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Comparitive Study of Transportation Problem and Fuzzy Transportation Problem by Reuben's Ranking System

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Abstract:
"Linear programming problem" was first used for the "transportation problem." Fuzzy numbers provided the best estimate of transportation unit costs and quantities, which are often impossible to determine in real-world scenarios. In many every day real world circumstances, although exact values for transportation cost and quantities cannot be determined, fuzzy numbers provide the closest possible estimates. The literature review includes several ways for resolving transportation difficulties in fuzzy environments, the parameters of each of these approaches are often fuzzy numbers (FN). This paper introduces an innovative strategy based on the triangular fuzzy set for determining the unit fuzzy cost of transportation of a unit product, with a focus on a general fuzzy transportation problem, and introduces a novel method that is based on the triangular fuzzy set for calculating the fuzzy cost of transportation of a product, and Reuben's ranking method is used for crisp values. To attain the best results in complex formulations and equations, they are in an understandable way. Reuben's ranking method is used for defuzzyfing triangular numbers. Toobtain a better optimal solution,It may be good to consider both criteria. Numerical examples are used to demonstrate the solution technique. The Reuben's ranking method helps decision makers solve real-world transportation scenarios where some or all of the parameters are fuzzy set integers.

Keywords: Transportation Problem, Fuzzy Transportation, Triangular Fuzzy Number, Reuben's Ranking, de fuzzi fication Crisp value.

1. Introduction

To determine the most effective solution, the study uses triangular fuzzy logic to solve the transportation problem by applying Reuben's Ranking Crisp value. A transportation problem (TP) is a specific kind of linear programming problem (LPP) in which a single commodity must be distributed from multiple supply sources to multiple demand locations while minimizing total transportation costs. Numerous efficient algorithms exist for addressing transportation-related problems in which every decision criterion-that is, the demand at each destination, the supply at each source, and the unit cost of transportation-is exactly defined. However, there are different circumstances in real life where ambiguity in one or more choice criteria exists, which is why they should be described precisely. Uncertainty may be caused by a variety of factors, including high information costs, computational errors, lack of evidence, and inaccurate measurements. Therefore, we are unable to tackle transportation difficulties using old, classical approaches to solve the transportation problems. When supply, demand, and transportation expenses are all unpredictable. The "Transportation Problem" refers to the "Fuzzy Transportation Problem." The notations of fuzzy sets was proposed by Zadehet al. [1], The conception of decision-making in fuzzy environments was first presented by Bellman et al. [2], and following this groundbreaking study, found the best solution for the transportation with fuzzy cost coefficient, fuzzy systems, and fuzzy sets of supply and demand, S. Chanaset al. [3],were introduced applying the extension principleto solving fuzzy transportation problems S.T.Liuet al. [4],were presented Extensions of traditional transportation problems using interval and fuzzy time interval S.Chanaset al. [5],we looked into an innovative approach for the fuzzy transportation problems and fuzzy optimum solution G. Nagarajanet al. [6] . Worked on a novel type of ranking function for the fuzzy transportation problem (TP) T. Anithakumari, et al. [7]. The two-stage fuzzy transportation problem has been presented by Nagoor Ganiet al. [8]. Zimmermann's fuzzy linear programming (FTP) has been adapted into numerous fuzzy optimization approaches to solving the TPs Zimmermann et al [9]. Numerous researchers have discussed several approachesfor solvingfuzzy transportation problems and offered different solutions for the classical, interval fuzzy TPs, and fuzzy transportation problems. In order to solve a symmetric fuzzy linear programming problem (FLPP), a new ranking function was suggested. To determine the optimal solution for a fully fuzzy transshipment Muralidaranet al. [10], developed methods for applying fuzzy linear programming and ranking methodology to identify the best solution for a fully fuzzy transhipment problem A. Kumar et al.[11].Has presented a novelapproach for fully interval integer transportation problems that finds the best optimal solution of Pandianet al. [12]. Were explained A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers A. Kaur et al. [13], however in this paper, the "fuzzy transportation problem" is resolved by the MODI method, Reuben's ranking, and the "triangular fuzzy number" for fuzzy costs. In this connection, the paper explains the result of FAP was discussed by Mohamed Muameret al. [14]. The optimization of fuzzy bottleneck cost transportation models using the triangular fuzzy transportation problem framework within the congruence modulo technique was explored by kapilkumar et al. [15].,The fuzzy multi-objective multi-item transportation problem in five dimensions , along with the milk transportation method, was introduced by byEkata Jain et al. [17]., A study of linear fractional transportation problem in bipolar fuzzy settings was explored by Nilima Akhtar et al. [18]., Using the "fuzzy transportation problem," this involves Reuben's ranking and the "triangular fuzzy number" for fuzzy costs. The contribution of the work is to present a methodologically sound and computationally tractable frame work for solving fuzzy transportation problems using a systematic de fuzzification technique based on Reuben's ranking.

2. Methods

The theory provided a mathematical approach to vague ideas and problems with multiple solutions. The fuzzy numbers, fuzzy sets, and ranking functions for crisp values, and some basic arithmetic operations, are defined below and could be useful.

Definition2.1: Let us assume that $X'$is a set, i.e., universal. The membership function of fuzzy subset $\bar{A}'$of $X'$is explained as below $\mu_A(x) : X' \rightarrow [0, 1]$.

$\bar{A}'= \{(x, \mu_A(x)) / x \in X', \mu_A(x) \in [0, 1]\}$

Definition 2.2: As the degree of membership function is $\bar{A}'$: R→ [0 1], a fuzzy number is a "triangular fuzzy number" in R, defined as the degree of the membership function. In accordance with the subsequent attributes. $\mu_A(x) = \begin{cases} \frac{x-b}{e-b}, & b < x < e \\ 1, & x = e \\ \frac{f-x}{f-e}, & e < x < f \\ 0, & otherwise \end{cases}$

It is represented as $\bar{A}' = (b, e, f)$ in which $'b'$is the core$\bar{A}'$, $'f'$ is the extent of the right side &$'e'$is the width of the left side. The following figure shows the geometric depiction of a "triangular fuzzy number." Triangular fuzzy arrangement is named as the shape of the "triangular fuzzy number,"$\bar{A}'$which is typically triangle. $\bar{A}' = (b, e, f)$ where $b, e, f \in$ R

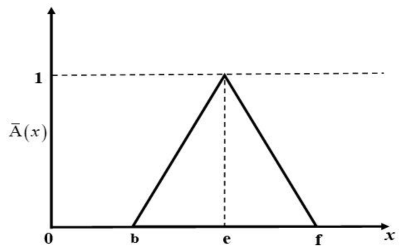

Fig 1: Triangular fuzzy number (TFN)$\bar{A}' = (b, e, f)$

A confidence interval for triangular fuzzy number (TFN)$\bar{A}' = (b, e, f)$at α-level set is defined as follows:$\bar{A}'_\alpha = [A^L_\alpha, A^U_\alpha] = [b+(e-b)\alpha, f-(f-e)\alpha]$, ∀ [0, 1].

Definition 2.3: This section introduces the "arithmetic operations" of fuzzy numbers (FN).

Let $\bar{A}' = (b, e, f)$ &$\bar{B}' = (g, j, k)$ be two TFN, then we will study some operations as follows:

Addition: $\bar{A}' + \bar{B}' = (b, e, f) + (g, j, k) = (b+g, e+j, f+k)$

The symmetric (image): $-\bar{A}' = (-b, -e, -f)$

Subtraction: $\bar{A}' - \bar{B}' = (b, e, f) - (g, j, k) = (b-g, e-j, f-k)$

Multiplication: $\bar{A}' * \bar{B}' = \begin{cases} (bg, ej, fk), b \geq 0, \\ (bk, ej, fk), b < 0, f \geq 0, \\ (bk, ej, fg), k < 0 \end{cases}$

Division: If $\bar{A}' = (b, e, f)$, $\bar{B}' = (g, j, k)$&$\bar{B}' \neq 0$ then If $\bar{A}' \div \bar{B}' = (b/k, e/j, f/g)$

Definition 2.4: The function for $\bar{A}' = (b, e, f)$ denoted R ($\bar{A}'$) proposed by the fuzzy assignment problem applied Reuben's ranking technique, Mohamed Muamer [16] et al. Which can be defuzzified using the Reuben's ranking is defined as

$$R(\bar{A}') = \frac{1}{2}\int_0^1 (r' + s')\,d\alpha$$

Where $\begin{cases} r' = b + (e - f)\alpha, \\ s' = f - (f - e)\alpha \end{cases}$ $, \forall \alpha \epsilon [0,1]$

"Triangular fuzzy number" of the form $\bar{A}' = (b, e, f)$ **,** the α- level set of such a fuzzy number (FN).

Definition 2.5: The literature has suggested a variety of methods for sorting numbers. The application of a ranking-function according to their graded averages provides an efficient technique to comparing fuzzy integers. i.e., for any two "triangular fuzzy numbers" $\bar{A}' = (b, e, f)$ and $\bar{B}' = (g, j, k)$ in F(R), Here is a comparison.

$\bar{A}'$ greater than $\bar{B}'$ ↔ R ($\bar{A}'$) > R ($\bar{B}'$)

$\bar{A}'$ less than $\bar{B}'$ ↔ R ($\bar{A}'$) < R ($\bar{B}'$)

$\bar{A}' - \bar{B}'$ ↔ R ($\bar{A}'$) - R ($\bar{B}'$)

$\bar{A}'$ Equal to $\bar{B}'$ ↔ R ($\bar{A}'$) = R ($\bar{B}'$) Then the "triangular fuzzy number" $\bar{A}' = (b, e, f)$ and $\bar{B}' = (g, j, k)$ are considered likewise in F(R).

3. FUZZY TRANSPORTATION PROBLEM

It has been explained in the literature that the most effective method to solve the transportation problem (TP) with a fuzzy cost coefficient where standard fuzzy integers are used to describe the costs associated with addressing such transportation challenges. The following is a description of the "fuzzy transportation problem," wherein the decision-maker is certain of the commodities' supply and demand but not of the actual cost of shipping from the i^{th} source to the j^{th} destination.

	Destination			**Supply**
Source	1	………………………	n	a'_1
	. . .		. . .	. . .
	m	………………………	$\bar{c}'_{mn}$	a'_m
Demand	b'_1	………………………	b'_n	Total

Minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}'_{ij}\, x_{ij}$

Subject to $\sum_{i=1}^{m} x_{ij} = a'_i$, i=1, 2, ----- m

$\sum_{j=1}^{n} x_{ij} = b'_j$, i=1, 2, -----n

Where,

a'_i**:** The total quantity available at the i^{th} source.

b'_j**:** The total amount of demand at the j^{th} destination.

x_{ij}**:** Total quantity of goods to be delivered from the i^{th} source to j^{th} destination.

$\bar{c}'_{ij}$**:** Fuzzy cost of transportation of product from i^{th} source to j^{th}destination.

Where $\bar{c}'_{ij}$are "Triangular Fuzzy Number" of the form $\bar{A}'$= (b, e, f).

The α-cut of the fuzzy number is a comprehensive and distinct representation of it. For every FN, the α-cut is a closed interval of real numbers α ∈(0, 1). FN of arithmetic operations (AO) can be described using their α-cut and arithmetic operations (AO).

4. A NUMERICAL ILLUSTRATION

Here, in this, we discuss an illustration that explains the comparative study in between the transportation problem (TP) and the "fuzzy transportation problem," Reuben's ranking, and the "triangular fuzzy number" for fuzzy costs.

A manufacturing company generates a product from three suppliers S_1,S_2,S_3.The merchandise will be shipped to four destinationsD_1,D_2,D_3,D_4. The minimal supply rangesS_1,S_2, andS_3whereas the highest demands are D_1,D_2,D_3,D_4 .

EXISTING METHOD

Table 1: Transportation Problem

	Destination				**Supply**
Source	6	1	9	3	70
	11	5	2	7	55
	10	12	4	7	90
Demand	85	35	50	45	215

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{c}'_{ij}\,x_{ij}$

Subject to $\sum_{i=1}^{m}x_{ij} = a'_i$, i=1,2, ----- m

$\sum_{j=1}^{n}x_{ij} = b'_j$, i=1, 2, -----n

Where,

a'_i**:** The total quantity available at the i^{th} source.

b'_j:The total amount of demand at thej^{th}destination.

x_{ij}:Total quantity of goods to be delivered from thei^{th} source to j^{th}destination.

$\bar{c}'_{ij}$**:** The fuzzy cost of transportation of a product from i^{th} source to j^{th}destination.

Where $\bar{c}'_{ij}$are "triangular fuzzy numbers" of the form $\bar{A}'$= (b, e, f).

Table 2: VAM method

	Destination				**Supply**
Source	6 70	1 30	9	3	70
	11	5 35	2 20	7	55
	10 15	12	4 30	7 45	90
Demand	85	35	50	45	215

Vogel's Approximation Method (VAM) is solved in the initial basic feasible solution (IBFS)

Minimum cost=70 × 6 + 35 × 5 + 20 × 2 + 15 × 10 + 45 × 7 + 30 × 4=1220

Table 3: MODI method

	Destination				**Supply**
Source	6 40	1 30	9	3	70
	11	5 5	2 50	7	55
	10 45	12	4	7 45	90
Demand	85	35	50	45	215

By applying the MODI method given in Table 3, the associated transportation cost (TC) optimization is provided by the MODI method.

Minimum cost=6 × 40 + 1 × 30 + 5 × 5 + 2 × 50 + 10 × 45 + 7 × 45=1160.

PROPOSED METHOD

Examine a balanced "fuzzy transportation problem" with four destinations and three suppliers, where the product's unit cost is the only fuzzy number represented.

Table 4: Fuzzy Transportation Problem

	Destination				**Supply**
Source	$\bar{c}'_{11}$	$\bar{c}'_{12}$	$\bar{c}'_{13}$	$\bar{c}'_{14}$	S_1
	$\bar{c}'_{21}$	$\bar{c}'_{22}$	$\bar{c}'_{23}$	$\bar{c}'_{24}$	S_2
	$\bar{c}'_{31}$	$\bar{c}'_{32}$	$\bar{c}'_{33}$	$\bar{c}'_{34}$	S_3
Demand	D_1	D_2	D_3	D_4	Total

Minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}'_{ij}\, x_{ij}$

Subject to $\sum_{i=1}^{m} x_{ij} = a'_i$, i=1, 2, ----- m

$\sum_{j=1}^{n} x_{ij} = b'_j$, i=1, 2, -----n

Where,

a'_i**:** The total quantity available at the i^{th} source.

b'_j: The total amount of demand at thej^{th} destination.

x_{ij}: Total quantity of goods to be delivered from thei^{th} source to j^{th}destination.

$\bar{c}'_{ij}$: The unit fuzzy cost of transportation of a unit product from i^{th} source to j^{th}destination.

Where $\bar{c}'_{ij}$are "triangular fuzzy numbers" of the form $\bar{A}'= (b, e, f)$.

To explain how the fuzzy transportation problem by fuzzy Reuben works, we consider the above illustration, which explains the VAM and MODI methods. The minimum and maximum unit cost ranges vary from source point to destination point, as shown below.

c_{11}=(5,6,7), c_{12}=(0,1,2), c_{13}=(7,9,10), c_{14} = (1,3,4),
c_{21}=(10,11,12),c_{22}=(4,5,6),c_{23}=(1,2,3),c_{24}=(7,8,9),

c_{31}=(9,10,11),c_{32}=(11,12,13),c_{33}=(2,4,5), c_{34}=(6,7,8),

S_1=70, S_2=55, S_3=90, D_1=85, D_2=35,D_3=50,D_4=45,Total=215.

Now, given that the total supply equals to the total demand, the problem is defined as "balanced transportation problem."

Table 5: Fuzzy Transportation Problem by fuzzy Reuben

	Destination				**Supply**
Source	(5,6,7) Rank=6	(0,1,2) Rank=1	(7,9,10) Rank=35/4	(1,3,4) Rank=11/4	70
	(10,11,12) Rank=11	(4,5,6) Rank=5	(1,2,3) Rank=2	(7,8,9) Rank=8	55
	(9,10,11) Rank=10	(11,12,13) Rank=12	(2,4,5) Rank=15/4	(6,7,8) Rank=7	90
Demand	85	35	50	45	215

Each and every number represented by a triangular fuzzy numbers (TFN) $\bar{A}'= (b, e, f)$. This is a fuzzy transported problem with balance (BFTP). It can be de-fuzzified using the Reuben's rankingfunction.For example, TFN of the formc_{11}= (5,6,7) , theα–level set of such a fuzzy number is [r′, s′] where r′ = $[5 + (6 - 5)\alpha]$, s′ = $[7 - (7 - 6)\alpha]$

$$R(\bar{A}') \text{ of } c_{11} = \frac{1}{2}\int_0^1 (5 + \alpha + 7 - \alpha)\,d\alpha = \frac{1}{2}\int_0^1 12\,d\alpha = 6$$

Similarly, c_{12}= 1, c_{13}= 35/4, c_{14} = 11/4, c_{21}= 11, c_{22}= 5, c_{23}= 2, c_{24}= 8, c_{31}= 10,

c_{32}= 12, c_{33}= 15/4, c_{34}= 7.The Reuben's ranking function for the fuzzy cost matrix are calculated in Table5. This reduces the transportation issue with every crisp coefficient.

The initial fuzzy basic feasible solution (IFBFS) is a solved fuzzy version of Vogel's Approximation Method (VAM) in Table 2.

Table 6: MODI method

	Destination				**Supply**
Source	6	1 30	8.75	2.75 40	70
	11	5 5	2 50	8	55
	10 85	12	3.75	7 5	90
Demand	85	35	50	45	215

The MODI method provides the associated fuzzy transportation cost optimization by using the fuzzy version of the MODI method, which is shown in Table 6.

Minimum cost=1×30+2×40+5×5+2×50+10×5+7×5=1120.

3. Results

A set of novel ranking methods on heptagonal fuzzy numbers were used in a novel approaching for solving the fuzzy transportation problem (FTP) by Balasundaram Baranidhara and Ghanshaym Singha Mahapatra[16] et al., but in a simple and understandable way, the fuzzy approach provides a minimum transportation cost, reflecting its ability to account for uncertainties in the cost data. This validation of the effectiveness of Reuben's ranking function may work well in real life transportation scenarios where costs are not always precise. The standard MODI method yielded a result of 1160, but optimal solution was obtained by applying Reuben's ranking function method. If you prefer a rapid, intuitive method, Reuben's ranking function method might be a good fit.

Classical MODI method	1160
Reuben's ranking MODI method	1120

4. Discussion

We present new method in this current paper that, a new approach is presented that, when compared to current approaches, maximizes the minimum cost. Inaccurate statistics are used to represent the transportation problem in the above method discussed. Additionally, Reuben's rankingwas applied to transform the fuzzy transportation problem (TP) into precise values, and a fuzzy version of Vogel's Approximation Method (VAM) wasapplied to solve the initial basic feasible answer, as well as an existing MODI approach for verifying the optimality of basic viable solutions that do not require conversion to classical ones. Although the figures employed are approximations, the obtained results are much more accurate than the previous technique.

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