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# Topological Cordial Labeling of some Graphs

**Dr. S. Selestin Lina**

Assistant Professor, Department of Mathematics, Nanjil Catholic College of Arts and Science, Kaliyakkavilai, Kanyakumari District, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, Tamil Nadu, India.

E-mail: selestinlina@gmail.com

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**Abstract**

B.D. Acharya [3] introduced the notion of set – valuation as set analogue of number valuation as introduced by A. Rosa [5]. Let G be a graph and X, a non-empty set. Define an injective function f:V(G)→2^X such that {f(V(G))} is a topology on X. If the induced function f^* on E(G) is defined by

f^* (uv)={■(1 if f(u)∩f(v) is not an empty set and singleton set@0 otherwise )┤ for every uv∈E(G) such that

|e_f (0)-e_f (1)|≤1 where e_f (0)= number of edges labeled with 0 and 〖 e〗_f (1)= number of edges labeled with 1 then f is a topological cordial labeling and a graph which admits such a labeling is called topological cordial graph. In this paper we proved Dodecahedral graph, Paley graph and some constructed graphs are topological cordial graph.

**Key words:-**Dodecahedral graph, Paley graph and topological cordial graph.

## Introduction

The graphs treated in this paper are simple. For standard terminology and notations we follow F. Harary [4]. Given a graph $G = (V, E)$, we can relate it to different topological structures. The relation between topology and graph theory is undergone many investigations. In 1983 Acharya [3] established another link between graph theory and point – set topology. He defined a set – indexer as follows: Let $G = (V, E)$ be a graph, X any non – empty set and $2^X$ denote the set of all subsets of $X$. A set – indexer of $G$ is an injective set valued function $f : V(G) \to 2^X$ such that the induced function $f^* : E(G) \to 2^X - \{\phi\}$ defined by $f^*(v_1v_2) = f(v_1) \Delta f(v_2)$ for every $v_1v_2 \in E(G)$ is also injective , where $\Delta$ denotes the symmetric difference of sets. A graph $G = (V, E)$ is said to be a bitopological graph if there exist a set indexer $f: V(G) \to 2^X$ such that $f(V)$ and $f^*(E) \cup \{\phi\}$ are both topologies on the corresponding ground set. Let $G$ be a graph and $X$, a non-empty set. Define an injective function $f: V(G) \to 2^X$ such that $\{f(V(G))\}$ is a topology on $X$. If the induced function $f^*$ on $E(G)$ is defined by

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1 then $f$ is a topological cordial labeling and a graph which admits such a labeling is called topological cordial graph. This definition is defined and introduced in [8]. In this paper we proved Dodecahedral graph, Paley graph and some constructed graphs are topological cordial graph.

## 1.Preliminaries

**Definition 1.1** The *Dodecahedral graph* is a 3-connected graph with 20 vertices and 30 edges.

**Definition 1.2** A *complete bipartite graph* or biclique is a special kind of bipartite graph where every vertex of the first set is connected to every vertex of the second set.

**Definition 1.3** The *double star* $S(n,m)$, where $n \geq m \geq 0$, is the graph consisting of the union of two stars $K_{1,n}$ *and* $K_{1,m}$ together with a line joining their centers.

**Definition 1.4** The *Paley graph* of order q with q a prime power is a graph on q nodes with two nodes adjacent if their difference is a square in the finite field GF(q). This graph is undirected when q=1 (mod 4). Simple Paley graphs therefore exist for orders 5, 9, 13, 17, 25,…..

## 2.Topological Cordial Labeling

**Definition 2.1** Let $G$ be a graph and $X$, a non-empty set. Define an injective function $f: V(G) \to 2^X$ such that $\{f(V(G))\}$ is a topology on $X$. If the induced function $f^*$ on $E(G)$ is defined by

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1 then $f$ is a topological cordial labeling and a graph which admits such a labeling is called topological cordial graph.

## 2.Topological Cordial Labeling of named graphs

**Theorem 2.1.** Dodecahedral graph is topological cordial graph.

**Proof:** Let $G$ be Dodecahedral graph with 20 vertices and 30 edges. Let $V(G) = \{v_i/1 \leq i \leq 5\} \cup \{u_i/1 \leq i \leq 10\} \cup \{w_i/1 \leq i \leq 5\}$ and $E(G) = \{v_iv_{i+1}/1 \leq i \leq i+1, where\ v_{i+1} = v_i\} \cup \{u_iu_{i+1}/1 \leq i \leq i+1, where\ u_{i+1} = u_i\} \cup \{w_iw_{i+1}/1 \leq i \leq i+1, where\ w_{i+1} = w_i\} \cup \{v_iu_{2i-2}/2 \leq i \leq 4\} \cup \{w_iu_{2i-1}/1 \leq i \leq 5\} \cup \{v_1u_{10}\}$. Let $X = \{1,2,\dots,20\}$. Now, define $f: V(G) \to 2^X$ by

$f(v_1) = \phi$, $f(v_2) = \{1\}$, $f(v_3) = \{2\}$, $f(v_4) = \{1,2,3\}$, $f(v_5) = \{4\}$, $f(v_6) = \{3,4\}$, $f(v_7) = \{1,2\}$, $f(v_8) = \{2,3\}$, $f(v_9) = \{3\}$, $f(v_{10}) = \{1,4\}$, $f(v_{11}) = \{1,3,4\}$, $f(v_{12}) = \{2,3,4\}$, $f(v_{13}) = \{1,2,3,4\}$, $f(v_{14}) = \{1,3\}$, $f(v_{15}) = \{2,4\}$, $f(v_{16}) = \{1,2,4\}$, $f(v_{17}) = \{1,2,3,4,5\}$, $f(v_{18}) = \{1,2,\dots.6\}$, $f(v_{19}) = \{1,2,\dots..7\}$, $f(v_{20}) = X$

Then the vertex labels are distinct and $\{f(V(G))\}$ is a topology on $X$.

The induced function $f^*$ on $E(G)$ is defined as follows:

$$f^*(uv) = \begin{cases} 1 \text{ if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 \text{ otherwise} \end{cases}$$

for every $uv \in E(G)$. Then, $|e_f(0) - e_f(1)| = 15 - 15 = 0 \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1. Hence $f$ is a topological cordial labeling. Thus $G$ is topological cordial graph.

**Illustration 2.1.** Dodecahedral graph is topological cordial graph.

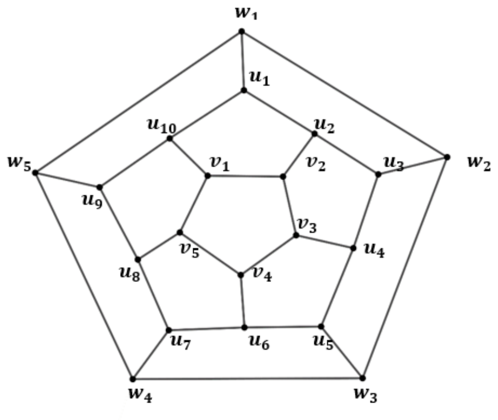



**Fig. 2.1**

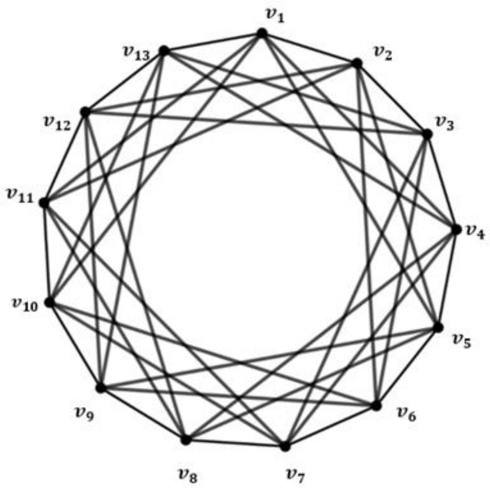
**Theorem 2.2** A Paley graph of order 13 is a topological cordial graph.

**Proof:** Let $G$ be a Paley graph of order 13. Thus it has 13 vertices and 39 edges. Let $V(G) = \{v_i/1 \le i \le 13\}$ an $E(G) = \{v_i v_{i+1}/1 \le i \le 13 \text{ where } v_{13} = v_1\} \cup \{v_i v_{i+3}/1 \le i \le 10\} \cup \{v_1 v_{i+4}/1 \le i \le 9\} \cup \{v_i v_{i+10}/1 \le i \le 3\} \cup \{v_i v_{i+9}/1 \le i \le 4\}$. Let $X = \{1,2,\ldots,13\}$. Define $f: V(G) \to 2^X$ by $f(v_1) = \phi$, $f(v_{i+1}) = \{1,2,\ldots,i\}$, $1 \le i \le 3$, $f(v_i) = \{1,2,\ldots,i+1\}$, $8 \le i \le 12, f(13) = X$. Then the vertex labels are distinct and $\{f(V(G))\}$ is a topology on $X$. The induced function $f^*$ on $E(G)$ is defined as follows:

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$. Then, $|e_f(0) - e_f(1)| \le 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1. Hence $f$ is a topological cordial labeling. Thus $G$ is topological cordial graph.

**Illustration 2.2** A Paley graph of order 13 is a topological cordial graph.




**ig 2.2**

## 3.Topological Cordial Labeling of generalized graphs

**Theorem 3.1.** The graph $K(m,n)$ is topological cordial graph, if $3 \leq n \leq 6$.

**Proof:** Let $K(m,n)$ be a complete bipartite graph.

Let $V(K) = \{v_i/1 \leq i \leq n\} \cup \{w_j/1 \leq j \leq n\}$ and $E(K) = \{v_i w_j/1 \leq i \leq n, 1 \leq j \leq n,$ where $3 \leq n \leq 6\}$ Let $X = \{1,2,3, \dots, n-1\}$, Define $f: V(K) \to 2^X$ . We label the vertices and edges satisfying the condition of topology. Therefore the vertex labels are distinct and $\{f(V(K))\}$ is a topology on $X$.The induced function $f^*$ on $E(K)$ is defined as follows:

$$f^*(uv) = \begin{cases} 1 \text{ if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 \text{ otherwise} \end{cases}$$

for every $uv \in E(K)$. Then, $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1. Hence $f$ is a topological cordial labeling. Thus $K$ is topological cordial graph.

**Illustration 3.1** $K(5,5)$ is topological cordial graph.

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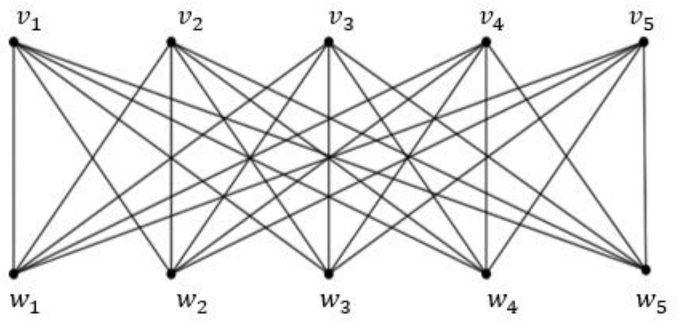


**Fig 3.1**

**Theorem 3.2** A double star $S(n, m)$ is a topological cordial graph.

**Proof:** Let $G$ be a double star $S(n, m)$.

Let $V(G) = \{u_0, u_1, u_2, \dots \dots . u_n\} \cup \{v_0, v_1, v_2, \dots . . v_m\}$ and $E(G) = \{u_0v_0\} \cup \{u_0u_i/1 \leq i \leq n\} \cup \{v_0v_i/1 \leq i \leq m\}$.

Then G has $m + n + 2$ $vertices\ and$ $n + m + 1$ $edges$ Let $X = \{1,2, \dots \dots n + m + 2\}$

Define $f: V(G) \to 2^X$ . We label the vertices and edges satisfying the condition of topology. Then the vertex labels are distinct and $\{f(V(G))\}$ is a topology on $X$. The induced function $f^*$ on $E(G)$ is defined as follows:

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$. Therefore, $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1. Hence $f$ is a topological cordial labeling. Thus $G$ is topological cordial graph.

**Illustration 3.2** A double star $S(2,4)$ is a topological cordial graph.

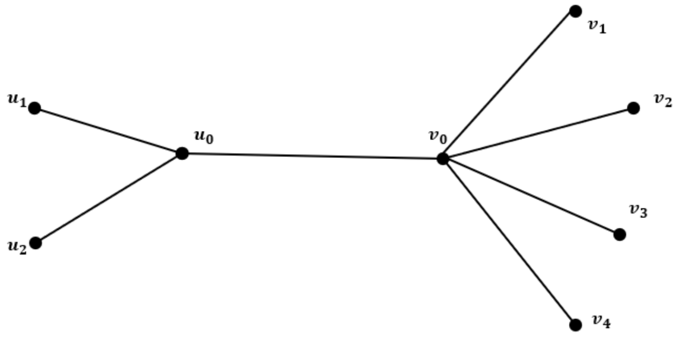


**Fig. 3.2**

## 4.Conclusion

In this paper deals with topological cordial graphs. The aim of this paper is to make some progress to a better understanding of topological cordial labeling.

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