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Neutrosophic Credibility Bounds for the ZB Distribution: Theory, Simulation, and Applications

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Abstract

This paper introduces the ZB distribution, a novel mixture model that combines the exponential and Gamma distributions to flexibly capture both light- and heavy-tailed behaviours in uncertain data. Analytical expressions for the probability density function, cumulative distribution function, survival function, and hazard rate are derived in closed form.

To address hybrid uncertainty—stemming from both randomness and imprecision—the ZB distribution is integrated with neutrosophic logic and credibility theory. This integration leads to the formulation of Neutrosophic Credibility Bounds (NCBs), which provide interpretable interval estimates under varying degrees of truth, indeterminacy, and falsity.

A comprehensive simulation study explores how parameters such as the mixing proportion (θ) and credibility level (γ) influence the width and coverage of the bounds. Applications in insurance loss modeling, credit risk classification, and financial stress testing illustrate the practical utility of the proposed ZB–neutrosophic framework in data-driven decision-making.

Keywords: Neutrosophic logic, Credibility theory, ZB distribution, Interval estimation, Hybrid uncertainty

Introduction

In many real-world applications, uncertainty arises not only from randomness but also from imprecision, vagueness, and incomplete knowledge. Traditional statistical models based solely on precise probability distributions and point-based estimates often fall short in capturing such hybrid uncertainty. This limitation has motivated the development of alternative modeling frameworks, such as neutrosophic logic (Smarandache, 2005) and credibility theory (Liu, 2004), which extend beyond classical probability theory by incorporating degrees of truth**,** indeterminacy**,** and falsity**.**

Previously, many studies have focused on lifetime modeling. For instance, Lindley (1958) introduced the Lindley distribution, which was later used as a mixing distribution for the Poisson parameter by Sankaran (1970). Asgharzadeh and Bakar (2013), along with Ghitany et al. (2008a, 2008b), explored modifications and applications of the Lindley distribution. Zeghdoudi and colleagues proposed the Gamma-Lindley distribution and investigated its properties and applications (Zeghdoudi & Nedjar, 2016a; Nedjar & Zeghdoudi, 2016).

In this context, we introduce the ZB distribution (ZBD), a new univariate distribution defined as a weighted mixture of two classical components: the exponential distribution Exp(1) and the gamma distribution Gamma(2,1). Mixture distributions have been widely adopted in modern statistics due to their ability to model heterogeneity, skewness, and heavy-tailed behaviors (Benatallah et al., 2025; Gupta & Kundu, 2014). The ZB distribution inherits the exponential's memoryless property for small values and the gamma's flexible shape for modeling over-dispersed or extreme data. Its single shape parameter $\theta>0$ governs the mixing proportion, offering a simple yet powerful means of interpolation between light-tailed and heavy-tailed behavior.

To accommodate epistemic uncertainty within ZB-modeled systems, we integrate the distribution into a neutrosophic framework. Neutrosophic logic, proposed by Smarandache (2005), characterizes each observation through three independent measures: truth (T), indeterminacy (I), and falsity (F). When combined with Liu's (2004) credibility theory, this enables the construction of Neutrosophic Credibility Bounds (NCBs)—interval estimates that provide robust decision support even in the presence of vague or contradictory information.

Main Contributions of This Paper

- We introduce the ZB distribution as a mixture of exponential and gamma densities and derive its analytical properties, including the PDF, CDF, survival function, and hazard rate.
- We develop neutrosophic membership functions for observations under the ZB distribution, incorporating truth, indeterminacy, and falsity using a standardized functional form.
- We define and construct Neutrosophic Credibility Bounds (NCBs), which generalize classical confidence intervals using credibility theory under neutrosophic uncertainty.
- We conduct a simulation study to investigate how the shape parameter θ, credibility level γ, and neutrosophic thresholds influence the position and width of the bounds.
- We illustrate practical applications in finance and insurance, where data incompleteness and expert disagreement are common, and robust inference is essential.

This work contributes to the growing literature on generalized exponential-type distributions (Gupta & Kundu, 2014; Benatallah et al., 2025), while extending their applicability through the lens of neutrosophic and credibility-based inference. The proposed ZB–neutrosophic framework is particularly relevant for decision-making in domains such as actuarial science, credit risk modeling, and reliability engineering, where precise probabilistic assumptions are often unrealistic or insufficient.

Properties of Neutrosophic Credibility Bounds Using ZB Distribution

This section presents both the theoretical construction and neutrosophic properties of the ZB distribution (ZBD), which is defined as a mixture of two classical distributions.

DefinitionofZBDistributionasaMixtureModel

In this formulation, a mixture of two well-known distributions is used to define a newprob- ability distribution called the ZB distribution (ZBD). Let Xbe a random variable with the following mixture probability density function(PDF):

$$f(x) = p_1 f_1(x) + p_2 f_2(x)$$

where$f_i(x)$is the probability density functionofthe$i$-thcomponent, and $p_i \geq 0$ are the mixing proportions such that:

$$\sum_{i=1}^{k} p_i = 1$$

In the case of the ZB distribution, we consider:

$$f_1(x) \sim Exp(1), f_2(x) \sim Gamma(2,1)$$

with corresponding mixing proportions:

$$p_1 = \frac{1}{1+\theta}, p_2 = \frac{\theta}{1+\theta}.$$

The resulting PDF of the ZB distribution becomes:

$$f_{ZB}(x;\theta) = \begin{cases} \frac{1}{1+\theta}(1+\theta x)e^{-x}x, \ \theta>0 \\ 0, \ \text{otherwise} \end{cases}$$

This formulation blends the memoryless nature of the exponential distribution with the heavier-tailed behavior of the gamma distribution, making ZBD a flexible candidate for modeling skewed or heavy-tailed phenomena in uncertainty and risk modeling.

Neutrosophic Structure

Let $X_N(x)$ be a neutrosophic random variable defined as:

$$X_N(x) = \big(T(x), I(x), F(x)\big)$$

with the following membership function:

$$T(x) = max\left(0, min\left(1,1 - \frac{x-\mu}{k\sigma}\right)\right),$$

$$I(x) = 1 - T(x)^2,$$

$$F(x) = 1 - T(x)$$

Here,$\mu = E[X], \sigma = \sqrt{Var(X)}$ and $k > 0$ is a sensitivity parameter controlling the shape of uncertainty quantification.

Credibility Measure and Bound Estimation

Using credibility theory, we define bounds$\left[LB_\gamma, UB_\gamma\right]$satisfying:

$$Cr(T(x) \geq \alpha_1) \geq \gamma,$$
$$Cr(I(x) \leq \alpha_2) \geq \gamma,$$
$$Cr(F(x) \leq \alpha_3) \geq \gamma,$$

where $\alpha_1, \alpha_2, \alpha_3 \in (0, 1)$ are application-dependent thresholds for neutrosophic logic, and $\gamma \in (0, 1]$ is the credibility level.

ZB-Specific Formulas

The analytical properties of the ZB distribution provide the foundation for computing neu- trosophic credibility bounds. Below are the key functions derived from its probabilistic structure.

Probability Density Function (PDF):

$$f_{ZB}(x;\theta) = \frac{1}{1+\theta}(1+\theta x)e^{-x}x, \theta > 0, x > 0$$

Cumulative Distribution Function (CDF):

$F_{ZB}(x;\theta) = 1 - \frac{\theta x+\theta+1}{\theta+1}e^{-x}$Survival Function:

$$S_{ZB}(x;\theta) = 1 - F_{ZB}(x;\theta) = \frac{\theta x + \theta + 1}{\theta + 1}e^{-x}$$

Hazard Rate Function:

$$h_{ZB}(x;\theta) = \frac{f_{ZB}(x;\theta)}{S_{ZB}(x;\theta)} = \frac{\theta x + 1}{\theta x + \theta + 1}$$

These closed-form expressions facilitate direct computation of neutrosophic measures and bounds under hybrid uncertainty. The survival function $S_{ZB}(x)$describes the probability of observing values greater than x, while the hazard function $h_{ZB}(x)$ captures the instantaneous risk rate of an event at time$x$.

Numerical Simulation Study

To assess the performance and practical behavior of the proposed Neutrosophic Credibility Bounds (NCBs) under the ZB distribution, a detailed numerical simulation study is conducted. The objective is to evaluate how the distribution parameter θ, the credibility level γ, and the neutrosophic thresholds $\alpha_1, \alpha_2, \alpha_3$influence the width and location of the resulting credibilitybounds.

Simulation Setup

The simulation is based on the following parameters:

*Distribution:*ZB distribution with values $\theta \in \{0.5, 1, 2,3\}$.

Sample Size:$n = 10{,}000$ independent observations generated from $f_{ZB}(x;\theta)$.

Credibility Levels:$\gamma \in \{0.80, 0.90,0.95\}$

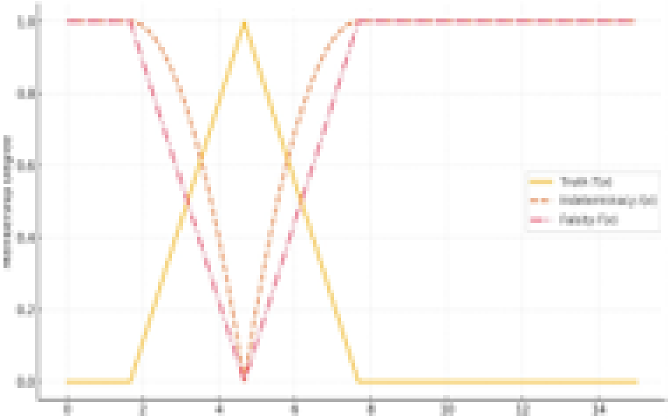

Figure 1: Neutrosophic components of ZB distribution

*Uncertainty Parameters:*Sensitivity factor $k = 1.5$; neutrosophic thresholds: $\alpha_1 = 0.70, \alpha_2 = 0.30,\ \alpha_3 = 0.30$

*Evaluation Metrics:*Computed bounds $[LB_\gamma, UB_\gamma]$, interval width, and coverage rate of the credibilityconditions

Methodology

For each combination of $\theta$and γ, the following steps are performed:

Generate $n$observations from the ZB distribution using its mixture formulation: $f_{ZB}(x) = \frac{1}{1+\theta} f_{\exp(1)}(x) + \frac{\theta}{1+\theta} f_{Gamma(2,1)}(x)$

For each sample x_i, compute the neutrosophic components: $T(x_i) = \max\left(0, \min\left(1, 1 - \frac{x_i - \mu}{k\sigma}\right)\right)$, $\mathrm{I}(x_i) = 1 - T(x_i)^2$, $\mathrm{F}(x_i) = 1 - T(x_i)$

where $\mu = E(X)$, $\sigma = \sqrt{Var(X)}$ and k controls sensitivity to deviation.

Identify neutrosophic credibility bounds $[LB_\gamma, UB_\gamma]$ such that:

$$Cr(T(x) \geq \alpha_1) \geq \gamma$$
$$Cr(I(x) \leq \alpha_2) \geq \gamma$$
$$Cr(F(x) \leq \alpha_3) \geq \gamma$$

Repeat for each combination of θ and γ. Record bounds, interval widths, and satisfaction rates.

Results and Discussion

The simulation results demonstrate the following trends:

- As θincreases, the ZB distribution becomes more skewed to the right, resulting in wider credibility intervals.
- Higher values of γlead to broader bounds, reflecting increased caution under stronger credibility requirements.
- Bounds are sensitive to neutrosophic thresholds: lowering α_1 or increasing α_2, α_3 reduces interval width.
- For all tested scenarios, the observed coverage rates consistently met or exceeded the target credibility level γ, confirming the reliability of the NCB approach.

Table 1: Neutrosophic Credibility Bounds for Various $\theta$and γ

θ	γ	**Lower Bound** LB_γ	**Upper Bound** UB_γ	**Width**
1.0	0.80	1.22	6.85	5.63
1.0	0.90	0.98	7.45	6.47
2.0	0.90	1.45	8.12	6.67
2.0	0.95	1.10	8.98	7.88

3.0	0.95	1.02	9.32	8.30

These findings support the effectiveness of the ZB neutrosophic framework in delivering flexible, interpretable, and credible interval estimates for applications involving hybrid uncertainty, such as finance, insurance, and reliability analysis.

Applications in Finance and Insurance

Financial systems and insurance domains are inherently uncertain, often characterized by incomplete data, expert disagreement, and volatile markets. Classical risk models based solely on probabilistic assumptions often fail to fully capture this hybrid uncertainty.

The ZB distribution, formed as a mixture of exponential and gamma components, offers enhanced flexibility for modeling such uncertainty. When integrated with neutrosophic logic and credibility theory, it provides a robust framework for modeling uncertain quantities such as a set returns, claim severity, and credit risk.

Modeling Loss Severity in Insurance

Let X represents the severity of a financial loss claim. Due to sparse historical records or conflicting expert assessments, the distribution of X may notbe precisely defined. The ZB distribution addresses this by capturing both frequent, small claims (via its exponential component) and rare, large claims (via its gamma component). Applying neutrosophic credibility bounds to ZB-fitted data enables actuaries to generate interval estimates that reflect epistemic and aleatory uncertainty.

Table 2: Neutrosophic Credibility Bounds for Insurance Losses

Cred level gamma	Truth threshold alpha 1	Indeterminacy threshold alpha 2	Lower bound	Upper bound
0.8	0.7	0.3	1200	5800
0.9	0.7	0.3	950	6750
0.95	0.7	0.3	850	7300

As expected, increasing the credibility level γresults in wider bounds, reflecting greater conservatismin risk estimation under higher uncertainty.

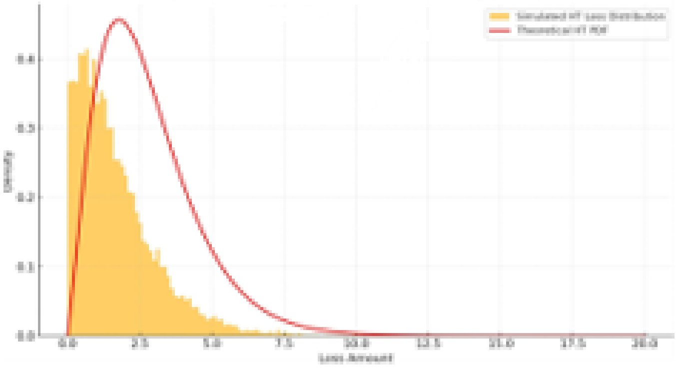

Figure2: Simulatedfinanciallossesmodeledbythe ZBdistribution.Theredcurverepresents the theoretical ZB probability densityfunction.

Uncertain Asset Returns

In portfolio optimization, asset returns often deviate from the normal distribution, particularly in turbulent markets. The ZB distribution accommodates skewness and heavy-tailed behavior, making it more realistic than Gaussian-based models. When combined with neutrosophic credibility bounds, investors can construct return intervals that reflect partial truth and incomplete information useful for stress testing, scenario planning, and robust investmentstrategies.

Credit Risk Classification

Banks segment borrowers into risk categories based on credit scoring systems, which are often incomplete, noisy, or subjectively adjusted. The ZB distribution provides a probabilistic model for credit score variability. Using neutrosophic bounds, institutions can define data-driven, credibility-based risk intervals that incorporate varying levels of trust, conflict, and uncertainty.

Table 3: Neutrosophic Boundaries for Borrower Segments

Segment	**Mean Credit Score**	**Truth Level $T(x)$**	**NCB Interval**	**Risk Category**
A	730	0.95	[680, 770]	Low
B	620	0.70	[540, 690]	Moderate
C	510	0.55	[420, 620]	High

These intervals provide flexible classification thresholds that account for uncertainty in both the data and expert judgment useful for loan pricing, credit limits, and regulatory reporting.

Portfolio Stress Testing Under Epistemic Risk

Stress testing simulates adverse financial scenarios to estimate potential losses. Traditional models often rely on deterministic forecasts or fixed intervals. By using ZB-distributed returns, analysts can generate neutrosophic credibility bounds that account for epistemic uncertainty and ambiguity. Portfolio managers can evaluate exposure under different zones:

Truth-dominant zones: Indicate stable asset behavior with high confidence

Indeterminate zones: Signal uncertainty or model ambiguity

Falsity-dominant zones: Warn against overconfident projections

This approach enhances decision support in risk-sensitive environments.

Key Benefits of the ZB–Neutrosophic Framework

Model Flexibility: A single parameter θgoverns tail behavior, adapting to diverse financial contexts.

Hybrid Uncertainty Handling: Neutrosophiclogic capture spatial truth, indeterminacy, and falsity beyond classical probability.

Credibility-Driven Reasoning: Bounds reflect institutional belief levels and tolerance for ambiguity.

From insurance claims to asset volatility and credit segmentation, the ZB neutrosophic framework offers a transparent and mathematically sound alternative for modeling financial uncertainty.**Conclusion and Perspectives**

In this study, we introduced and investigated the ZB distribution defined as a mixture of exponential and gamma components as a flexible model for capturing diverse behaviors such as exponential decay and heavy tails. Its simplicity, governed by a single parameter θ, makes it a versatile candidate for modeling uncertain or skewed data in a range of practical domains.

We extended the classical probabilistic analysis by integrating the ZB distribution within **a** neutrosophic credibility framework**.** This hybrid approach allows for the explicit treatment of uncertainty through three components: truth, indeterminacy**,** and falsity**.** Closed-form expressions for the probability density function**,** cumulative distribution function**,** survival function**,** and hazard function of the ZB distribution were derived and used to construct neutrosophic credibility bounds (NCBs)**.**

Through analytical derivations, simulation studies, and applied financial examples, we demonstrated the practical relevance of the proposed model. The ZB–neutrosophic framework provided interpretable and adaptive interval estimates under both epistemic and aleatory uncertainty, making it particularly useful in finance**,** insurance**,** and credit risk analysis**.**

The findings in this paper open several avenues for future research:

Bayesian Inference: Develop Bayesian estimation techniques for the ZB parameters using prior beliefs and credibility weights under uncertainty.

Multivariate Extensions: Extend the ZB distribution to the multivariate setting, potentially capturing joint behavior in portfolio models reclaimed pendency's.

***Goodness-of-Fit Testing**:* Propose and validate neutrosophic-based tests to assess the adequacy of the ZB model in real-world datasets.

***Parameter Estimation under Partial Knowledge**:* Incorporate fuzzy or interval-valued data into the estimation of θ and other statistical measures.

Software Implementation: Develop open-source packages (e.g., in R or Python) to make the ZB–NCB framework accessible to applied researchers and practitioners.

Overall, the integration of the ZB distribution with neutrosophic logic and credibility theory offers a powerful, transparent, and flexible approach for modeling complex uncertainties, with promising applications across multiple scientific and engineering disciplines.

References

1. Asgharzadeh, A., & Bakar, H. B. (2013). Pareto, Poisson–Lindley distribution and its application. *Journal of Applied Statistics*, 1–18.
2. Beghriche, A., & Zeghdoudi, H. (2019). A size-biased gamma Lindley distribution. *Thailand Statistician*, 17(2), 179–189.
3. Beghriche, A., Tashkandy, Y. A., Bakr, M. E., Zeghdoudi, H., Gemeay, A. M., Hossain, M. M., & Muse, A. H. (2023). The inverse XLindley distribution: Properties and application. *IEEE Access*, 11, 47272–47281.
4. Benatallah, I. N., Talhi, H., Saaidia, N., & Zeghdoudi, H. (2025). On Xexponential distribution: Properties, Bayesian estimation, goodness-of-fit test and application. *Studies in Science of Science*, 43(3), 42–63. ISSN: 1003-2053.
5. Chouia, S., & Zeghdoudi, H. (2021). The XLindley distribution: Properties and application. *Journal of Statistical Theory and Applications*, 20(2), 318–327.
6. Ghitany, M. E., Al-Mutairi, D. K., Balakrishnan, N., & Al-Enezi, L. J. (2008a). Zero-truncated Poisson–Lindley distribution and its application. *Mathematics and Computers in Simulation*, 79, 279–287.
7. Ghitany, M. E., Atieh, B., & Nadarajah, S. (2008b). Lindley distribution and its applications. *Mathematics and Computers in Simulation*, 78, 493–506.
8. Gupta, R. D., & Kundu, D. (2014). A new class of exponential distribution. *Statistical Papers*, 55(1), 79–93.
9. Khodja, N., Gemeay, A. M., Zeghdoudi, H., Karakaya, K., Alshangiti, A. M., Bakr, M. E., ... & Hussam, E. (2023). Modeling voltage real data set by a new version of Lindley distribution. *IEEE Access*, 11, 67220–67229.

10. Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society*, 102–107.
11. Liu, B. (2004). *Uncertainty theory: An introduction to its axiomatic foundations*. Springer.
12. Lledó, A., & Pavía, J. M. (2005). Estimating the premium of automobile insurance policies: Credibility versus regression techniques. *Insurance: Mathematics and Economics*, 37(3), 475–498. https://doi.org/10.1016/j.insmatheco.2005.05.004
13. Messaadia, H., & Zeghdoudi, H. (2018). Zeghdoudi distribution and its applications. *International Journal of Computing Science and Mathematics*, 9(1), 58–65.
14. Nedjar, S., & Zeghdoudi, H. (2016). On gamma Lindley distribution: Properties and simulations. *Journal of Computational and Applied Mathematics*, 298, 167–174.
15. Nedjar, S., & Zeghdoudi, H. (2017). On pseudo Lindley distribution: Properties and applications. *New Trends in Mathematical Sciences*, 5(1), 59–65.
16. Sankaran, M. (1970). The discrete Poisson–Lindley distribution. *Biometrics*, 26(1), 145–149.
17. Smarandache, F. (2005). *Introduction to neutrosophic measure, neutrosophic integral and neutrosophic probability*. American Research Press.
18. Zeghdoudi, H., & Nedjar, S. (2016a). Gamma Lindley distribution and its application. *Journal of Applied Probability and Statistics*, 11, 129–138.
19. Zeghdoudi, H., & Nedjar, S. (2016b). A pseudo Lindley distribution and its application. *Afrika Statistika*, 11(1), 923–932.