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Reflection at the Free Surface of Generalized Thermoelastic Medium with Voids Under Three Phase Lag Effect

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Abstract:
In this paper, propagation of plane waves in an isotropic, homogeneous, generalized magneto-thermo-viscoelastic semi-infinite medium (having stress-free insulated boundary) containing a distribution of vacuous pores (voids) has been investigated. Basic governing equations are based on the linear theory of thermoelasticity, namely, three-phase-lag (3PL) thermoelasticity to account for finite velocity of the temperature. It is found that three compressional waves and a shear vertical (SV) wave can exist in the medium adopted. Reflection phenomena of compressional and shear waves from the free surface of the medium is considered. The formulae for reflection coefficients of various reflected waves have been obtained in the closed form. The numerical values of the modulus of reflection coefficients are presented graphically.

Keywords: Three-phase-lag thermoelasticity, Reflection, Voids.

Introduction

Concept of generalized thermoelasticity has drawn the attention of many researchers during the last decades. Series of generalized theories of thermoelasticity has been developed to overcome the shortcomings inherent in the classical coupled dynamical theory of elasticity. These theories are characterized by the finite speed of propagation of thermal disturbance. Generalized thermoelastic model known as dual-phase-lag model was developed by Tzou (1995) by considering micro-structural effects into the delayed response in time at the macroscopic level by taking into account that the increase in the lattice temperature is delayed due to phonon-electron interactions on the macroscopic level. Tzou introduced two different phase lags, one for the heat flux vector,τ_q and the other for the temperature gradient,τ_T. For this model, classical Fourier's law is thus modified to $\vec{q}(p, t+\tau_q) = -K[\vec{\nabla}T(p, t+\tau_T)]$. The delay time τ_q is interpreted as the relaxation time due to fast transient effects of thermal inertia. The other delay time $\tau_T$is interpreted as that caused by the microstructural interactions. Stability of the dual-phase-lag heat conduction is discussed by Quintanilla and Racke (2006). Sixth and the most recent development in generalized thermoelasticity is three-phase-lag thermoelastic model. Roychoudhuri (2007) established this model by modifying heat conduction law to the form $\vec{q}(p, t+\tau_q) = -[K\vec{\nabla}T(p, t+\tau_T) + K^*\vec{\nabla}v(p, t+\tau_v)]$. Here$\tau_v$, the delay time in thermal displacement gradient is also introduced in addition to τ_q and τ_T. Stability of three-phase-lag heat conduction equation and the relations among the three material parameters are discussed by Quintanilla and Racke (2008). Mukhopdhyay and Kumar (2010) analysed the effects of phase lags on wave propagation in a thick plate under axisymmetric temperature distribution. A magneto-thermoelastic problem in an unbounded, perfectly conducting medium with three phase lags has been discussed by Das and Kanoria (2012).

Wave propagation in a porous media has its significance in diversified fields of science and engineering. The general non-linear theory of elastic materials with voids has been formulated by Nunziato and Cowin (1979). Linearized version of the said theory is developed by Cowin and Nunziato (1983), where void volume has been included as an additional kinematic variable. This theory reduces to the classical theory of elasticity in the limiting case when the void volume vanishes. Iesan (1986) established a theory of linear thermoelastic materials with voids Singh and Tomar (2007) investigated the propagation of plane waves in a thermoelastic material with voids. Many problems of waves and vibrations are studied by several researchers in an elastic material with voids. Some of them are Abo-Dahab *et al.* (2013) , Sharma and Kumar (2013), Malik et. al.(2022), and Kundu et al. (2022). The two-dimensional deformation of a generalized thermoelastic half-space with voids and microtemperatures under the action of a mechanical force was investigated by L. Rani (2023).

In this paper, we have studied reflection of plane waves at the free surface of generalized thermoelastic medium with voids. The study is in the context of three phase lag thermoelasticity. It is found that there exist three sets of coupled dilatational waves and one set of coupled transversal waves. The reflection coefficients of various reflected waves are computed numerically for a specific model and their variations with angle of incidence are presented graphically. The effect of viscosity on reflection coefficients is demonstrated graphically.

Nomenclature

τ_{ij}	Components of stress tensor
λ, μ	Lame's constants
β	$(3\lambda + 2\mu)\alpha_t$
α_t	Coefficient of linear thermal expansion
c_e	Specific heat at constant strain
K	Thermal conductivity
K^*	$\frac{c_e(\lambda+2\mu)}{4}$, material constant
T	Absolute temperature
T_0	Reference temperature
Θ	Temperature deviation from the reference temperature

$$\Theta = T - T_0, \left|\frac{\Theta}{T_0}\right| \ll 1$$

u_i	Components of displacement vector
ρ	Density of the medium
e_{ij}	Components of strain tensor
e_{kk}	e, cubical dilatation
ϕ	Change in volume fraction field
δ_{ij}	Kronecker delta function
h_i	Components of equilibrated stress vector
q_i	Components of heat flux vector
α, b, ξ_1	Void material parameters
m	Thermo-void coefficient
χ	Equilibrated inertia
t	Time variable

Basic equations and Problem Formulation

In this section, a resumé of the basic equations has been presented for the analysis of wave propagation in a generalized thermo-viscoelastic medium containing voids. The constitutive relation is

$$\tau_{ij} = \lambda\delta_{ij}u_{k,k} + \mu\left(u_{i,j} + u_{j,i}\right) + b\phi\delta_{ij} - \beta\Theta\delta_{ij}, \tag{1}$$

The balance of linear momentum in the absence of body forces may be written as

$$\rho\ddot{u}_i = \tau_{ji,j} \tag{2}$$

Again, the volume fraction field ϕ satisfies the following equation [Iesan (1986)]

$$\alpha\nabla^2\phi - b(\nabla\cdot\vec{u}) - \xi_1\phi + m\Theta = \rho\chi\ddot{\phi} \tag{3}$$

The heat equation corresponding to generalized thermoelasticity theory with three phase lags [Roychoudhuri (2008)] is

$$\left[K^*\left(1 + \tau_v\frac{\partial}{\partial t}\right) + K\frac{\partial}{\partial t}\left(1 + \tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2\Theta = \left(1 + \tau_q + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e\ddot{\Theta} + \beta T_0\ddot{e} + mT_0\ddot{\phi}\right)$$
(4)

In the above, the usual summation convention on repeated indices has been followed. Indices following comma indicate partial derivative with respect to those indices.

We consider the propagation of plane waves in a homogeneous, isotropic, generalized semi-infinite solid occupying the region $z \geq 0$. Solid is assumed to be composed of material possessing voids. The surface $z = 0$ is assumed to be unstressed, unstrained, thermally insulated and initially at uniform temperature T_0. Rectangular Cartesian coordinate system has been chosen with origin on the surface $z = 0$. The z-axis is pointing vertically downward into the medium.

Assuming xz-plane as the plane of incidence, the wave motion will be the same in every plane parallel to xz-plane which in turn implies that all physical quantities will be independent of y-coordinate. Thus the displacement vector ($\vec{u} = (u, 0, w)$) components, change in void volume fraction ϕ and thermal parameter Θ are $u = u(x, z, t), v = 0, w = w(x, z, t)$, $\phi = \phi(x, z, t)$ and $\Theta = \Theta(x, z, t)$.

Hence the dynamical equations for xz-plane may be obtained as follows

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u - \beta\frac{\partial \Theta}{\partial x} + b\frac{\partial \phi}{\partial x}, \tag{5}$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial z} + \mu\nabla^2 w - \beta\frac{\partial \Theta}{\partial z} + b\frac{\partial \phi}{\partial z}. \tag{6}$$

For convenience, we will make use of the following non-dimensional quantities

$$(x', z') = \frac{\bar{\omega}}{c_1}(x, z), \left(t', \tau_q', \tau_v', \tau_T', \beta'\right) = \bar{\omega}(t, \tau_q, \tau_v, \tau_T, \beta) ,$$

$$(u', w') = \frac{\rho\bar{\omega}c_1}{\beta T_0}(u, w),\ \Theta' = \frac{\Theta}{T_0},\ \phi' = \frac{\bar{\omega}^2\chi}{c_1^2}\phi, (\tau_{zx}', \tau_{zz}') = \frac{1}{\beta T_0}(\tau_{zx}, \tau_{zz}). \tag{7}$$

where $\bar{\omega} = \frac{\rho c_e c_1^2}{K}, c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$ are the characteristic frequency and longitudinal wave velocity in the medium respectively.

For investigation of plane waves, the potentials $\psi_1(x, z, t)$,$\psi_2(x, z, t)$ are introduced. They are related to displacement components u and w by the relation

$$u = \frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial z}, w = \frac{\partial\psi_1}{\partial z} + \frac{\partial\psi_2}{\partial x} . \tag{8}$$

Plugging the above potentials from Eq.(8) into non-dimensional form of the Equations (1)-(4), we get

$$\left(1 + \alpha_1\frac{\partial}{\partial t}\right)\nabla^2\psi_2 = a_1\frac{\partial^2\psi_2}{\partial t^2}, \tag{9}$$

$$\left[\frac{\lambda_e}{\mu_e} + 2\right]\nabla^2\psi_1 - a_1\frac{\partial^2\psi_1}{\partial t^2} - \left(1 + \beta\frac{\partial}{\partial t}\right)\gamma^2\Theta - a_2\phi = 0, \tag{10}$$

$$\nabla^2\phi - a_3(\nabla^2\psi_1) - a_4\phi + a_5\Theta - a_6\ddot{\phi} = 0, \tag{11}$$

$$\left[a_7\left(1 + \tau_v\frac{\partial}{\partial t}\right) + \frac{\partial}{\partial t}\left(1 + \tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2\Theta = \left(1 + \tau_q + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\ddot{\Theta} + a_8(1 + \beta\frac{\partial}{\partial t})\ddot{\psi}_1 + a_9\ddot{\phi}\right). \tag{12}$$

where

$$\gamma^2 = \frac{\rho c_1^2}{\mu_e}, a_1 = \gamma^2,\ \ a_2 = \frac{\rho b c_1^4}{\beta T_0\mu\bar{\omega}^2\chi}, a_3 = \frac{b\chi\beta T_0}{\alpha\rho c_1^2}, a_4 = \frac{\xi_1 c_1^2}{\bar{\omega}^2\alpha},\ a_5 = \frac{mT_0\chi}{\alpha},$$

$$a_6 = \frac{\rho c_1^2 \chi}{\alpha}, a_7 = \frac{K^*}{K\bar{\omega}},\ a_8 = \frac{\beta^2 T_0}{K\rho\bar{\omega}},\ a_9 = \frac{m c_1^4}{K\chi\bar{\omega}^3}\,.$$

Eq. (9) is uncoupled while equations (10)-(12) are coupled in ψ_1, Θ and ϕ.

To suit the actual situation of the problem, we seek solutions of differential equations (9)-(12) in the following forms:

$$[\psi_1, \psi_2, \Theta, \phi](x, z, t) = [\bar{\psi}_1, \bar{\psi}_2, \bar{\Theta}, \bar{\phi}]\, exp\{\iota k(x \sin\,\theta - z \cos\,\theta) - \iota\omega t\}, \tag{13}$$

where k is the wave number and ω is angular frequency connected by the relation $\omega = kV$, V being the phase velocity and $(sin\,\theta\,, -\,cos\,\theta)$ denotes the projection of wave normal of incident wave onto the *xz*-plane. Barred quantities are the amplitudes of the field quantities.

Injecting Eq.(13) into Eqs.(10)-(12), we get respectively

$$(a_1\omega^2V^2 + \iota\omega^3 a_{10})\bar{\psi}_1 + (\iota\omega V^2\gamma^2\beta_{00})\bar{\Theta} + a_2V^2\bar{\phi} = 0, \tag{14}$$

$$\omega^2 a_3\bar{\psi}_1 + a_5V^2\bar{\Theta} + (a_6\omega^2V^2 - \omega^2 - a_4V^2)\bar{\phi} = 0, \tag{15}$$

and $$\iota\omega^3\beta_{00}a_8a_{11}\bar{\psi}_1 + (\omega^2\tau_{T0} + \iota\omega a_7\tau_{v0} + V^2a_{11})\bar{\Theta} + a_9a_{11}V^2\bar{\phi} = 0, \tag{16}$$

where $\beta_{00} = \beta + \frac{\iota}{\omega}, \quad \tau_{v0} = \tau_v + \frac{\iota}{\omega}, \quad \tau_{T0} = \tau_T + \frac{\iota}{\omega}\,,$

$$a_{10} = \left(\frac{\lambda+2\mu}{\mu}\right),\ a_{11} = 1 - \iota\omega\tau_q - \frac{\tau_q^2\omega^2}{2}.$$

The condition for the existence of non-trivial solution of the system of equations (14)-(16) provides us

$$V^6 + AV^4 + BV^2 + C = 0 \tag{17}$$

where $A = \frac{A'}{F}, B = \frac{B'}{F}, C = \frac{C'}{F}$,

$$F = a_1a_6a_{11}\omega^4 - a_1a_4a_{11}\omega^2 - a_1a_5a_9a_{11}\omega^2,$$

$$A' = a_1a_6\tau_{T0}\omega^6 + (\iota a_1a_6a_7\tau_{v0} + \iota a_6a_{10}a_{11})\omega^5 - (a_1a_4\tau_{T0} + a_1a_{11})\omega^4$$
$$-(\iota a_1a_4a_7\tau_{v0} + \iota a_4a_{10}a_{11} - \iota a_3a_9a_{11}\beta_{00}\gamma^2 + \iota a_5a_9a_{10}a_{11})\omega^3 - a_2a_3a_{11}\omega^2\,,$$

$$B' = \iota a_6a_{10}\tau_{T0}\omega^7 - (a_1\tau_{T0} + a_6a_7a_{10}\tau_{v0})\omega^6 - (\iota a_4a_{10}\tau_{T0} + \iota a_1a_7\tau_{v0} +$$
$$\iota a_{10}a_{11})\omega^5$$
$$-(a_2a_3\tau_{T0} - a_4a_7a_{10}\tau_{v0})\omega^4 - \iota a_2a_3a_7\tau_{v0}\omega^3,$$

$$C' = a_7a_{10}\tau_{v0}\omega^6 + (\iota a_8a_{11}\beta_{00} - \iota a_{10}\tau_{T0})\omega^3.$$

Using the transformation $V^2 = Y$ in Eq. (17), we obtain

$$Y^3 + AY^2 + BY + C = 0. \tag{18}$$

Eq. (18) is a cubic in V^2, which implies that there shall be three dilatational waves travelling with three different velocities.

Using Cardan's method in Eq.(18), we get

$$Z^3 + 3HZ + G = 0, \tag{19}$$

where $Z = Y + \frac{A}{3},\ H = \frac{B}{3} - \frac{A^2}{9}$ and $G = \frac{2A^3}{27} - \frac{AB}{3} + C.$

Since the quantities A, B and C are complex, therefore the coefficients H and G are complex. Thus the roots of Eq.(19) and hence of Eq.(18) are given by

$$V_1^2 = S - \frac{A}{3},\quad V_2^2 = -\frac{1}{2}S + \iota\frac{\sqrt{3}}{2}T - \frac{A}{3},\quad V_3^2 = -\frac{1}{2}S - \iota\frac{\sqrt{3}}{2}T - \frac{A}{3} \tag{20}$$

where $S = U + V,\ T = U - V,\ \ U^3 = \frac{1}{2}\left[-G + \sqrt{G^2 + 4H^3}\right]$ and $V = \frac{-H}{U}$.

$V_{1,2,3}$ are the speeds of propagation of three coupled dilatational waves namely longitudinal displacement wave (P_1), thermal wave (P_2) and longitudinal void volume fraction wave (P_3). It can be easily observed that speeds of all the coupled longitudinal waves are influenced by three phase lags (τ_q,$\tau_\upsilon$andτ_T) and void parameters. Eq. (9) corresponds to the uncoupled transverse displacement wave (SV) whose velocity is given by

$$V_4 = \sqrt{\frac{1}{a_1}}. \tag{21}$$

Reflection at the free surface

Here, we shall consider the problem of incidence of a coupled longitudinal wave on the free and thermally insulated boundary of a thermoelastic half-space with voids.

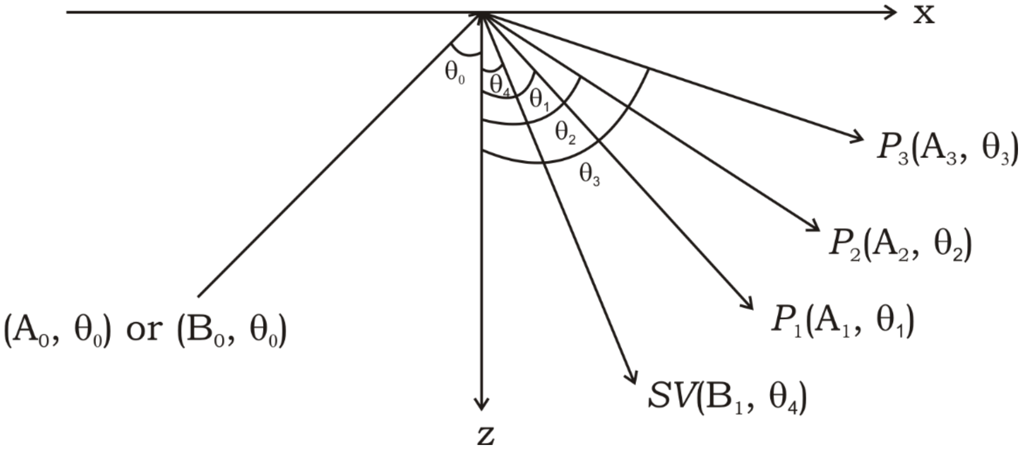

Fig. 1 Schematic diagram for the problem

We assume that a set of coupled longitudinal waves of amplitude A_0 propagating with the phase velocity V_1 becomes incident obliquely at the free plane surface, making an angle θ_0 with the normal. In order to satisfy the boundary conditions, this incident coupled longitudinal wave gives rise to the following reflected waves:

(i) Three coupled longitudinal waves with amplitudes A_1, A_2 and A_3 propagating with speeds $V_{1,2,3}$ and making angles $\theta_{1,2,3}$ respectively with the normal.

(ii) A transverse wave of amplitude B_1 propagating with speed V_4 making an angle θ_4 with the normal.

Full structure of the wave field consisting of the incident and reflected waves can be written as:

$$\begin{aligned}\psi_1 &= A_0\, exp\{\iota k_1(x \sin\theta_0 - z\cos\theta_0) - \iota\omega t\} \\ &\quad + \textstyle\sum_{i=1}^{3} A_i\, exp\{\iota k_i(x\sin\theta_i + z\cos\theta_i) - \iota\omega t\},\end{aligned} \tag{22}$$

$$\begin{aligned}\Theta &= a_1^* A_0\, exp\{\iota k_1(x\sin\theta_0 - z\cos\theta_0) - \iota\omega t\} \\ &\quad + \textstyle\sum_{i=1}^{3} a_i^* A_i\, exp\{\iota k_i(x\sin\theta_i + z\cos\theta_i) - \iota\omega t\},\end{aligned} \tag{23}$$

$$\begin{aligned}\phi &= b_1^* A_0\, exp\{\iota k_1(x\sin\theta_0 - z\cos\theta_0) - \iota\omega t\} \\ &\quad + \textstyle\sum_{i=1}^{3} b_i^* A_i\, exp\{\iota k_i(x\sin\theta_i + z\cos\theta_i) - \iota\omega t\},\end{aligned} \tag{24}$$

$$\psi_2 = B_1\, exp\{\iota k_4(x\sin\theta_4 + z\cos\theta_4) - \iota\omega t\}, \tag{25}$$

where a_i^* and b_i^* $(i = 1,2,3)$ are the coupling parameters. Their expressions are given by

$$a_i^* = \frac{(a_1a_6\omega^4 - a_1a_4\omega^2)V_i^4 + (\iota a_6a_{10}\omega^5 - a_1\omega^4 - \iota a_4a_{10}\omega^3 - a_2a_3\omega^2)V_i^2 - \iota a_{10}\omega^5}{(g_1V_i^4 + g_2V_i^2)},$$

$$b_i^* = \frac{(-a_1a_5\omega^2)V_i^4 + \iota\omega^3(a_3\beta_{00}\gamma^2 - a_5a_{10})V_i^2}{(g_1V_i^4 + g_2V_i^2)},$$

where $g_1 = \iota\omega a_4\beta_{00}\gamma^2 - \iota\omega^3 a_6\beta_{00}\gamma^2 + a_2a_5$, $g_2 = \iota\omega^3\beta_{00}\gamma^2$.

The amplitudes A_1, A_2, A_3 and B_1 can be determined from the boundary conditions at $z = 0$. The surface $z = 0$ is assumed to be traction free and thermally insulated so that there is no variation of temperature and volume fraction field on it. Therefore, the boundary conditions are written as

$$\tau_{zz} = 0,\ \tau_{zx} = 0,\ \frac{\partial\Theta}{\partial z} = 0, \frac{\partial\phi}{\partial z} = 0 \quad \text{at}\ z = 0 \tag{26}$$

The above boundary conditions are identically satisfied if and only if

$$k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 = k_4\sin\theta_4 \text{ and } k_1V_1 = k_2V_2 = k_3V_3 = k_4V_4\,. \tag{27}$$

Now, substituting the values of potentials ψ_1, Θ, ϕ and ψ_2 from Eqs.(22)-(25) into the above boundary conditions one can obtain the following system of four simultaneous equations

$$\sum A_{ij}Z_j = C_i, \qquad (i,j = 1,2,3,4), \tag{28}$$

where $A_{1j} = \left[a_{12} - \delta_4 + a_{13}\cos^2\theta_j + b_1\frac{b_j^*}{k_j^2} + a_{14}\frac{a_j^*}{k_j^2}\right]\frac{k_j^2}{k_1^2}$,

$$A_{2j} = \sin 2\theta_j \frac{k_j^2}{k_1^2}, \quad A_{3j} = \frac{a_j^*\cos\theta_j}{k_j}\frac{k_j^2}{k_1^2}, \quad A_{4j} = \frac{b_j^*\cos\theta_j}{k_j}\frac{k_j^2}{k_1^2}. \ (j = 1,2,3).$$

$$A_{14} = a_{13}\sin\theta_4\cos\theta_4\frac{k_4^2}{k_1^2},\ A_{24} = -\cos 2\theta_4\frac{k_4^2}{k_1^2},\ A_{34} = 0,\ A_{44} = 0,$$

$$a_{12} = -\omega\delta_2,\ \ a_{13} = -2\delta_3,\quad a_{14} = \iota\omega\beta_{00},$$

$$C_1 = -A_{11},\ \ C_2 = A_{21},\ \ C_3 = A_{31},\ \ C_4 = A_{41},$$

$$Z_1 = \frac{A_1}{A_0},\quad Z_2 = \frac{A_2}{A_0},\quad Z_3 = \frac{A_3}{A_0},\quad Z_4 = \frac{B_1}{A_0}.$$

Here, $Z_i\ (i = 1,2,3,4)$ are the reflection coefficients for the incidence of coupled longitudinal wave travelling with speed V_1 .

Numerical results and discussion

Following Dhaliwal and Singh (1980), we consider an example where magnesium crystal like material is modeled as an isotropic thermoelastic solid with voids for numerical computations. For this test material, elastic and thermal constants are

$\lambda = 2.17 \times 10^{10} Nm^{-2},\ \mu = 3.278 \times 10^{10} Nm^{-2},\ \beta = 2.68 \times 10^{6}\ Nm^{-2}\, degree^{-1}\ ,\ T_0 = 298K,$
$K = 1.7 \times 10^{2} Wm^{-1}\, degree^{-1},\ c_e = 1.04 \times 10^{3} JKg^{-1}\, degree^{-1},\ \rho = 1.74 \times 10^{3} Kgm^{-3}.$

Void parameters are given by

$\alpha = 3.688 \times 10^{-5} N,\quad \xi_1 = 1.475 \times 10^{10} Nm^{-2},\ \ \chi = 1.753 \times 10^{-15} m^2,$
$b = 1.13849 \times 10^{10} Nm^{-2},\quad m = 2 \times 10^{6} Nm^{-2}\, degree^{-1}.$

Other constants involved in the problem are:

$\tau_v = 0.1,\ \tau_q = 0.2,\ \tau_T = 0.15,\ \omega = 3.5 \times 10^{12}.$

To discuss the nature of dependence of reflection coefficients on the angle of incidence, we have computed their expressions using Matlab software. All the relative amplitudes are found to be complex valued.

All the figures have been taken in the context of thermoelastic theory based on:

(i) Three-phase-lag model with voids (3PLV)

(ii) GN-III model with voids (GN3V)

(iii) Three-phase-lag model without voids (3PLWV).

Figures 2-5 are drawn for incidence of a P-wave of speed V_1. Considered range for angle of incidence is $0° \leq \theta \leq 90°$. Figure 2 shows the variations in the absolute values of reflection coefficient Z_1. It is clear that $|Z_1|$has value almost equal to unity during the whole range of incidence for all the three models considered. Despite of this, difference in trends of variations of $|Z_1|$ for 3PLV, GN3V and 3PLWV models is easily noticeable from the graph. Presence of voids increases the values of $|Z_1|$in the entire range of angle of incidence. Absence of three phase lags increases the values till $\theta = 45°$and after that causes a decrement in the values.

In Figure 3, the reflection coefficients $|Z_2|$ decrease with increasing angle of incidence, which are affected due to void parameters and relaxation times. In the absence of voids and three relaxation times τ_q,$\tau_v$andτ_T, the values of $|Z_2|$ rise at each angle of incidence. Variations in the values of $|Z_3|$ against the angle of incidence are depicted in the figure 4. Magnitude of Z_3 is very small for all the three cases. Also pattern of variations in the graph is similar to that for $|Z_2|$.

Figure 5 characterizes the behaviour of $|Z_4|$with increasing angle of incidence. Its trend of variations is different from the remaining reflection coefficients. $|Z_4|$ starts from zero value, increases till it attains its maximum value at $\theta = 45°$and then decreases smoothly to zero. Absence of relaxation times (three phase lags) causes an increment in the numerical values of Z_4 while absence of voids decreases the numerical values.

Concluding remarks

The facts extracted from our study can be concluded as:

(i) It can be checked out from the calculatory part that amplitude ratios are independent of the wavelength of the incident wave but depend only upon angle of incidence and wave numbers of the incident wave.

(ii) At grazing incidence ($\theta = 90°$) of longitudinal wave of speed V_1, no other reflected wave appears except the longitudinal wave of the same amplitude as that of incident wave. In case of normal incidence ($\theta = 0°$), the reflected longitudinal wave having maximum amplitude is the wave of amplitude equal to that associated with incident *P*-wave. Thus at normal and grazing incidence, it appears that incident *P*-wave is reflected as a *P*-wave.

(iv) Voids and three phase lag parameters are having a pronounced effect on reflection coefficients.

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Figure Captions:

Fig. 2 Variation of the modulus of reflection coefficient Z_1 with the angle of incidence of coupled longitudinal wave with speed V_1

Fig. 3 Variation of the modulus of reflection coefficient Z_2 with the angle of incidence of coupled longitudinal wave with speed V_1.

Fig. 4 Variation of the modulus of reflection coefficient Z_3 with the angle of incidence of coupled longitudinal wave with speed V_1.

Fig. 5 Variation of the modulus of reflection coefficient Z_4 with the angle of incidence of coupled longitudinal wave with speed V_1.

Fig. 2

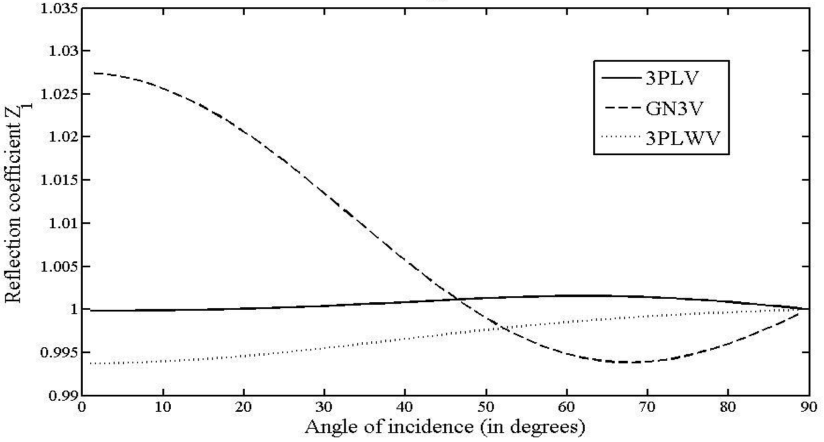

Fig. 3

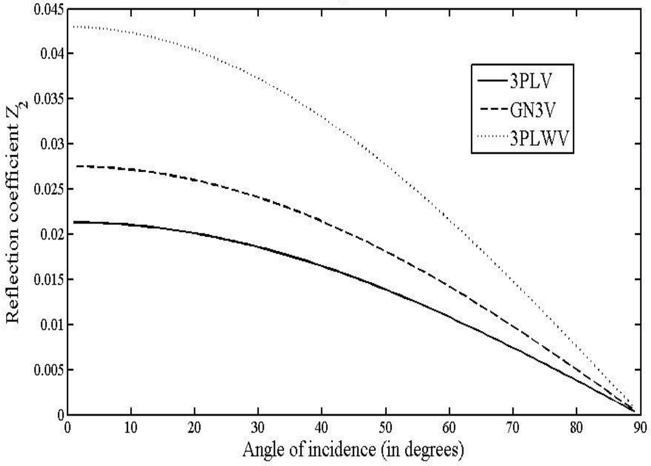

Fig. 4

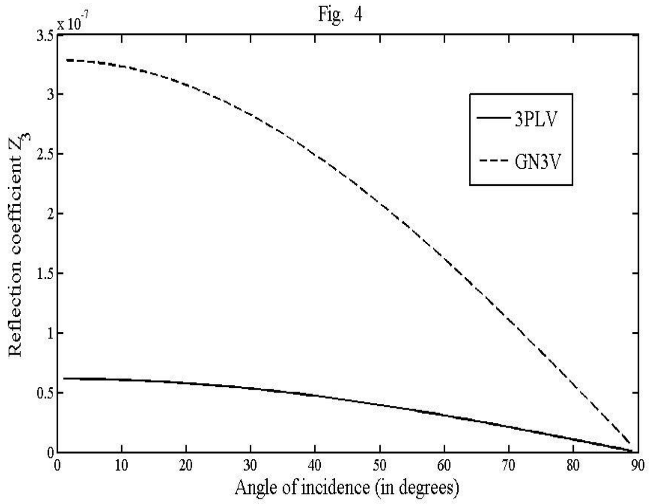

Fig. 5