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Distance Pair Antimagic Labeling on Cycle Related Graphs

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Abstract:

A distance pair antimagic labeling of a graph G with p vertices is a bijection $f: V(G) \longrightarrow P$

where $P = \begin{cases} \pm 1, \pm 2, \cdots, \pm \frac{p}{2}, & if \quad p \ is \ even \\ 0, \pm 1, \pm 2, \cdots, \pm \frac{p-1}{2}, & if \quad p \ is \ odd \end{cases}$ such that the induced weight function $w: V(G) \longrightarrow W$ defined by $w(v) = \sum_{u \in N(v)} f(u) = k_i$is one-one, where $N(v) = \{u \in V: uv \in E\}$ is the open neighborhood of v and the set of all weights W is either of the form $\{\pm k_1, \pm k_2, \pm k_3, \cdots, \pm k_{\frac{p}{2}}\}$ or $\{0, \pm k_1, \pm k_2, \pm k_3, \cdots, \pm k_{\frac{p-1}{2}}\}$ according as p is even or odd. In this paper, we explored the result on distance pair antimagic labeling of cycle related graphs. Also we investigated the closed distance magic labeling of circulant graph and its complement.

Keywords: Graph labeling, distance antimagic, pair sum labeling, distance pair antimagic labeling.

AMS Subject Classification(2010): 05C12, 05C78.

1. Introduction

A magic square of order n is an $n \times n$ array whose entries are an arrangement of the integers $1,2,3, \ldots, n^2$ in which all elements in any row, any column, the main diagonal or the main back diagonal add to the same sum r. Vilfred [9] in his doctoral thesis introduced the concept of Σ-labeling. Following this, Miller et al. [5] and B.D. Acharya et al [1] studied the concepts under the name of neighborhood magic graphs. Further, Sugeng et al. [7] used the term distance magic labeling for the same concept. A *distance magic labeling* of a graph $G = (V, E)$ of order n is a bijection $f: V(G) \rightarrow \{1,2, \ldots, n\}$ such that $\sum_{u \in N(v)} f(u) = k$ for all $v \in V$. The constant k is called the *magic constant* of the labeling f. A graph which admits a distance magic labeling is called a *distance magic graph*. In 2013, Kamatchi and Arumugam [3] introduced the concept of a distance antimagic graph. Motivated by these works and pair sum labeling [6] we introduced the concept of distance pair antimagic labeling [4]. In this paper, we present several results of distance pair antimagic labeling on cycle related graphs and complement of circulant graph.

2. Preliminaries

Definition 2.1 *A graph G is said to be distance antimagic if there is a bijection $f: V(G) \rightarrow \{1,2, \ldots, p\}$ such that for every pair of distinct vertices x and y applies $w(x) \neq w(y)$.*

Definition 2.2 *A injective map $f: V(G) \to \{\pm 1, \pm 2, \cdots, \pm p\}$ is said to be pair sum labeling if the induced edge function $f_e: E(G) \to Z\backslash\{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \pm k_3, \cdots, \pm k_{\frac{q}{2}}\}$ or $\{\pm k_1, \pm k_2, \pm k_3, \cdots, \pm k_{\frac{q-1}{2}}\} \cup \{\pm k_{\frac{q+1}{2}}\}$ according as q is even or odd.*

Definition 2.3 *Let G be a (p, q) graph. Let $f: V(G) \longrightarrow P$ be a bijection where*

$$P = \begin{cases} \pm 1, \pm 2, \cdots, \pm \frac{p}{2}, & if \quad p \ is \ even \\ 0, \pm 1, \pm 2, \cdots, \pm \frac{p-1}{2}, & if \quad p \ is \ odd \end{cases}$$

Then f is called a distance pair antimagic (DPAM) labeling if the induced weight function $w: V(G) \longrightarrow W$ defined by $w(v) = \sum_{u \in N(v)} f(v) = k_i$ is one-one, where $N(v) = \{u \in V: uv \in E\}$ is the open neighborhood of v and the set of all weights W is either of the form $\{\pm k_1, \pm k_2, \pm k_3, \cdots, \pm k_{\frac{p}{2}}\}$ or $\{0, \pm k_1, \pm k_2, \pm k_3, \cdots, \pm k_{\frac{p-1}{2}}\}$ according as p is even or odd. A graph which admits distance pair antimagic labeling is called a distance pair antimagic graph.

Definition 2.4 *A function f is called closed distance pair antimagic labeling, if we take closed neighborhood $n[v]$ instead of open neighborhood $N(v)$ in the previous definition.*

Definition 2.5 *The n-sunlet graph $C_n \odot K_1$ is the graph on $2n$ vertices obtained by attaching a pendant edge to each vertices of a cycle C_n and it is denoted by S_n.*

Definition 2.6 *The helm graph H_n is the graph obtained from an wheel graph W_n by adjoining a pendant edge at each vertex of the cycle.*

Definition 2.7 *The triangular snake T_n is obtained from a path P_n by replacing each edge of the path by a triangle C_3.*

Definition 2.8 *The Gear graph G_n is formed by adding a vertex between each pair of adjacent vertices of a wheel graph W_n.*

Definition 2.9 *The n-book graph is defined as the graph Cartesian product $B_n = S_{n+1} \times P_2$, where S_{n+1} is a star graph and P_2 is the path graph on two vertices.*

Definition 2.10 *The friendship graph is defined as the graph F_n which consisting of n triangles with a common vertex.*

Definition 2.11 *The Prism graph $C_n \times K_2$ is constructed by the Cartesian product of a cycle of length $n \geq 3$ and an edge.*

Definition 2.12 *Resty [8] defined the following graph. For $n \geq 4$, the n-crossed prism graph obtained by taking two disjoint cycle graphs namely C_n^1 and C_n^2, where $V(C_n^1) = \{v_1, v_2, \ldots, v_n\}$ and $V(C_n^2) = \{u_1, u_2, \ldots, u_n\}$, and adding edges $u_s v_{s+1}$ for $s \in \{1,3, \ldots, n-1\}$ and $u_t v_{t-1}$ for $t \in \{2,4, \ldots, n\}$. The n-crossed prism graph denoted by CP_n.*

Definition 2.13 *The circulant graph $C(n; D)$, where $D \subseteq \left\{1,2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor\right\}$ is the graph with vertex set $\{v_1, v_2, \ldots, v_n\}$, where v_i and v_j are adjacent if and only if there is a number $d \in D$ such that $i + d \equiv j (mod\ n)$ or $j + d \equiv i (mod\ n)$.*

3. Main Results

Theorem 3.1 *The sunlet graph S_n is a distance pair antimagic graph if $n = 4,6,8, \ldots$*

Proof. Let $V(S_n) = \{v_i, u_i: 1 \leq i \leq n\}$ be the vertex set and

$E(S_n) = \{v_i u_i : 1 \le i \le n\} \cup \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_1 v_n\}$ be the edge set of S_n.

Define $f: V(S_n) \longrightarrow \{\pm 1, \pm 2, \cdots, \pm n\}$ by

$$f(v_i) = \begin{cases} (-1)^i (i), & if \quad 1 \le i \le \frac{n}{2} \\ (-1)^i (n+1-i), & if \quad \frac{n}{2} < i \le n \end{cases}$$

$$f(u_i) = \begin{cases} (-1)^{i+1} \left(\frac{n}{2} + i\right), & if \quad 1 \le i \le \frac{n}{2} \\ (-1)^{i+1} \left(\frac{3n}{2} + 1 - i\right), & if \quad \frac{n}{2} < i \le n \end{cases}$$

Then the induced vertex weight labeling are as follows.

$$w(v_i) = \begin{cases} \frac{n}{2} + 4, & if \quad i = 1 \\ (-1)^{i+1} \left(\frac{n}{2} + 3i\right), & if \quad 1 < i < \frac{n}{2} \\ (-1)^{i+1} 2n - 1, & if \quad i = \frac{n}{2}, \frac{n}{2} + 1 \\ (-1)^{i+1} 2n + 3\left(\frac{n}{2} + 1 - i\right), & if \quad i = \frac{n}{2} + 2, \frac{n}{2} + 3, \ldots, n-1 \\ -\left(\frac{n}{2} + 4\right), & if \quad i = n \end{cases}$$

$$w(u_i) = \begin{cases} (-1)^i (i), & if \quad 1 \le i \le \frac{n}{2} \\ (-1)^i (n+1-i), & if \quad \frac{n}{2} < i \le n \end{cases}$$

Hence S_n is a distance pair antimagic graph if $n = 4,6,8, \ldots$

Corollary 3.2 *The Helm graph H_n is a distance pair antimagic graph if $n = 4,6,8, \ldots$*

By using Theorem 2.10 [4], *If G is a distance pair antimagic graph with even number of vertices, then the join graph $G + K_1$ is a distance pair antimagic graph.*

Theorem 3.3 *The triangular snake graph T_n is a distance pair antimagic graph for all $n \ge 2$.*

Proof. Let $V(T_n) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n-1\}$ be the vertex set and $E(T_n) = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i, u_i v_{i+1} : 1 \le i \le n-1\}$ be the edge set of T_n.

Consider following two cases.

Case(i): n =is odd

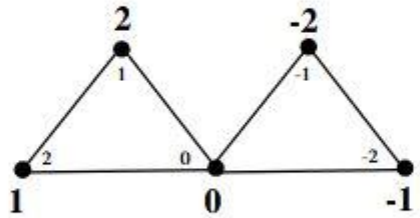

Figure 3.1 DPAM Labeling of T_3. (The vertex labels are in usual font and weights are in bold font)

Clearly from figure 3.1 T_3 is a distance pair antimagic graph.

For $n = 5,7,9, \dots$ define $f: V(T_n) \longrightarrow \{0, \pm 1, \pm 2, \cdots, \pm n-1\}$ by

$$f(v_i) = (n+1) - 2i, \text{ if } i = 1,2,3, \dots, n$$

$$f(u_i) = n - 2i, \text{ if } i = 1,2,3, \dots, n-1$$

Then the induced vertex weight labeling are as follows.

$$w(v_i) = \begin{cases} 2n-5, & if \quad i = 1 \\ 4(n-3) - 8(i-2), & if \quad i = 2,3,4, \dots, n-1 \\ -(2n-5), & if \quad i = n \end{cases}$$

$$w(u_i) = 2(n-2i) \text{ if } i = 1,2,3, \dots, n-1$$

Case(ii): n is even

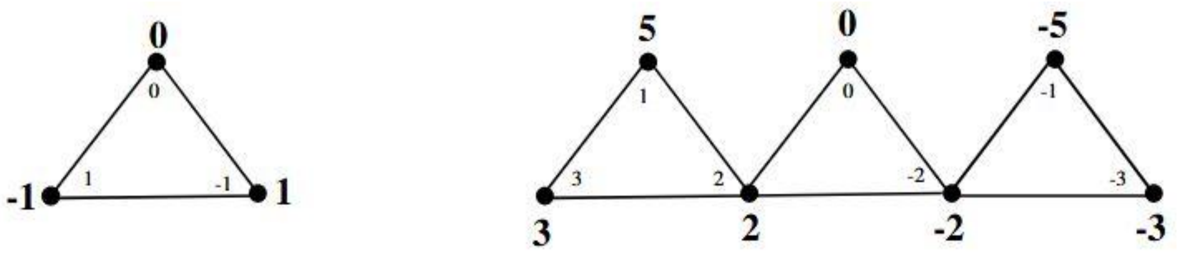

Figure 3.2 DPAM Labeling of T_2 and T_4.

Clearly from figure 3.2 T_2 and T_4 are distance pair antimagic graphs.

For $n = 6,8,10, \dots$ define $f: V(T_n) \longrightarrow \{0, \pm 1, \pm 2, \cdots, \pm n-1\}$ by

$$f(v_1) = n - 1 = -f(v_n)$$

and remaining vertices has following labeling

$$f(v_i) = \begin{cases} n - 2(i-1), & if \quad 1 < i < \frac{n}{2} \\ n - 2i, & if \quad \frac{n}{2} \leq i < n \end{cases}$$

$$f(u_i) = \begin{cases} n - 1 - 2i, & if \quad i = 1,2,3, \dots, \frac{n}{2} - 1 \\ 0, & if \quad i = \frac{n}{2} \\ n - 1 - 2(i-1) & if \quad i = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n-1 \end{cases}$$

Then the induced vertex weight labeling are as follows.

v_i	$w(v_i)$
$i = 1$	$2n - 5$
$i = 2$	$3 + 4(n-4)$
$i = 3,4,5, \dots, \frac{n}{2} - 1$	$4(n + 1 - 2i)$

$i = \frac{n}{2}$	3
$i = \frac{n}{2} + 1$	-3
$i = \frac{n}{2} + 2, \frac{n}{2} + 3, \ldots, n-2$	$4(n+1-2i)$
$i = n-1$	$-(3+4(n-4))$
$i = n$	$-(2n-5)$

u_i	$w(u_i)$
$i = 1$	$2n-3$
$i = 2,3,4,\ldots,\frac{n}{2}-1$	$2(n+1)-4i$
$i = \frac{n}{2}$	0
$\frac{n}{2}+1, \frac{n}{2}+2, \ldots, n-2$	$2(n-1)-4i$
$i = n-1$	$-(2n-3)$

Hence T_n is a distance pair antimagic graph for all $n \geq 2$.

Theorem 3.4 *The n-gear graph G_n is a distance pair antimagic graph, if $n = 4,6,8,\ldots$*

Proof. Let $V(G_n) = \{u_0, u_i, v_i : i = 1,2,3,\ldots,n\}$ and $E(G_n) = \{u_i v_i, v_i u_{i+1} : i = 1,2,3,\ldots,n-1\} \cup \{v_n u_1\} \cup \{u_0 u_i : i = 1,2,3,\ldots,n\}$ be vertex set and edge set of G_n.

Define $f: V(G_n) \longrightarrow \{0, \pm 1, \pm 2, \cdots, \pm n\}$ by the following two cases

Case : (i) $n \equiv 2 \; mod \; 4$

u_i	$f(u_i)$
$i = 0$	0
$i = 1,2,3,4,\ldots,\frac{n}{2}-1$	i
$i = \frac{n}{2}$	n
$i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n-1$	$\frac{n}{2} - i$
$i = n$	$-n$

v_i	$f(v_i)$
$i = 1,2,3,4,\ldots,\frac{n}{2}$	$\frac{n}{2} + (i-1)$

$i = \frac{n}{2} + 1, \frac{n}{2} + 2, \ldots, n$	$-(i-1)$

Then the induced vertex weight labeling are as follows.

u_i	$w(u_i)$
$i = 1$	$-\left(\frac{n}{2} - 1\right)$
$i = 2,3,4, \ldots, \frac{n}{2}$	$(n-1) + 2(i-1)$
$i = \frac{n}{2} + 1$	$\frac{n}{2} - 1$
$i = \frac{n}{2} + 2, \frac{n}{2} + 3, \ldots, n$	$3 - 2i$

v_i	$w(v_i)$
$i = 1,2,3, \ldots, \frac{n}{2} - 2$	$2i + 1$
$i = \frac{n}{2} - 1$	$\frac{3n}{2} - 1$
$i = \frac{n}{2}$	$n - 1$
$i = \frac{n}{2} + 1, \frac{n}{2} + 2, \ldots, n - 2$	$n - 1 - 2i$
$i = n - 1$	$-\left(\frac{3n}{2} - 1\right)$
$i = n$	$-(n-1)$

Case : (ii) $n \equiv 0 \ mod\ 4$

u_i	$f(u_i)$
$i = 0$	0
$i = 1,2,3,4, \ldots, \frac{n}{4}$	$2i - 1$
$i = \frac{n}{4} + 1, \frac{n}{4} + 2, \ldots, \frac{n}{2}$	$2\left(i - \frac{n}{4}\right)$
$i = \frac{n}{2} + 1, \frac{n}{2} + 2, \ldots, \frac{3n}{4}$	$n + 1 - 2i$
$i = \frac{3n}{4} + 1, \frac{3n}{4} + 2, \ldots, n$	$\frac{3n}{2} - 2i$

v_i	$f(v_i)$
$i = 1,2,3, \ldots, \frac{n}{4}$	$n - 2(i-1)$
$i = \frac{n}{4} + 1, \frac{n}{4} + 2, \ldots, \frac{n}{2}$	$2(i-1) + 1$

$i = \frac{n}{2}+1, \frac{n}{2}+2, \dots, \frac{3n}{4}$	$-2(n+1-i)$
$i = \frac{3n}{4}+1, \frac{3n}{4}+2, \dots, n$	$n+1-2i$

Then the induced vertex weight labeling are as follows.

u_i	$w(u_i)$
$i = 0$	0
$i = 1$	1
$i = 2,3, \dots, \frac{n}{4}$	$2(n+1) - 4(i-1)$
$i = \frac{n}{4}+1$	$n+3$
$i = \frac{n}{4}+2, \frac{n}{4}+3, \dots, \frac{n}{2}$	$4(i-1)$
$i = \frac{n}{2}+1$	-1
$i = \frac{n}{2}+2, \frac{n}{2}+3, \dots, \frac{3n}{4}$	$-(4(n-i)+6)$
$i = \frac{3n}{4}+1$	$-(n+3)$
$i = \frac{3n}{4}+2, \frac{3n}{4}+3, \dots, n$	$-\left(4\left(i-\frac{n}{2}-1\right)\right)$

v_i	$w(v_i)$
$i = 1,2,3, \dots, \frac{n}{4}-1$	$4i$
$i = \frac{n}{4}$	$\frac{n}{2}+1$
$i = \frac{n}{4}+1, \frac{n}{4}+2, \dots, \frac{n}{2}-1$	$4\left(i-\frac{n}{4}\right)+2$
$i = \frac{n}{2}$	$\frac{n}{2}-1$
$i = \frac{n}{2}+1, \frac{n}{2}+2, \dots, \frac{3n}{4}-1$	$4\left(\frac{n}{2}-i\right)$
$i = \frac{3n}{4}$	$-(\frac{n}{2}+1)$
$i = \frac{3n}{4}+1, \frac{3n}{4}+2, \dots, n-1$	$-\left(4\left(i-\frac{3n}{4}\right)+2\right)$
$i = n$	$-\left(\frac{n}{2}-1\right)$

Hence the n-gear graph G_n is a distance pair antimagic graph, if $n = 4,6,8, \dots$

Theorem 3.5 *The book graph* $B_n = S_{n+1} \times P_2$ *is a distance pair antimagic graph if* $n \geq 2$.

Proof. Let the vertex set of book graph $V(B_n)$ be $\{u_i, v_i: i = 0,1,2,3,\ldots,n\}$ and the edge set be $\{u_0u_i, v_0v_i: i = 1,2,3,\ldots,n\} \cup \{u_iv_i: i = 0,1,2,3,\ldots,n\}$.

Define $f: V(S_{n+1} \times P_2) \longrightarrow \{\pm 1, \pm 2, \cdots, \pm n+1\}$ by

$$f(u_0) = 1;\ f(v_0) = -1;$$

$$f(u_i) = i + 1,\ \ if \quad i = 1,2,3,\ldots,n;$$

$$f(v_i) = -(i+1),\ if \quad i = 1,2,3,\ldots,n$$

Then the induced vertex weight labeling are as follows.

$$w(u_0) = \frac{(n+1)(n+2)}{2} - 1$$

$$w(u_i) = -i, \quad if \quad i = 1,2,3,\ldots,n$$

$$w(v_0) = 1 - \frac{(n+1)(n+2)}{2}$$

$$w(v_i) = i, \quad if \quad i = 1,2,3,\ldots,n$$

Hence B_n is a distance pair antimagic graph if $n \geq 2$.

Theorem 3.6 *The friendship graph* F_n *is a distance pair antimagic graph.*

Proof. Let the vertex set of friendship graph $V(F_n)$ be $\{v_0, v_1, v_2, \ldots, v_{2n}\}$ and the edge set be $\{v_iv_{i+1}: i = 1,3,\ldots,2n-1\} \cup \{v_0v_i: i = 1,2,3,\ldots,2n\}$.

Define $f: V(F_n) \longrightarrow \{0, \pm 1, \pm 2, \cdots, \pm n\}$ by

$$f(v_i) = \begin{cases} 0, & if \quad i = 0 \\ \frac{i+1}{2}, & if \quad i \quad is \quad odd \\ -\left(\frac{i}{2}\right), & if \quad i \quad is \quad even \end{cases}$$

Then the induced vertex weight labeling are as follows.

$$w(v_i) = \begin{cases} 0, & if \quad i = 0 \\ -\left(\frac{i+1}{2}\right), & if \quad i \quad is \quad odd \\ \frac{i}{2}, & if \quad i \quad is \quad even \end{cases}$$

Hence F_n is a distance pair antimagic graph.

Theorem 3.7 *The prism graph* $C_n \times K_2$ *is a distance pair antimagic graph.*

Proof. Let $V(C_n \times K_2) = \{v_i, u_i: 1 \leq i \leq n\}$ be the vertex set and $E(C_n \times K_2) = \{v_iv_{i+1}, u_iu_{i+1}, v_iu_i: 1 \leq i \leq n-1\} \cup \{u_nu_1, v_nv_1, v_nu_n\}$ be the edge set of $C_n \times K_2$.

Define $f: V(C_n \times K_2) \longrightarrow \{\pm 1, \pm 2, \cdots, \pm n\}$ by $f(v_i) = i,\ \textit{if}\ i = 1,2,3,\ldots,n$

$$f(u_i) = \begin{cases} -n, & \textit{if}\ \ i = 1 \\ -(i-1), & \textit{if}\ \ i = 2,3,4,\ldots,n \end{cases}$$

Then the induced vertex weight labeling are as follows.

$$w(v_i) = i + 1, \quad \textit{if} \quad i = 1,2,3,\ldots,n-1; \quad w(v_n) = 1$$

$$w(u_i) = \begin{cases} -(n-1), & \textit{if} \quad i = 1 \\ -n, & \textit{if} \quad i = 2 \\ -(i-2), & \textit{if} \quad i = 3,4,5,\ldots,n \end{cases}$$

Hence $C_n \times K_2$ is a distance pair antimagic graph.

Theorem 3.8 *The crossed prism CP_n is a distance pair antimagic graph if $n \equiv 0\ (mod\ 4)$.*

Proof. Let $V(CP_n) = \{v_i, u_i : 1 \le i \le n\}$ be the vertex set of CP_n and edge sets as follows:

$E(CP_n) = \{v_i v_{i+1}, v_1 v_n : 1 \le i \le n-1\} \cup \{u_i u_{i+1}, u_1 u_n : 1 \le i \le n-1\}$ and adding edges $u_s v_{s+1}$ for $s \in \{1,3,\ldots,n-1\}$ and $u_t v_{t-1}$ for $t \in \{2,4,\ldots,n\}$, where v_i, u_i are inner and outer vertices of CP_n.

Define $f: V(CP_n) \longrightarrow \{\pm 1, \pm 2, \cdots, \pm n\}$ by

v_i	$f(v_i)$	u_i	$f(u_i)$
$i = 1$	2	$i = 1$	1
$i = 2$	-2	$i = 2$	-1
$i = 3,4,5,\ldots,\frac{n}{2}$	$(-1)^{i+1}(2i-3)+1$	$i = 3,4,5,\ldots,\frac{n}{2}$	$(-1)^{i+1} \quad (2i-3)$
$i = \frac{n}{2}+1$	n	$i = \frac{n}{2}+1$	$n-1$
$i = \frac{n}{2}+2$	$-n$	$i = \frac{n}{2}+2$	$-(n-1)$
$i = \frac{n}{2}+3, \frac{n}{2}+4, \ldots, n$	$(-1)^{i+1}[2(n-i)+4]$	$i = \frac{n}{2}+3, \frac{n}{2}+4, \ldots, n$	$(-1)^{i+1} \quad 2(n-i)+3$

Then the induced vertex weight labeling are as follows.

v_i	$w(v_i)$	u_i	$w(u_i)$
$i = 1$	7	$i = 1$	6
$i = 2$	-7	$i = 2$	-6
$i = 3,5,\ldots,\frac{n}{2}-1$	$-[6(i-1)+1]$	$i = 3,5,\ldots,\frac{n}{2}-1$	$-[6(i-1)]$
$i = 4,6,\ldots,\frac{n}{2}$	$6i-9$	$i = 4,6,\ldots,\frac{n}{2}$	$6i-10$
$i = \frac{n}{2}+1$	$3-6(i-1)$	$i = \frac{n}{2}+1$	$4-6(i-1)$

$i=\frac{n}{2}+2$	$6(i-2)-3$	$i=\frac{n}{2}+2$	$6(i-2)-4$
$i=\frac{n}{2}+3,\frac{n}{2}+5,\ldots,n-1$	$-[6(n-i+1)+3]$	$i=\frac{n}{2}+3,\frac{n}{2}+5,\ldots,n-1$	$-[6(n-i+1)+2]$
$i=\frac{n}{2}+4,\frac{n}{2}+6,\ldots,n$	$6(n-i+2)+1$	$i=\frac{n}{2}+4,\frac{n}{2}+6,\ldots,n$	$6(n-i+2)$

Hence the crossed prism CP_n is a distance pair antimagic graph if $n \equiv 0\ (mod\ 4)$.

Theorem 3.9 *The circulant graph* $C(2n;\{d_1,d_2,n\})$ *is a distance pair antimagic if* $n=d_1+d_2, d_1<d_2$ *and* $n>3$.

Proof. Let $G=C(2n;\{d_1,d_2,n\})$ and $V(G)=\{u_1,u_2,u_3,\ldots,u_{2n}\}$

Define $f:V(G)\longrightarrow\{\pm1,\pm2,\cdots,\pm n\}$ by following two cases.

Case (i): n is odd

$$f(u_i)=\begin{cases}-i, & if \quad i=1,3,5,\ldots,n\\ i, & if \quad i=2,4,6,\ldots,n-1\\ n-i, & if \quad i=n+2,n+4,n+6,\ldots,2n-1\\ i-n, & if \quad i=n+1,n+3,n+4,\ldots,2n\end{cases}$$

Then the induced vertex weight labeling are as follows.

$$w(u_i)=\begin{cases}i, & if \quad i=1,3,5,\ldots,n\\ -i, & if \quad i=2,4,6,\ldots,n-1\\ i-n, & if \quad i=n+2,n+4,n+6,\ldots,2n-1\\ n-i, & if \quad i=n+1,n+3,n+4,\ldots,2n\end{cases}$$

Case (ii): n is even

$$f(u_i)=\begin{cases}-i, & if \quad i=1,3,5,\ldots,n-1\\ i, & if \quad i=2,4,6,\ldots,n\\ i-n, & if \quad i=n+1,n+3,n+5,\ldots,2n-1\\ n-i, & if \quad i=n+2,n+4,n+6,\ldots,2n\end{cases}$$

Then the induced vertex weight labeling are as follows.

$$w(u_i)=\begin{cases}i, & if \quad i=1,3,5,\ldots,n-1\\ -i, & if \quad i=2,4,6,\ldots,n\\ n-i, & if \quad i=n+1,n+3,n+5,\ldots,2n-1\\ i-n, & if \quad i=n+2,n+4,n+6,\ldots,2n\end{cases}$$

Hence $C(2n;\{d_1,d_2,n\})$ is a distance pair antimagic if $n=d_1+d_2,\ d_1<d_2$ and $n>3$.

Corollary 3.10 *The circulant graph* $\bar{C}(2n;\{d_1,d_2,n\})$ *is a closed distance pair antimagic, if* $n=d_1+d_2,\ d_1<d_2$ *and* $n>3$.

Corollary 3.11 *The circulant graph* $\bar{C}(2n;\{d_1,d_2\})$ *is a distance pair antimagic, if* $n=d_1+d_2,\ d_1<d_2$ *and* $n>3$.

Corollary 3.12 *The circulant graph* $C(2n; D)$ *is a distance pair antimagic, if* $n > 3$ *where* $D \subseteq D_n = \{(d_i, d_j): d_i + d_j = n\}$ *and* $d_i < d_j$.

4. Conclusion

The distance pair antimagic labeling on cycle related graphs is discussed in this paper. In particular the distance pair antimagicness of prism and crossed prism is investigated for particular n. Also it is proved that the circulant graph $C(2n; \{d_1, d_2, n\})$ is a distance pair antimagic if $n = d_1 + d_2$, $\mathrm{d}_1 < \mathrm{d}_2$ and $n > 3$ while its complement is closed distance pair antimagic with the same n.

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