

Next-To-Leading Order Scalars Dispersion Relations in HTL Scalar QED

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ABSTRACT

We investigate the next-to-leading order (NLO) contributions to scalar dispersion relations in hot scalar quantum electrodynamics (SQED) within the framework of hard-thermal-loop (HTL) resummed perturbation theory. Using the real-time formalism, we derive analytic expressions for the effective propagators and compute the complete NLO retarded scalar self-energy for slow-moving scalars with soft momentum. The results provide compact expressions for both the real and imaginary parts of the scalar self-energy, corresponding respectively to corrections to the energy and damping rates. Contributions from both longitudinal and transverse photon modes are evaluated.

Keywords: Soft scalars, Energy and Damping Rate; Resummation; Hard Thermal Loop.

1. Introduction

Difficulties arise when standard perturbation theory is used within the context of gauge theories at high temperature, leading to gauge dependence and infrared divergence of physical quantities that exhibit [1-4]. The problem was resolved in [5-12], which demonstrated that for consistent calculations at high temperatures, it is imperative to utilize an effective perturbation approach that incorporates the so-called hard thermal loops (HTL) into the dressed propagators and vertices [5-12]. This method is also necessary in abelian theories such as quantum electrodynamics (QED) [12], scalar quantum electrodynamics (SQED) [13], and in ϕ^4 -theory [14]. Within this theoretical framework, it was established that the zero-momentum transverse gluon damping rate is finite and positive [15]. In a series of works, we have used the imaginary time formalism to study the infrared behavior of the gluon and quark damping rates [16-24]. The results have indicated that there are difficulties in the infrared sector. A similar observation has been done in the context of scalar electrodynamics [25]. In order to investigate the infrared characteristics more comprehensively, we have determined the next-to-leading order contributions to both the longitudinal gluon and the retarded quark self-energy within the framework of hard thermal loop (HTL) summed perturbation theory of massless Quantum Chromodynamics (QCD) at high temperatures, using real-time formalism [26-28]. Furthermore, the retarded fermion self-energy has been computed within the context of HTL summed perturbation theory of massless Quantum Electrodynamics (QED) [29].

To look further into the infrared behavior, we propose to calculate the next-to-leading order dispersion relations for slow-moving scalar with soft momentum at high-temperature scalar quantum electrodynamics (Scalar QED). We determine the analytic expression of the next-to-leading order for slow-moving scalars with soft momentum. We use the real time formalism (RTF) of finite-temperature quantum field theory [30-33] in the context of hard-thermal-loop summed perturbation of massless SQED at high temperature. We derive the expressions of the effective propagators in RTF that contribute to the complete next-to-leading order contribution of retarded scalar self-energy. A compact analytic expression for the complete next-to-leading retarded scalar self-energy is given. The real part and the opposite of the imaginary part of the retarded scalars self-energy are related to the next-to-leading order contributions of energy and damping rate respectively.

2. Effective expansion

In the Landau gauge, the effective photon propagator followed from the resummation of the HTL photon self-energy. To leading order the effective photon propagator is given by:

$$\Delta_{ra/ar}^{\mu\nu}(K) = P_T^{\mu\nu} \frac{1}{\delta\Pi_T^{r/a} - K^2 \mp i\text{sgn}(k_0)\varepsilon} + P_L^{\mu\nu} \frac{1}{\delta\Pi_L^{r/a} - K^2 \mp i\text{sgn}(k_0)\varepsilon}, \quad (1)$$

where $P_T^{\mu\nu}, P_L^{\mu\nu}$ are the usual transverse and longitudinal projectors respectively, and $\delta\Pi_T^{r/a}, \delta\Pi_L^{r/a}$ are the transverse and longitudinal Hard Thermal Loop in photon self-energy obtained by summing the contribution of one loop diagrams (Fig. 1) at the limit $P \sim eT, K \sim T$, they are given by:

$$\begin{aligned} \delta\Pi_L^{r/a}(K) &= -3m^2 \left[1 - \frac{k_0}{2k} \ln \frac{k_0 + k \pm i\varepsilon}{k_0 - k \pm i\varepsilon} \right] \\ \delta\Pi_T^{r/a}(K) &= \frac{3}{2} m^2 \frac{k_0^2}{k^2} \left[1 - \left(1 - \frac{k^2}{k_0^2} \right) \frac{k_0}{k} \ln \frac{k_0 + k \pm i\varepsilon}{k_0 - k \pm i\varepsilon} \right] \end{aligned} \quad (2)$$

with $m = \frac{1}{6} e^2 T^2$ the photon thermal mass.

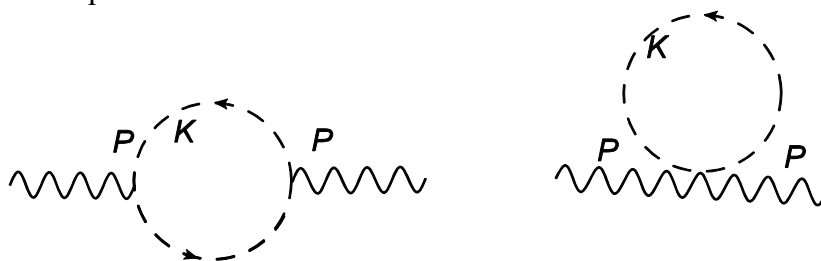


Figure1: Hard thermal loop contribution to photon self-energy

The HTL contribution to the scalar self-energy is obtained by summing the one loop diagrams (Fig.2)

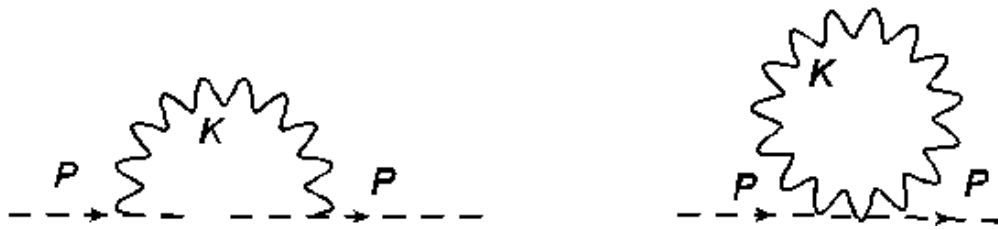


Figure2: Hard thermal loop contribution to scalar self-energy

and it is given by:

$$\delta\Sigma_{hl}(K) = m_s^2, \quad (3)$$

with $m_s = eT/2$ the scalar thermal mass.

The HTL resummed scalar propagator is given by:

$$\begin{aligned} \Delta_{R,A}^{-1}(K) &= K^2 - m_s^2 \pm i \text{sgn}(k_0) \varepsilon \\ \Delta_F^{-1}(K) &= -2\pi i [1 + 2n_B(|k_0|)] \delta(K^2 - m_s^2), \end{aligned} \quad (4)$$

where $K = (k_0, k)$ and $n_B(k_0) = 1/(\exp(k_0/T) - 1)$ the Bose distribution function.

The dressed vertices are equal to the tree amplitudes, i.e., unaffected by the hard thermal loops.

The vertex with one photon and two scalar external lines undressed (Q incoming, P outgoing) is:

$$\Gamma^\mu(P, Q) = -e(P + Q)^\mu, \quad (5)$$

and the vertex between two photons and two scalars is momentum independent and writes:

$$\Gamma^{\mu\nu}(P, Q) = -ie^2 g^{\mu\nu}, \quad (6)$$

3. Damping rate and energy for scalars in hot SQED

The dispersion relations are defined by:

$$\omega^2 = p^2 - \Sigma_{ret}(\omega, p), \quad (7)$$

where $\Sigma_{ret}(\omega, p)$ is the full retarded scalar self-energy. The next- to-leading order scalar energy and damping rates are given by:

$$\omega_s(p) = \omega_{s_0} - \frac{\text{Re} \delta\Sigma(\omega_{s_0}, p)}{2\omega_{s_0}} + O(e^3 T) \quad \gamma_s(p) = \frac{1}{2\omega_{s_0}} \text{Im} \Sigma(\omega_{s_0}, p) + O(e^3 T)$$

where ω_{s_0} is the leading-order scalar energy. So, to obtain the next-to leading-order dispersion relations for slow moving scalars, we have to determine the next-to leading-order 'NLO' scalar self-energy. The diagrams that contribute to next-to-leading-order scalar self-energy are the following two diagrams, in which the internal momenta are soft.



Figure3: NLO HTL-summed scalar self-energy

The contribution of the first diagram in the Keldysh basis is given by:

$$\Sigma^{(1)}(P) = Tr_{soft} [\Gamma^\mu(P, Q) \Delta(Q) \Gamma^\nu(Q, P) \Delta_{\mu\nu}(K)] \quad (9)$$

With $Q=P-K$ and the contribution of the second diagram is:

$$\Sigma^{(2)}(P) = Tr_{soft} [\Gamma^{\mu\nu}(P, K) \Delta_{\mu\nu}(K)] \quad (10)$$

Doing the sum over the indices in the Keldysh basis. We find the following expression for the first contribution to the NLO HTL-dressed scalar self-energy:

$$\begin{aligned} \Sigma^{(1)}(P) &= e^2 \int \frac{d^4 K}{(2\pi)^4} (P+Q)^\mu (P+Q)^\nu \\ &\times [\Delta^R(Q) \Delta_{\mu\nu}^S(K) + \Delta^S(Q) \Delta_{\mu\nu}^A(K)] \end{aligned} \quad (11)$$

and for the second contribution the following expressions:

$$\Sigma^{(2)}(P) = -ie^2 g^{\mu\nu} \int \frac{d^4 K}{(2\pi)^4} [\Delta_{\mu\nu}^A(K) + \Delta_{\mu\nu}^R(K) + \Delta_{\mu\nu}^S(K)] \quad (12)$$

where the retarded (R), advanced (A), and symmetric (S) propagators are given by:

$$\begin{aligned} \Delta_{B,F}^{R,A}(K) &\equiv \Delta_{B,F}^{ra}(K) = \Delta_{B,A}(k_0 \mp i\epsilon, \vec{k}); \\ \Delta_{B,F}^S(K) &\equiv \Delta_{B,F}^{rr}(K) = (1 \pm 2n_{B,F}(k_0)) \text{sign}(k_0) [\Delta_{B,F}^R(K) - \Delta_{B,F}^A(K)], \end{aligned} \quad (13)$$

We perform the Lorentz summation and decompose the NLO HTL-dressed scalar self-energy to two parts. The first part involves longitudinal photons:

$$\Sigma_l(P) = N \left\{ (2p^0 - k^0)^2 (\Delta^S(Q) \Delta_l^A(K) + \Delta^R(Q) \Delta_l^S(K)) - (\Delta_l^A(K) + \Delta_l^R(K) + \Delta_l^S(K)) \right\}, \quad (14)$$

and the second part involves transverse photons

$$\Sigma_t(P) = N \left\{ 4p^2 (1 - x^2) (\Delta^S(Q) \Delta_t^A(K) + \Delta^R(Q) \Delta_t^S(K)) - 2 (\Delta_t^A(K) + \Delta_t^R(K) + \Delta_t^S(K)) \right\},$$

with $\Delta_{l,t}^{A/R}(K) = -1 / (k_0^2 - k^2 - \delta \Pi_{l,t}^{a/r}(K))$. (15)

For the first two terms in Eq. (14) the integration over k_0 can be done easily by means of δ function, we get the following result for $\Delta^S(Q) \Delta_l^A(K)$:

$$I_{SL} = \frac{-ie^2}{4\pi^2} \int_0^\infty dk \int \frac{d\Omega}{4\pi} \frac{(1 + 2n_B(\omega_q)) k^2}{\omega_q} \times \left\{ (p_0 + \omega_q)^2 \Delta_l^A(p_0 - \omega_q, k) - (p_0 - \omega_q)^2 \Delta_l^A(p_0 + \omega_q, k) \right\} \quad (16)$$

and the following result for $\Delta^R(Q) \Delta_l^S(K)$ contribution:

$$I_{RL} = \frac{-ie^2}{4\pi^2} \int_0^\infty dk \int \frac{d\Omega}{4\pi} \frac{k^2}{\omega_l} (1 + 2n_B(\omega_l)) \times \left\{ (2p_0 - \omega_l)^2 \Delta^R(p_0 - \omega_l, q) - (2p_0 + \omega_l)^2 \Delta^R(p_0 + \omega_l, q) \right\} \quad (17)$$

The first two terms in Eq. (15) are also done easily means of δ function, we get the following result for $\Delta^S(Q) \Delta_t^A(K)$:

$$I_{ST} = \frac{-ie^2 p^2}{\pi^2} \int_0^\infty dk \int \frac{d\Omega}{4\pi} (1 - x^2) \frac{(1 + 2n_B(\omega_q)) k^2}{\omega_q} \times \left\{ \Delta_t^A(p_0 - \omega_q, k) - \Delta_t^A(p_0 + \omega_q, k) \right\} \quad (18)$$

and the following contribution for $\Delta^R(Q) \Delta_t^S(K)$ contribution:

$$I_{RT} = \frac{-ie^2}{\pi^2} p^2 \int_0^\infty dk \int \frac{d\Omega}{4\pi} (1-x^2) \frac{(1+2n_B(\omega_t))k^2}{\omega_t} \times \{ \Delta^R(p_0 - \omega_t, q) - \Delta^R(p_0 + \omega_t, q) \} \quad (19)$$

The sum of the three terms given by (16), (17), (18) and (19) gives the NLO scalar self-energy as function of p . We use expression (8) to plot the variation of NLO scalar energy (Fig.4) and damping (Fig.5) energy with p .

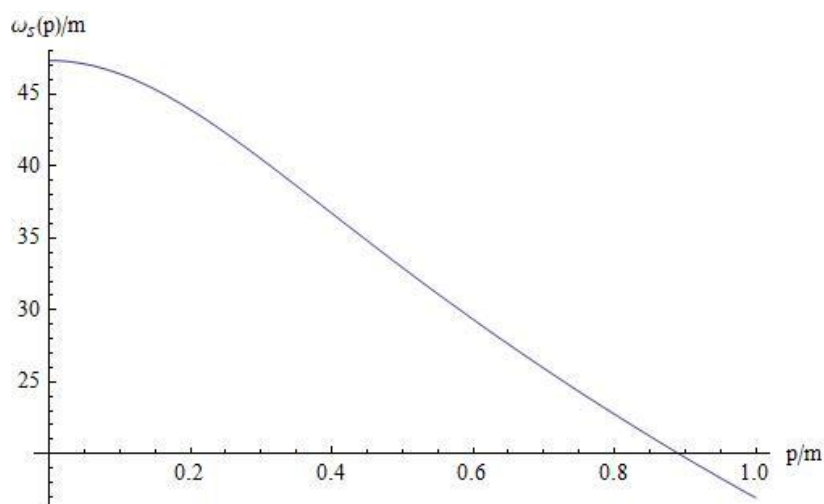


Figure4: The scalar energy variation with p

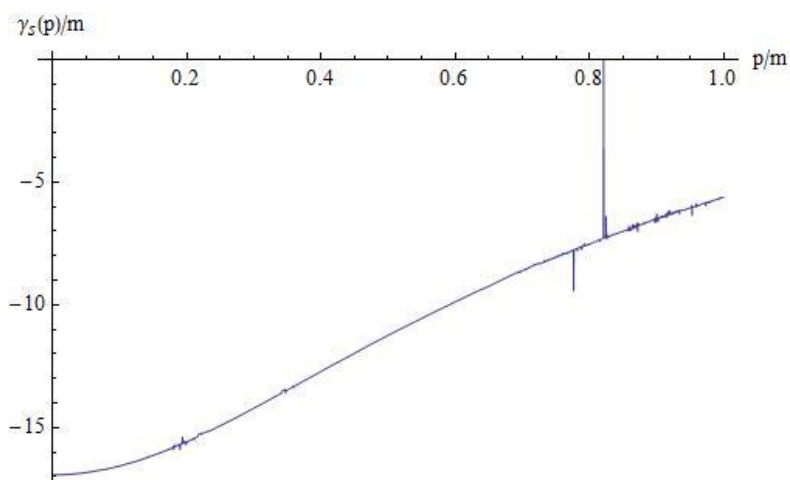


Figure5: The scalar damping variation with p

Figure 4 displays the momentum dependence of the NLO scalar energy. The results indicate that the next-to-leading order corrections introduce finite modifications to the leading-order

dispersion relation, particularly in the soft momentum regime. In the limit $p \rightarrow 0$, the scalar energy approaches a finite value consistent with the earlier findings of Kraemmer et al. [13], confirming the reliability of the HTL-resummed framework. For finite momenta, the NLO corrections cause a deviation from the leading-order behavior, reflecting the contributions from both longitudinal and transverse photon modes in the medium. This behavior demonstrates the importance of higher-order effects in accurately describing the energy spectrum of scalars in hot SQED.

Figure 5 illustrates the variation of the scalar damping rate with momentum. In contrast to the energy corrections, the damping rate is directly linked to medium-induced interactions that control the lifetime of scalar excitations. The results show a monotonic increase in the damping rate with momentum, implying that scalars become more unstable as their momentum grows. This feature originates from enhanced scattering with thermal photons and highlights the essential role of NLO corrections in regularizing infrared-sensitive contributions. The smooth behavior of the damping rate across the momentum range further illustrates the effectiveness of HTL resummation in controlling infrared divergences inherent in thermal field theory.

Together, Figures 4 and 5 underline that NLO effects are indispensable for a consistent description of scalar quasiparticles in a thermal medium. While the scalar energy corrections remain moderate, the damping rate is significantly shaped by NLO contributions, emphasizing that quasiparticle lifetimes are highly sensitive to thermal fluctuations.

Conclusion

In this work, we have derived the complete next-to-leading order contribution to the scalar self-energy in hot scalar QED using the real-time formalism in the framework of HTL-resummed perturbation theory. Compact analytic expressions for both the real and imaginary parts of the retarded scalar self-energy have been obtained, corresponding respectively to energy corrections and damping rates.

The analysis of the dispersion relations, as presented in Figures 4 and 5, demonstrates that the NLO contributions yield non-trivial modifications to both the scalar energy and damping rate. The energy spectrum remains consistent with known results in the infrared limit, while the damping rate shows a pronounced dependence on the scalar momentum, confirming the strong influence of medium effects on quasiparticle stability. These findings reinforce the necessity of resummation techniques for achieving infrared-safe results in thermal field theory.

Extension of the present analysis to higher-loop contributions and different gauge choices, in order to test the robustness and gauge invariance of the results.

Outlooks

- Since resummed perturbation theory has inherent limitations in strongly coupled regimes, it would be valuable to compare the present results with non-perturbative approaches, such as lattice simulations or functional renormalization group methods. This comparison would provide insight into the applicability of HTL methods beyond the weak-coupling domain.
- Scalar fields coupled to gauge bosons appear in many cosmological and astrophysical scenarios. The methods and results presented here could be adapted to study scalar damping and energy shifts in the early universe or in dense astrophysical environments, potentially impacting models of reheating, scalar dark matter, or stellar plasma dynamics.

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