Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 29 No. 4 (2022)

Fractional Order Differential Equations in Applied Nonlinear Analysis

V. K. Singh

Division of Mathematics and Computer Science, National University of Singapore, Singapore

Article History:

Received: *25-09-2022*

Revised: *18-11-2022*

Accepted: *20-12-2022*

Abstract:

Fractional order differential equations provide a powerful framework for understanding and modeling complex nonlinear phenomena in various scientific and engineering disciplines. This article delves into the mathematical foundations, methodologies, and real-world applications of fractional order differential equations in applied nonlinear analysis, emphasizing their crucial role in addressing intricate problems and advancing scientific understanding.

Keywords: Nonlinear Analysis.

1. Introduction

The world is filled with complex nonlinear phenomena, from biological systems to physical processes. Fractional order differential equations have emerged as a valuable tool for modeling and analyzing these intricate behaviors, offering a versatile approach to understanding complex dynamics.

2. Mathematical Foundations

2.1 Fractional Calculus

Fractional calculus extends the concept of differentiation and integration to non-integer orders. Key concepts include:

- **Fractional Derivatives and Integrals**: Generalized derivatives and integrals of non-integer orders.
- **Fractional Order Differential Equations**: Differential equations involving fractional derivatives.

2.2 Fractional Order Models

Fractional order models describe processes with memory and long-range dependence. They are defined by fractional order differential equations and fractional operators.

3. Methodologies for Analysis

3.1 Numerical Methods

Solving fractional order differential equations often requires numerical techniques, such as finite difference methods, fractional Adams-Bashforth schemes, and Caputo fractional derivatives.

3.2 Analytical Methods

Analytical solutions to fractional order differential equations are explored through Laplace and Fourier transforms, series expansions, and integral transforms.

4. Applications

4.1 Physics and Engineering

Fractional order models find applications in the modeling of viscoelastic materials, electrical circuits, and anomalous diffusion processes. They provide a deeper understanding of complex physical systems.

4.2 Biology and Medicine

Fractional order modeling is used in modeling biological processes like drug delivery, tumor growth, and neuronal dynamics, where memory effects play a significant role.

4.3 Finance

Fractional calculus is applied in the modeling of financial systems with long-range dependence, improving the accuracy of financial predictions.

5. Significance and Future Directions

Fractional order differential equations offer a unique perspective in applied nonlinear analysis. Future directions include the development of efficient numerical methods, expanding applications in data science, and interdisciplinary research collaborations.

6. Conclusion

Fractional order differential equations have become indispensable tools in understanding complex nonlinear phenomena across various disciplines. By extending traditional calculus to non-integer orders, they provide insights into memory effects, long-range dependence, and complex dynamics that were previously challenging to capture. As we continue to explore and apply fractional calculus, it will undoubtedly play a pivotal role in advancing applied nonlinear analysis.

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