. Mohamed Rilwan and K. Kalaimurugan, Antimagic and magic labelings in Cayley digraphs, Australian Journal Of Combinatorics, 55 (2013), 65–71.
[6] K. Thirusangu, A. K. Nagar and R. Rajeswari, Labelings in Cayley digraphs, European J. Combin., 32(1) (2011), 133–139.
[7] R. Yilmaz and I. Cahit, E-cordial graphs, Ars Combin., 46(1997), 251–266.