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# Bayesian Bonus-Malus Premiums Under Different Loss Functions in Car Insurance

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**Abstract—**

In the traditional bonus-malus system, automobile insurance premiums are typically calculated based solely on claim frequency. This study introduces an alternative approach, incorporating both claim frequency and claim severity into the determination of premiums. Furthermore, the research explores the premiums under various loss functions, including quadratic, linex, and entropy, applied to both frequency and severity. The Bayesian approach is employed to compute the bonus-malus premiums. Additionally,this work relies on the R program to provide a numerical application based on a real automobile insurance dataset. By incorporating diverse loss functions, this approach aims to enhance flexibility, achieve balance, and provide greater control over premiums, all while ensuring the solvency of the insurance company.

## 1. Introduction

In actuarial science, a key responsibility involves designing a pricing framework that fairly distributes the burden of claims among insured individuals. This is achieved by using the most appropriate models to calculate insurance premiums. One such method is the **bonus-malus system (BMS)**, which adjusts premiums based on an individual's claim history. It is widely implemented, particularly in automobile insurance, to ensure that all policyholders pay premiums commensurate with their risk levels. Safe driving without claims results in a premium reduction as a reward, whereas a reported incident leads to a premium increase.

The BMS serves two primary purposes for insurance companies. First, it encourages policyholders to operate their vehicles more carefully throughout the year, aiming to reduce the number of claims. Second, it ensures that policyholders pay premiums aligned with their individual risk levels as reflected in their claims history [1]. The fundamental concept of this system is that higher claim frequencies lead to higher premiums. Traditionally, BMSs have relied solely on the stochastic nature of claim frequency [4]. However, since claims may vary significantly in size, it is important to design a BMS that accounts for both the **frequency** and **severity** of claims. Although several authors have already proposed this idea [2, 7, 9], their models typically determine premiums using only the classical **quadratic loss function**.

In insurance, actuaries use **loss functions** as analytical tools to evaluate the costs associated with deviations from expected outcomes. These functions are critical in risk assessment, premium calculation, and decision-making processes. Measuring different forms of loss enhances an insurer's ability to manage and mitigate risk. Loss functions help insurers select models and strategies that align with their risk tolerance. By accurately evaluating risks, insurers can set fair and financially sustainable premium rates.

There are several types of loss functions used in statistics. The most common is the **quadratic loss function**, also known as the mean squared error. In automobile insurance, it calculates the squared difference between actual and estimated values, assigning greater weight to larger errors. Another important loss function is the **linex loss function**, which is asymmetric. This asymmetry allows for an unequal assessment of underestimation versus overestimation of risk—particularly useful when underestimating significant claims has more serious consequences than overestimating minor ones. Lastly, the **entropy loss function** measures the proportional gap between observed and expected values. It is particularly sensitive to rare but important deviations, making it suitable for capturing infrequent but high-impact events.

In this study, we aim to calculate bonus-malus premiums using multiple loss functions while incorporating both claim frequency and severity. While most existing research focuses on a single loss function, we examine the use of **different loss functions across various scenarios**, and we compare the resulting premiums with those obtained using only one loss function. Additionally, we provide a numerical application using a real automobile insurance dataset and employ R software to compute the premiums, allowing for a detailed comparison and identification of the most beneficial approach for insurance providers.

This article is structured as follows:

- The first two sections address **claim frequency** and **claim severity**, each divided into five subsections: the selected distribution, the Bayesian approach, and the premiums computed using the quadratic, linex, and entropy loss functions.
- The third section presents the **final premiums** calculated based on both claim frequency and severity.
- The fourth section offers a **numerical application**, using a real dataset and R software, and analyzes the outcomes based on the final premiums discussed in the third section.
- The final section provides a **summary of the study's key findings**.

## 2. CLAIMFREQUENCYDISTRIBUTIONUSINGPOISSON-AKASH

**2.1. Poisson-Akash Distribution.** We generallyusethePoissondistribution in car insurance to explain the random frequency of the claim.The claims numberkissupposedtofollowthePoissondistributionwithparameter$\theta$and probability mass function as

$$P(l|\theta) = \frac{e^{-\theta}\theta^{l}}{l!}, l = 0,1,2, \dots, \theta > 0,$$

withthefollowingexpectedvalue

$$E[l|\theta] = Var[l|\theta] = \theta.$$

$\theta$ is assumed to follow the Akash distribution with parameter $\gamma$and a probability density function expressed as follows:

$$\pi(\theta) = \frac{\gamma^3}{\gamma^2 + 2}(1 + \theta^2)e^{-\gamma\theta}.$$

Then,themixeddistributionofthePoissonwithAkashdistribution(seeShanker in 2016 [11]) is:

$$f(l) = \int_0^\infty P(l|\theta)\pi(\theta)d\theta = \frac{\gamma^3}{\gamma^2 + 2}\frac{\gamma^2 + 2\gamma + l^2 + 3l + 3}{(1 + \gamma)^{l+3}} (1)$$

2.2. **Bayesian method.** The Bayesian technique is one of the most commonly employed computational solutions for the bonus-malus premium calculation, whichhasbeenwellexploredin [5].Thismethod'sprimarygoal istodeterminetheposteriordistributionfunction.

$l_1, l_2 \ldots, l_t$is asample ofsize$t$.Theclaimstotalnumbergeneratedbyaninsured over tyearsis$N = \sum_{i=1}^{t} l_i$,where$l_i$istheclaim'snumberestablishedbyaninsuredinyears$i, i = 1, 2, \ldots, t$.Thelikelihoodfunctionis:

$$L(\theta; l_{1,}l_2, \ldots, l_n) = \prod_{i=1}^{t}\frac{e^{-\theta}\theta^{l_i}}{l_i!} = \frac{1}{\prod_{i=1}^{t} l_i\,!}e^{-t\theta}\theta^{\sum l_i} \propto e^{-t\theta}\theta^N. (2)$$

ThePriordistributionis:

$$\pi(\theta) = \frac{\gamma^3}{\gamma + 2}(1 + \theta^2)e^{-\gamma\theta} \propto (1 + \theta^2)e^{-\gamma\theta}. (3)$$

Inorder to determine the posterior distribution for an insured with$k_{1,}k_2, \ldots, k_t$ claimhistory,weemploytheBayes'theorem.Theproductofthepriordistribution in (3) and the likelihood function in (2) represents the posterior distribution function as follows:

$$\begin{aligned}\pi^*(\theta|l_1, \ldots, l_n) \propto P(l_1, \ldots, l_n|\theta)\pi(\theta) &= e^{-\theta t}\theta^N(1 + \theta^2)e^{-\gamma\theta}\\ &= \theta^N(1 + \theta^2)e^{-\theta(t+\gamma)}.\end{aligned}$$

Finaly,fortheclaimfrequency,theposteriordistributionfunctionis(for moredetailsreferto[9])

$$\pi^*(\theta|l_1, \ldots, l_n) = \frac{(t + \gamma)^{n+3}}{\Gamma(n + 1)[(n + 2)(n + 1) + (t + \gamma)^2]}\theta^N(1 + \theta^2)e^{-\theta(t+\gamma)}.$$

2.3. **Premium under quadratic loss function.** In this paper, we aim to determinethenetpremiumwhichcorrespondstothemeanoftheclaimnumber ofeachinsured.Assumethattheclaimhistoryis $l_1, l_2 \ldots, l_t$.Themeanof theposteriordistributionfunctionforPoisson-Akashdistribution(whichisthe expected number of claims) is obtained by:

$$\begin{aligned}\widehat{\theta_{t+1}} = E[\theta|l_{1,} \ldots, l_t] = E[l_{1,} \ldots, l_n|\theta] &= \int_0^\infty \theta\pi^*(\theta_i|l_1, \ldots, l_n)\\ &= \frac{(n + 1)(n + 2)(n + 3) + (n + 1)(t + \gamma)^2}{(t + \gamma)^3 + (t + \gamma)(n + 2)(n + 1)}.\end{aligned}$$

Consider that 100 represents the initial premium at time $t$=0.Consequently, thepremiumattime$t$+1isdefinedthereby:

$$\begin{aligned}Premium_{t+1} &= 100\frac{\widehat{\theta_{t+1}}}{E[x]}\\ &= 100\frac{\theta(\theta^2 + 2)}{\theta^2} + 6\frac{(n + 1)(n + 2)(n + 3) + (n + 1)(t + \gamma)^2}{(t + \gamma)^3 + (t + \gamma)(n + 2)(n + 1)}. (4)\end{aligned}$$

2.4. **Premium under Linex Loss function:**The following subsection examinestheasymmetriclinexlossfunction.Itexhibitsanexponentialincrease on one side of zero and a linear increase on the other one, as demonstrated in [8, 10, 12].Itisarticulatedas:

$$L(\hat{\theta}, \theta) = \exp\left(a(\hat{\theta} - \theta)\right) - a(\hat{\theta} - \theta) - 1, \quad a \neq 0,$$

where $\hat{\theta}$ denotes the estimator of $\theta$under the linex loss function that minimizes the aforementioned equation (see [13]), $\hat{\theta}$can be defined as:

$$\widehat{\theta_L} = -\frac{1}{\alpha}\ln\left(E\left[e^{-\alpha\theta}\middle|X\right]\right).$$

Then

$$\begin{aligned}\mathrm{E}\left[e^{-\alpha\theta}\middle|l_{1,}\dots,l_n\right] &= \int_0^\infty e^{-\alpha\theta}\,\pi^*(\theta_i|l_1,\dots,l_n)\mathrm{d}\theta \\ &\propto \int_0^\infty \theta^{N+2}\,e^{-(t+\gamma+\alpha)\theta}\mathrm{d}\theta + \int_0^\infty \theta^N\,e^{-(t+\gamma+\alpha)\theta}\mathrm{d}\theta \\ &= \frac{(t+\gamma)^{n+3}(n+1+t+\gamma+\alpha)}{[(n+2)(n+1)+(t+\gamma)^2](t+\gamma+\alpha)^{n+2}}.\end{aligned}$$

Finally

$$\widehat{\theta_L} = -\frac{1}{a}\ln\left(\frac{(n+2)(n+1)+(t+\gamma+a)^2}{(t+\gamma+a)^{n+3}}\frac{(t+\gamma)^{n+3}}{(n+2)(n+1)+(t+\gamma)^2}\right).$$

Attime$t$=0,thepremiumis100.Thepremiumattime$t$+1isexpressedas

$$\begin{aligned}Premium_{t+1} = 100\,&\frac{\gamma(\gamma^2+2)}{\gamma^2+6} \\ &* \left[-\frac{1}{a}ln\left(\frac{(n+2)(n+1)+(t+\gamma+a)^2}{(t+\gamma+a)^{n+3}} * \right.\right. \\ &\left.\left. * \frac{(t+\gamma)^{n+3}}{(n+2)(n+1)+(t+\gamma)^2}\right)\right]. \quad (5)\end{aligned}$$

**PremiumunderEntropyLossfunction:**Thispartinterpretstheentropylossfunction.Inv ariousactualscenarios,itappearsmoresensibleto express the loss as the ratio $\frac{\hat{\theta}}{\theta}$;forthiscontext,[6]presenteda loss function referred to as entropy.This loss function serves as a powerful asymmetric loss which assigns credit for both underestimation and overestimation, defined by the following structure.

$$\mathrm{L}(\widehat{\theta_L} - \theta) = \left(\frac{\widehat{\theta_E}}{\theta}\right)^q - \mathrm{q}\ln\left(\frac{\widehat{\theta_E}}{\theta}\right) - 1; \quad \mathrm{q} \neq 0$$

where$\widehat{\theta_E}$denotestheestimateof$\theta$thatminimizestheaforementionedequationundertheentropylossf unction. Theexpressionfor$\widehat{\theta_E}$isprovidedas follows:

$$\widehat{\theta_E} = \left(E\left[\theta^{-p}\middle|l_{1,}\dots,l_n\right]\right)^{-\frac{1}{p}}$$

$$E\left[\theta^{-p}\middle|l_{1,}\dots,l_n\right] = \int_0^\infty \theta^{-p}\,\Pi^*\left(\theta\middle|l_{1,}\dots,l_n\right)d\theta$$

$$= \int_0^\infty \theta^{-p}\ \theta^{N}(1+\theta^2)\, e^{-\theta(t+\gamma)}\, d\theta$$
$$= \left[\frac{\Gamma(n-p+3)}{(t+\theta)^{n-p+3}} + \right.$$
$$\left. + \frac{\Gamma(n-p+1)}{(t+\theta)^{n-p+1}}\right] \frac{(t+\gamma)^{n+3}}{\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]}$$
$$E\left[\theta^{-p}\middle|l_{1,}\dots,l_n\right] = \left[\frac{\Gamma(n-p+3)+\Gamma(n-p+1)(t+\gamma)^2}{(t+\theta)^{n-p+3}}\right]$$
$$* \frac{(t+\gamma)^{n+3}}{\left[\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]\right]}$$

Finally

$\widehat{\theta_E} =$

$$= \left[\left(\frac{\Gamma(n-p+3)+\Gamma(n-p+1)(t+\gamma)^2}{(t+\theta)^{n-p+3}}\right) * \frac{(t+\gamma)^{n+3}}{\left[\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]\right]}\right]^{-\frac{1}{p}} \quad (6)$$

$$Premium_{t+1} = 100\frac{\gamma(\gamma^2+2)}{\gamma^2+6} * \left[\left(\frac{\Gamma(n-p+3)+\Gamma(n-p+1)(t+\gamma)^2}{(t+\theta)^{n-p+3}}\right)\right.$$
$$\left. * \frac{(t+\gamma)^{n+3}}{\left[\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]\right]}\right]^{-\frac{1}{p}} . (7)$$

## 3. CLAIMSEVERITYDISTRIBUTIONUSINGINVERS-GAMMALINDLEY

3.1. **Mixing Distribution.** $X$ denote a random variable that signifies the claimsizeassociatedwitheachinsurer.Letusconsiderthat$X$adherestoan Inverse-Gamma distribution, characterized by the following probability density function:

$$f(x|\lambda) = \frac{\lambda^\alpha x^{-\alpha-1}}{\Gamma(\alpha)} e^{-\frac{\lambda}{x}}.$$

The Invers-Gamma expected value is

$$E[X] = \frac{\lambda}{\alpha-1}.$$

Let λ be a random variable that adheres to a Lindley distribution characterized by the parameter β. Then, the representation of the PDF of λ is provided as:

$$\pi(\lambda) = \frac{\beta^2}{\beta+1}(\lambda+1)e^{-\beta\lambda}.$$

Thus, the combination of the Invers-Gamma and Lindley distributions is derived thereby ([9]):

$$f(x) = \int_0^\infty \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\frac{\lambda}{x}} \frac{\beta^2}{\beta+1}(\lambda+1)e^{-\beta\lambda}\, d\lambda$$
$$= \frac{x^{-\alpha-1}\beta^2(\alpha+1)}{(\beta+1)\left(\beta+\frac{1}{x}\right)^{\alpha+2}}\left[\alpha+1+\beta+\frac{1}{x}\right]. (8)$$

3.2. **BayesianMethod.**$N = \sum_{i=1}^{t} l_i$ representsthetotalnumberofclaims submitted by an insured over a duration of t years. Define x as the amount of claim$l$where $l$ranges from 1 to $N$.The subsequent expression identifies the likelihood function:

$$L(\lambda \mid x_{1,\dots,}x_n) = f(x_{1,\dots,}x_n \mid \lambda) = \prod_{i=1}^{n} \frac{\lambda^{\alpha} x^{-\alpha-1}}{\Gamma(\alpha)} e^{-\lambda \frac{1}{x_i}}$$
$$= \frac{\lambda^{n\alpha}}{(\Gamma(\alpha))^n} \prod_{i=1}^{n} x^{-\alpha-1} e^{-\lambda \sum_{i=1}^{n} \frac{1}{x_i}}$$
$$\propto \lambda^{n\alpha} e^{-\lambda \sum_{i=1}^{n} \frac{1}{x_i}}.$$

The following $\pi(\lambda)$ denotes the prior distribution

$$\pi(\lambda) \propto (\lambda+1)e^{-\beta\lambda}.$$

To determine the posterior distribution function, we utilize Bayes' Theorem in the following manner:

$$\pi^*(\lambda \mid x_{1,\dots,}x_n) \propto f(x_{1,\dots,}x_n \mid \lambda)\pi(\lambda) \propto \lambda^{n\alpha}(1+\lambda)e^{-\lambda\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}.$$

Thus, the posterior distribution function for the claim severity is represented as (for more detail see[9]):

$$\pi^*(\lambda \mid x_{1,\dots,}x_n) = \frac{\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n+2}}{\Gamma(\alpha n+1)\left(\alpha n+2+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)} \lambda^{n\alpha}(1+\lambda)e^{-\lambda\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}. \quad (9)$$

3.3. **Premium under quadratic loss function.** For an insured given a claim history $l1, l2, \dots, lt$, the mean of the posterior distribution function for Poisson- Akash distribution (or the predicted claim number) is:

$$\widehat{\lambda_{t+1}} = E[\lambda \mid x_{1,} \dots, x_n] = \int_0^{\infty} \lambda \pi^*(\lambda \mid x_{1,} \dots, x_n) d\lambda$$
$$= \frac{(\alpha n+2)\left(\alpha n+3+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}{\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left(\alpha n+2+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}.$$

According to

$$E[\lambda \mid x_1, \dots, x_n] = \hat{\lambda}.$$

We have

$$E[x_1, \dots, x_n \mid \lambda] = \frac{\hat{\lambda}}{\alpha-1}.$$

Consequently

$$E[x_1, \dots, x_n \mid \lambda] = \frac{(\alpha n+2)\left(\alpha n+3+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}{(\alpha-1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left(\alpha n+2+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}. \quad (10)$$

3.4. **PremiumunderLinexLossfunction:**

Inthissection,also,weusedtheVarian'sasymmetriclinexlossfunction, definedbythefollowingequation:

$$L(\hat{\lambda},\lambda) = \exp\left(a(\hat{\lambda}-\lambda)\right) - a(\hat{\lambda}-\lambda) - 1, \quad a \neq 0.$$

Consider $\hat{\lambda}$ is the estimator of $\lambda$ under the Linex loss fonction which minimizes the precedent equation, $\hat{\lambda}$ is given by:

$$\widehat{\lambda_L} = -\frac{1}{c} \ln E\left(e^{-c\lambda} \mid X\right)$$

Then

$$E\left(e^{-c\lambda}\middle|X\right)=\int_0^{\infty}Be^{-c\lambda}\,\pi^*(\lambda_i|x_1,\dots,x_n)d\lambda=B\int_0^{\infty}e^{-c\lambda}\,\lambda^{\alpha n}(1+\lambda)e^{-\lambda\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}d\lambda$$

$$=\left[\frac{\Gamma(\alpha n+1)}{\left(c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n+1}}+\frac{\Gamma(\alpha n+2)}{\left(c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n+2}}\right]$$

$$*\frac{\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n+2}}{\Gamma(\alpha n+1)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}E\left(e^{-c\lambda}\middle|X\right)$$

$$=\left(\frac{\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)^{\alpha n+2}\left(\frac{\alpha n+1+c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)$$

Finally

$$\widehat{\lambda_L}=\frac{1}{\alpha-1}\left(-\frac{1}{c}\right)\ln\left[\left(\frac{\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)^{\alpha n+2}\left(\frac{\alpha n+1+c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)\right]$$

3.5. **Premium under Entropy Loss function:** In this part, we also, utilized the Entropy loss function in the following manner:

$$\widehat{\lambda_E}=\left[E(\lambda^{-p}|X)\right]^{-\frac{1}{p}}$$

Then

$$E(\lambda^{-p}|X)=\int_0^{\infty}B\lambda^{-p}\,\pi^*\,(\lambda_i|x_1,\dots,x_n)d\lambda=B\int_0^{\infty}\lambda^{-p}\,\lambda^{\alpha n}(1+\lambda)e^{-\lambda\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}d\lambda$$

$$=\left[\frac{\Gamma(\alpha n-p+1)}{\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n-p+1}}+\frac{\Gamma(\alpha n+2)}{\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n-p+2}}\right]$$

$$*\frac{\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)^{\alpha n+2}}{\Gamma(\alpha n+1)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}$$

$$E(\lambda^{-p}|X)=\frac{\Gamma(\alpha n-p+1)\left[\alpha n-p+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}{\Gamma(\alpha n+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}$$

Finally

$$\widehat{\lambda_E}=\frac{1}{\alpha-1}\left[\frac{\Gamma(\alpha n-p+1)\left[\alpha n-p+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}{\Gamma(\alpha n+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}\right]^{-\frac{1}{p}}$$

## 4. FINALPREMIUMSUSINGBOTHFREQUENCYANDCLAIMSEVERITY

The tables bellow represents the final premiums that will be used in the numerical application

TABLE1.FinalpremiumsusingInvers-GammaLindleydistribution under Quadratic loss function for the claim severity

| Claimfrequencydistribution | Finalpremium |
|---|---|
| PoissonAkashunder Quadraticloss function(4) | $Premium_{t+1} = 100\frac{\gamma(\gamma^2+2)}{\gamma^2+6} * \frac{(n+1)(n+2)(n+3)+(n+1)(t+\gamma)^2}{(t+\gamma)^3+(t+\gamma)(n+2)(n+1)} * \frac{(\alpha n+2)\left(\alpha n+3+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}{(\alpha-1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left(\alpha n+2+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}. \quad (11)$ |
| PoissonAkashunder Linexlossfunction(5) | $Premium_{t+1} = 100\frac{\gamma(\gamma^2+2)}{\gamma^2+6} * \left[-\frac{1}{a}ln\left(\frac{(n+2)(n+1)+(t+\gamma+a)^2}{(t+\gamma+a)^{n+3}} * * \frac{(t+\gamma)^{n+3}}{(n+2)(n+1)+(t+\gamma)^2}\right)\right] * \frac{(\alpha n+2)\left(\alpha n+3+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}{(\alpha+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left(\alpha n+2+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}. \quad (12)$ |
| PoissonAkashunder Entropylossfunction(7) | $\text{Premium}_{t+1} = = 100\frac{\gamma(\gamma^2+2)}{\gamma^2+6} * \left[\left(\frac{\Gamma(n-p+3)+\Gamma(n-p+1)(t+\gamma)^2}{(t+\theta)^{n-p+3}}\right) * \frac{(t+\gamma)^{n+3}}{[\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]]}\right]^{-\frac{1}{p}} * \frac{(\alpha n+2)\left(\alpha n+3+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}{(\alpha+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left(\alpha n+2+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)}. \quad (13)$ |

TABLE2.FinalpremiumsusingInvers-GammaLindleydistribution under Linex loss function for the claim severity

| Claimfrequencydistribution | Finalpremium |
|---|---|
| Poisson Akash under Quadraticlossfunction(4) | $Premium_{t+1} = 100\frac{\theta(\theta^2+2)}{\theta^2} +6\frac{(n+1)(n+2)(n+3)+(n+1)(t+\gamma)^2}{(t+\gamma)^3+(t+\gamma)(n+2)(n+1)}$ |

| | |
|---|---|
| | $$*\frac{1}{\alpha-1}-$$ $$-\frac{1}{c}\ln\left[\left(\frac{\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)^{\alpha n+2}\left(\frac{\alpha n+1+c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)\right]. ($$ |
| Poisson Akash under Linexlossfunction(5) | $$Premium_{t+1}=100\frac{\gamma(\gamma^2+2)}{\gamma^2+6}$$ $$*\left[-\frac{1}{a}ln\left(\frac{(n+2)(n+1)+(t+\gamma+a)^2}{(t+\gamma+a)^{n+3}}*\right.\right.$$ $$\left.\left.*\frac{(t+\gamma)^{n+3}}{(n+2)(n+1)+(t+\gamma)^2}\right)\right]$$ $$*\frac{1}{\alpha-1}-$$ $$-\frac{1}{c}\ln\left[\left(\frac{\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)^{\alpha n+2}\left(\frac{\alpha n+1+c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)\right]. ($$ |
| Poisson Akash under Entropy loss function (7) | $$Premium_{t+1}=$$ $$=100\frac{\gamma(\gamma^2+2)}{\gamma^2+6}*$$ $$*\left[\left(\frac{\Gamma(n-p+3)+\Gamma(n-p+1)(t+\gamma)^2}{(t+\theta)^{n-p+3}}\right)\right.$$ $$\left.*\frac{(t+\gamma)^{n+3}}{[\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]]}\right]^{-\frac{1}{p}}$$ $$*\frac{1}{\alpha-1}-$$ $$-\frac{1}{c}\ln\left[\left(\frac{\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)^{\alpha n+2}\left(\frac{\alpha n+1+c+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}{\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}}\right)\right].(1$$ |

TABLE3.FinalpremiumsusingInvers-GammaLindleydistribution under Entropy loss function for the claim severity

| Claimfrequencydistribution | Finalpremium |
|---|---|
| Poisson Akash under Quadraticlossfunction(4) | $$Premium_{t+1}=100\frac{\theta(\theta^2+2)}{\theta^2}$$ $$+6\frac{(n+1)(n+2)(n+3)+(n+1)(t+\gamma)^2}{(t+\gamma)^3+(t+\gamma)(n+2)(n+1)}$$ |

| | |
|---|---|
| | $* \frac{1}{\alpha-1}\left[\frac{\Gamma(\alpha n-p+1)\left[\alpha n-p+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}{\Gamma(\alpha n+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}\right]^{-\frac{1}{p}}$ . (1 |
| Poisson Akash under Linexlossfunction(5) | $Premium_{t+1} = 100\frac{\gamma(\gamma^2+2)}{\gamma^2+6} * \left[-\frac{1}{a}ln\left(\frac{(n+2)(n+1)+(t+\gamma+a)^2}{(t+\gamma+a)^{n+3}} * * \frac{(t+\gamma)^{n+3}}{(n+2)(n+1)+(t+\gamma)^2}\right)\right] * * \frac{1}{\alpha-1}\left[\frac{\Gamma(\alpha n-p+1)\left[\alpha n-p+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}{\Gamma(\alpha n+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}\right]^{-\frac{1}{p}}$ . ( |
| Poisson Akash under Entropy loss function (7) | $Premium_{t+1} = = 100\frac{\gamma(\gamma^2+2)}{\gamma^2+6} * * \left[\left(\frac{\Gamma(n-p+3)+\Gamma(n-p+1)(t+\gamma)^2}{(t+\theta)^{n-p+3}}\right) * \frac{(t+\gamma)^{n+3}}{[\Gamma(n+1)[(n+2)(n+1)+(t+\gamma)^2]]}\right]^{-\frac{1}{p}} * \frac{1}{\alpha-1}\left[\frac{\Gamma(\alpha n-p+1)\left[\alpha n-p+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}{\Gamma(\alpha n+1)\left(\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right)\left[\alpha n+1+\beta+\sum_{i=1}^{n}\frac{1}{x_i}\right]}\right]^{-\frac{1}{p}}$ . (1 |

## 5. NUMERICALAPPLICATION

For the numerical analysis, we utilized a real data set to evaluate the premiums determined solely by the number of claims and those calculated usingboth claim frequency and severity.We employed the different loss functions aforementioned. We focused on the data set related to one-year car insurance policiesissuedin2004or2005.Thisdatasetcanbefoundonthewebsiteof the Faculty of Business and Economics at Macquarie University in Sydney, Australia.Forfurtherreference,see[3].Theportfolio comprises a total of 67,856 policies, with 4,624 of them having at least one claim recorded.Among these policies, 4,333 submitted one claim, 271 filed two claims,18presentedthreeclaims,and2submittedfourclaims.

5.1. **Premiums using only claim frequency.**The following tables [4-8] represents the premiums based only on the claim frequency and using different loss functions (Quadratic, Linex (a=1.1 and a=-0.3), Entropy (p=0.2 and p=-0.2)).

TABLE4.PremiumsusingPoisson-Akashwith$\gamma$=14.0125

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 93.10335 | 187.7328 | 283.7319 | 380.8901 | 478.9653 |
| 2 | 87.10775 | 175.4826 | 265.0100 | 355.5334 | 446.8701 |
| 3 | 81.84590 | 164.7550 | 248.6416 | 333.3880 | 418.8542 |
| 4 | 77.18976 | 155.2800 | 234.2057 | 313.8769 | 394.1853 |
| 5 | 73.03964 | 146.8484 | 221.3763 | 296.5537 | 372.2957 |
| 6 | 69.31671 | 139.2954 | 209.8972 | 281.0674 | 352.7389 |
| 7 | 65.95779 | 132.4893 | 199.5640 | 267.1384 | 335.1592 |

TABLE5.Premiums using Poisson-Akash under linex with $\gamma = 14.0125$ and$a$ =1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 89.74638 | 180.8718 | 273.2438 | 366.6832 | 460.9817 |
| 2 | 84.16501 | 169.4815 | 255.8516 | 343.1409 | 431.1913 |
| 3 | 79.24454 | 159.4601 | 240.5731 | 322.4814 | 405.0633 |
| 4 | 74.87319 | 150.5725 | 227.0418 | 304.2025 | 381.9597 |
| 5 | 70.96317 | 142.6349 | 214.9718 | 287.9125 | 361.3823 |
| 6 | 67.44461 | 135.5014 | 204.1363 | 273.3010 | 342.9363 |
| 7 | 64.26111 | 129.0547 | 194.3536 | 260.1196 | 326.3050 |

TABLE6.Premiums using Poisson-Akash under linex with $\gamma = 14.0125$ et $a = -0.3$

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 94.07916 | 189.7294 | 286.7865 | 385.0298 | 484.2065 |
| 2 | 87.95956 | 177.2214 | 267.6655 | 359.1284 | 451.4195 |
| 3 | 82.59612 | 166.2833 | 250.9721 | 336.5396 | 422.8403 |
| 4 | 77.85570 | 156.6341 | 236.2678 | 316.6628 | 397.7068 |
| 5 | 73.63484 | 148.0569 | 223.2142 | 299.0345 | 375.4296 |
| 6 | 69.85195 | 140.3807 | 211.5459 | 283.2909 | 355.5462 |
| 7 | 66.44175 | 133.4695 | 201.0515 | 269.1430 | 337.6885 |

TABLE7.PremiumsusingPoisson-Akashunderentropywith $\gamma$=14.0125et$p$=−0.2

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 60.54909 | 151.9319 | 246.6078 | 342.9572 | 440.4815 |
| 2 | 56.67214 | 142.0671 | 230.4023 | 320.1982 | 411.0317 |
| 3 | 53.26630 | 133.4216 | 216.2259 | 300.3148 | 385.3215 |
| 4 | 50.25001 | 125.7803 | 203.7168 | 282.7908 | 362.6792 |
| 5 | 47.55958 | 118.9764 | 192.5945 | 267.2267 | 342.5842 |
| 6 | 45.14460 | 112.8783 | 182.6386 | 253.3087 | 324.6273 |
| 7 | 42.96458 | 107.3807 | 173.6732 | 240.7868 | 308.4824 |

TABLE8.PremiumsusingPoisson-Akashunderentropywith $\gamma$=14.0125et$p$=+0.2

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 43.30648 | 133.28359 | 227.5163 | 323.5812 | 420.8996 |
| 2 | 40.54181 | 124.65254 | 212.5987 | 302.1461 | 392.7958 |
| 3 | 38.11183 | 117.08492 | 199.5447 | 283.4153 | 368.2582 |
| 4 | 35.95882 | 110.39385 | 188.0226 | 266.9037 | 346.6460 |
| 5 | 34.03769 | 104.43409 | 177.7753 | 252.2361 | 327.4629 |
| 6 | 32.31269 | 99.09100 | 168.6004 | 239.1175 | 310.3188 |
| 7 | 30.75509 | 94.27286 | 160.3365 | 227.3127 | 294.9030 |

- *Discussion.* We observe that the premiums using the linex loss function in the under estimation are the highest for good drivers but also with bad drivers. The tables show that both good and bad drivers have lower premiums when employing the entropy loss function.

## 5.2. Premiums using both frequency and claim severity.

### 5.2.1. Premiums using Inverse-Gamma Lindley under Quadratic loss function for the claim severity.

**Premiums using Inverse-Gamma Lindley under Quadratic loss function for the claim severity.** This first part shows the tables [9-13] that represents the premiums using the Invers-Gamma Lindley distribution (IGL) under the quadratic loss function for the severity claim. For the claim frequency we will use Poisson- Akash distribution (PA) under different loss functions (Quadratic, Linex (a=1.1 and a=-0.3) and Entropy (p=0.2 and p=-0.2)).These tables were calculated using formulas (11), (12) and (13) from table 1.

TABLE 9. Premiums using PA under quadratic loss function and IGL under quadratic loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 958.7095 | 873.0867 | 1318.0627 | 1898.941 | 2592.533 |
| 2 | 896.9712 | 816.1151 | 1231.0910 | 1772.524 | 2418.809 |
| 3 | 842.7886 | 766.2240 | 1155.0526 | 1662.118 | 2267.165 |
| 4 | 794.8431 | 722.1587 | 1087.9913 | 1564.844 | 2133.638 |
| 5 | 752.1083 | 682.9460 | 1028.3929 | 1478.479 | 2015.154 |
| 6 | 713.7722 | 647.8194 | 975.0670 | 1401.271 | 1909.298 |
| 7 | 679.1846 | 616.1665 | 927.0647 | 1331.828 | 1814.142 |

TABLE 10. Premiums using PA under linex loss function and IGL under quadratic loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $a$=1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 924.1418 | 841.1782 | 1269.3408 | 1828.112 | 2495.192 |
| 2 | 866.6690 | 788.2056 | 1188.5459 | 1710.741 | 2333.943 |
| 3 | 816.0017 | 741.5993 | 1117.5704 | 1607.742 | 2192.518 |
| 4 | 770.9887 | 700.2660 | 1054.7118 | 1516.612 | 2067.464 |

| | | | | | |
|---|---|---|---|---|---|
| 5 | 730.7263 | 663.3507 | 998.6409 | 1435.398 | 1956.083 |
| 6 | 694.4947 | 630.1748 | 948.3052 | 1362.552 | 1856.238 |
| 7 | 661.7134 | 600.1930 | 902.8603 | 1296.835 | 1766.217 |

TABLE11.Premiums using PA under linex loss function and IGL under quadratic loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $a$=-0.3

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 968.7576 | 882.3723 | 1332.253 | 1919.579 | 2620.903 |
| 2 | 905.7425 | 824.2015 | 1243.427 | 1790.447 | 2443.434 |
| 3 | 850.5138 | 773.3316 | 1165.878 | 1677.830 | 2288.741 |
| 4 | 801.7004 | 728.4566 | 1097.571 | 1578.734 | 2152.699 |
| 5 | 758.2371 | 688.5663 | 1036.931 | 1490.847 | 2032.117 |
| 6 | 719.2837 | 652.8667 | 982.726 | 1412.356 | 1924.493 |
| 7 | 684.1680 | 620.7250 | 933.975 | 1341.822 | 1827.833 |

TABLE12. Premiums using PA under Entropy loss function and IGL under quadratic loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $p$=-0.2

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 89.91753 | 562.9445 | 1419.3163 | 2661.842 | 4294.249 |
| 2 | 84.16011 | 526.3932 | 1326.0473 | 2485.199 | 4007.143 |
| 3 | 79.10233 | 494.3594 | 1244.4574 | 2330.875 | 3756.495 |
| 4 | 74.62302 | 466.0466 | 1172.4629 | 2194.864 | 3535.755 |
| 5 | 70.62765 | 440.8366 | 1108.4504 | 2074.064 | 3339.849 |
| 6 | 67.04132 | 418.2415 | 1051.1506 | 1966.041 | 3164.787 |
| 7 | 63.80391 | 397.8714 | 999.5514 | 1868.852 | 3007.392 |

TABLE13. Premiums using PA under Entropy loss function and IGL under quadratic loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $p$=0.2

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 64.31164 | 493.8480 | 1309.4378 | 2511.456 | 4103.346 |
| 2 | 60.20601 | 461.8679 | 1223.5813 | 2345.089 | 3829.361 |
| 3 | 56.59740 | 433.8280 | 1148.4512 | 2199.711 | 3590.144 |
| 4 | 53.40011 | 409.0360 | 1082.1373 | 2071.558 | 3379.447 |
| 5 | 50.54717 | 386.9536 | 1020.1601 | 1957.716 | 3192.432 |
| 6 | 47.98548 | 367.1562 | 970.3553 | 1855.896 | 3025.294 |
| 7 | 45.67238 | 349.3038 | 922.7940 | 1764.274 | 2875.006 |

- *Discussion.*We note that the premiums using the entropy loss function are the most punitive with bad drivers and the lightest with good drivers in these tables. In addition to attracting drivers and requiring them to behave well in order to receive some very interesting premiums, this can assist insurers in establishing balance and solvability within the insurance companies. We observe that premiums using linex loss function in the over estimation are the most severe with bad drivers, and those in the under estimation are the most severe with good drivers.

5.3.2. **PremiumsusingInverse-GammaLindleyunderlinexlossfunctionfortheclaimseverity.**The premiums employing the Invers-Gamma Lindley distribution under the linex loss function for the severity claim are provided in this section's tables [14–16]. The Poisson-Akash distribution under several loss functions (Quadratic, Linex (a=1.1 and a=-0.3), and Entropy (p=0.2 and p=-0.2)) will be used to determine the claim frequency. Table 2's formulas (14), (15), and (16) were used to create the tables which follow.

TABLE14.Premiums using Poisson-Akash under Quadratic loss function and Inverse-Gamma Lindley Linex loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766 et a=1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 831.8621 | 2293.440 | 4652.594 | 7848.582 | 11874.183 |
| 2 | 778.2924 | 2143.785 | 4345.595 | 7326.086 | 11078.501 |
| 3 | 731.2787 | 2012.731 | 4077.189 | 6869.759 | 10383.949 |
| 4 | 689.6770 | 1896.979 | 3840.471 | 6467.716 | 9772.374 |
| 5 | 652.5964 | 1793.975 | 3630.097 | 6110.755 | 9229.703 |
| 6 | 619.3326 | 1701.703 | 3441.863 | 5791.645 | 8744.865 |
| 7 | 589.3213 | 1618.557 | 3272.421 | 5504.627 | 8309.040 |

TABLE15. Premiums using Poisson-Akash under Entropy loss function and Inverse-Gamma Lindley Linex loss function with $\gamma$=14.0125,$\alpha$=1.08,$\beta$=0.001766,a=1.1etp=-0.2

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 564.8014 | 1856.730 | 3993.547 | 6943.714 | 10703.656 |
| 2 | 528.6372 | 1736.175 | 3731.115 | 6482.922 | 9988.027 |
| 3 | 496.8676 | 1630.520 | 3501.545 | 6080.351 | 9363.274 |
| 4 | 468.7316 | 1537.137 | 3298.973 | 5725.549 | 8813.068 |
| 5 | 443.6353 | 1453.988 | 3118.860 | 5410.430 | 8324.761 |
| 6 | 421.1084 | 1379.464 | 2957.635 | 5128.638 | 7888.409 |
| 7 | 400.7732 | 1312.278 | 2812.450 | 4875.112 | 7496.091 |

TABLE16. Premiums using Poisson-Akash under Entropy loss function and Inverse-Gamma Lindley Linex loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, a=1.1 et p=0.2

| | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 403.9624 | 1628.833 | 3684.381 | 6551.416 | 10227.818 |
| 2 | 378.1736 | 1523.355 | 3422.806 | 6117.428 | 9544.896 |
| 3 | 355.5068 | 1430.872 | 3201.411 | 5738.193 | 8948.636 |
| 4 | 339.4235 | 1349.102 | 3044.823 | 5403.891 | 8423.462 |

| | | | | | |
|---|---|---|---|---|---|
| 5 | 317.5033 | 1276.269 | 2878.878 | 5106.922 | 7957.314 |
| 6 | 301.4125 | 1210.972 | 2730.301 | 4841.313 | 7540.715 |
| 7 | 286.8832 | 1152.090 | 2596.477 | 4602.308 | 7166.113 |

- *Discussion.*According to these tables, premiums that use the entropy loss function are the gentler with all drivers, whereas those that use the quadratic loss function are the harshest with both good and bad drivers.

5.3.3. **Premiums using Inverse-Gamma Lindley under Entropy loss functionfortheclaimseverity.**This part introduces the tables [17-22] which represents the premiums using the Invers-Gamma Lindley distribution undertheentropylossfunctionfortheseverityclaim.Poisson-Akash distribution under different loss functions (Quadratic, Linex (a=1.1 and a=-0.3) and Entropy (p=0.2 and p=-0.2)) will be employed fortheclaimfrequency.The formulas (17), (17) and (18) from table 3 were used to create the tables bellow.

TABLE17.Premiums using Poisson-Akash under quadratic loss functionandInverse-GammaLindleyunderentropylossfunction with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $p$= -1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 551.1474 | 556.9714 | 860.9861 | 1255.7779 | 1728.308 |
| 2 | 515.6550 | 520.6273 | 804.1743 | 1172.1782 | 1612.496 |
| 3 | 484.5062 | 488.8001 | 754.5044 | 1099.1657 | 1511.402 |
| 4 | 456.9431 | 460.6893 | 710.6986 | 1034.8385 | 1422.386 |
| 5 | 432.3755 | 435.6742 | 671.7677 | 977.7245 | 1343.400 |
| 6 | 410.3367 | 413.2658 | 636.9341 | 926.6668 | 1272.831 |
| 7 | 390.4528 | 393.0733 | 605.5780 | 880.7437 | 1209.395 |

TABLE18.Premiums using Poisson-Akash under linex loss function and Inverse-Gamma Lindley under entropy loss function with$\gamma$=14.0125,$\alpha$=1.08,$\beta$=0.001766,$a$=1.1,$p$=-1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 531.2749 | 536.6159 | 829.1599 | 1208.9383 | 1663.416 |
| 2 | 498.2347 | 502.8229 | 776.3830 | 1131.3205 | 1555.920 |
| 3 | 469.1068 | 473.0912 | 730.0203 | 1063.2070 | 1461.639 |
| 4 | 443.2296 | 446.7232 | 688.9597 | 1002.9424 | 1378.272 |
| 5 | 420.0833 | 423.1737 | 652.3331 | 949.2348 | 1304.020 |
| 6 | 399.2543 | 402.0097 | 619.4528 | 901.0616 | 1237.459 |
| 7 | 380.4086 | 382.8833 | 589.7672 | 857.6029 | 1177.446 |

TABLE19.Premiums using Poisson-Akash under linex loss function and Inverse-Gamma Lindley under entropy loss function with$\gamma$=14.0125,$\alpha$=1.08,$\beta$=0.001766,$a$=-0.3,$p$=-1.1

| | Claimnumber |
|---|---|

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | 556.9239 | 562.8950 | 870.2552 | 1269.4263 | 1747.221 |
| 2 | 520.6975 | 525.7859 | 812.2324 | 1184.0306 | 1628.9912 |
| 3 | 488.9473 | 493.3343 | 761.5761 | 1109.5563 | 1525.1786 |
| 4 | 460.8853 | 464.7070 | 716.9560 | 1044.0235 | 1435.7094 |
| 5 | 435.8989 | 439.2596 | 677.3449 | 985.9903 | 1352.9708 |
| 6 | 413.5051 | 416.4857 | 641.9372 | 933.9975 | 1282.9960 |
| 7 | 393.3177 | 396.0918 | 610.0919 | 887.3526 | 1218.5522 |

TABLE20. Premiums using Poisson-Akash under entropy loss function and Inverse-Gamma Lindley under entropy loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $s$ = -1.1, $p$= -1.1

| T | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 574.6295 | 569.9507 | 874.8278 | 1271.1963 | 1745.464 |
| 2 | 537.5994 | 532.7376 | 817.0750 | 1186.5394 | 1628.472 |
| 3 | 505.1048 | 500.1524 | 766.5856 | 1112.6062 | 1526.351 |
| 4 | 476.3537 | 471.3745 | 722.0595 | 1047.4702 | 1436.431 |
| 5 | 450.7294 | 445.7675 | 682.4906 | 989.6401 | 1356.644 |
| 6 | 427.7445 | 422.8320 | 647.0879 | 937.9439 | 1285.361 |
| 7 | 407.0082 | 402.1624 | 615.2208 | 891.4481 | 1221.286 |

TABLE21. Premiums using Poisson-Akash under entropy loss function and Inverse-Gamma Lindley under entropy loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $s$= -0.2, $p$= -1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 358.4347 | 450.7562 | 748.3329 | 1130.7149 | 1589.443 |
| 2 | 335.4842 | 421.4892 | 699.1569 | 1055.6795 | 1483.175 |
| 3 | 315.3225 | 395.8393 | 656.1387 | 990.1247 | 1390.402 |
| 4 | 297.4668 | 373.1690 | 618.1797 | 932.3488 | 1308.699 |
| 5 | 281.5402 | 352.9830 | 584.4292 | 881.0347 | 1236.188 |
| 6 | 267.2442 | 334.8908 | 554.2180 | 835.1478 | 1171.392 |
| 7 | 254.3390 | 318.5802 | 527.0123 | 793.8635 | 1113.135 |

TABLE22. Premiums using Poisson-Akash under entropy loss function and Inverse-Gamma Lindley under entropy loss function with $\gamma$=14.0125, $\alpha$=1.08, $\beta$=0.001766, $s$= 0.2, $p$= -1.1

| t | Claimnumber | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 1 | 256.3630 | 395.4298 | 690.3996 | 1066.8330 | 1518.783 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | 239.9969 | 369.8230 | 645.1319 | 996.1624 | 1417.373 |
| 3 | 225.6120 | 347.3711 | 605.5196 | 934.4077 | 1328.831 |
| 4 | 212.8668 | 327.5199 | 570.5557 | 879.9699 | 1250.845 |
| 5 | 201.4942 | 309.8383 | 539.4600 | 831.6114 | 1181.624 |
| 6 | 191.2827 | 293.9862 | 511.6187 | 788.3598 | 1119.761 |
| 7 | 182.0620 | 279.6916 | 486.5421 | 749.4401 | 1064.134 |

- *Discussion.*Referring to the tables, the premiums with the entropy loss function with "p=-1.1" and those with the linex loss function in the underestimation are nearly identical; these premiums are highest for poor drivers. The results also demonstrate that premiums with good and bad drivers are lower when applying the entropy loss function with "p=-0.2" and "p=0.2."

## 6. Conclusion

This study presented the Bayesian bonus-malus premiums for claim frequency and claim severity. Using the Bayesian approach, we employed the Poisson-Akash distribution for claim frequency and the Invers-Gamma Lindley distribution for claim severity. Different loss functions (quadratic, linex, and entropy) were additionally introduced in this work. Using the R software and an actual car insurance dataset, we computed the premiums for the frequency only and for both the frequency and the severity of the claim. We established the premiums using the several loss functions.

We found from the numerical application that premiums that use distinct loss functions for the frequency and severity of claims provide greater outcomes than those that apply the same loss function. This numerical research demonstrated that using various loss functions might provide flexible outcomes that could be very beneficial to the insurer. The insurer has a wide range of choices thanks to the wide variety of these outcomes. The insurance company selects the loss function based on its goal, offering excellent drivers the highest rewards or penalizing bad drivers most severely, deciding how to draw clients and make them attractive offers, maintaining the organization's solvability, and maintaining equilibrium.

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