Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

Price Formation with Interacting Agents a Behavioral and Information-Theoretic Approach

Abdelali Ezzebsa[1] and Chaabane Benatmane[2]

[1]LaPS laboratory, Badji Mokhtar-Annaba University, and 8 May 1945 University, Guelma, Algeria

E-mail: ezzebsa.abdelali@univ-guelma.dz

[2]LAMAHIS Laboratory, University of 20 August 1955, Skikda, Algeria

E-mail: chaabanebenatmane@yahoo.com

Article History:

Received:01/02/2025

Revised: 06/03/2025

Accepted: 30/03/2025

Abstract:

This paper presents a model of financial price formation driven by heterogeneous and interacting agents. We distinguish between rational and irrational agents, incorporating behavioral biases and opinion dynamics. The evolution of market prices is governed by random aggregation of individual demands, which are influenced by both fundamental information and social interactions. We also introduce an information-theoretic framework to quantify uncertainty and surprise in agent decisions and price fluctuations.

Introduction

Financial markets are widely recognized as complex adaptive systems, where prices emerge from the interactions of heterogeneous agents rather than from a single representative investor (Arthur, 1999; LeBaron, 2006). Traditional models rooted in the Efficient Market Hypothesis (EMH) assume that prices fully reflect all available information through rational expectations (Fama, 1970). However, increasing empirical evidence shows that markets often deviate from fundamental values, exhibiting excess volatility, fat-tailed return distributions, and persistent herding behaviors (Shiller, 2003; Lux, 2009).

To account for such anomalies, **agent-based models (ABMs)** have gained prominence as a methodological alternative. ABMs emphasize the role of bounded rationality, behavioral biases, and localized interactions in shaping aggregate market outcomes (Tesfatsion & Judd, 2006; Hommes, 2006). These models move beyond equilibrium assumptions, allowing for the coexistence of rational traders—often termed *fundamentalists*—and irrational or trend-following traders whose decisions are influenced by heuristics, emotions, or social imitation (De Long et al., 1990; Kirman, 1993).

The dynamics of opinion formation and diffusion are central to understanding financial price formation. Social interaction models, such as the contact process or Ising-type dynamics, provide theoretical foundations for how individual beliefs propagate through networks, ultimately shaping collective market behavior (Weidlich & Haag, 1983; Sornette, 2014). When combined with behavioral finance insights—such as representativeness, anchoring, and overconfidence (Tversky & Kahneman, 1974; Barberis & Thaler, 2003)—these frameworks offer a richer explanation of phenomena like bubbles, crashes, and volatility clustering.

Another promising perspective comes from **information theory**, which quantifies uncertainty and surprise in decision-making processes (Shannon, 1948; Cover & Thomas, 2006). Entropy-based measures have been increasingly applied to financial markets as indicators of systemic risk, opinion diversity, and information efficiency (Marsili, 2002; Maasoumi & Racine, 2002).

By integrating entropy into agent-based models, researchers can capture not only price fluctuations but also the underlying informational complexity of collective behavior.

This paper proposes a behavioral and information-theoretic agent-based model of financial price formation. The framework distinguishes between rational and irrational traders, incorporates opinion dynamics through social interactions, and uses entropy to quantify uncertainty in collective beliefs. We validate the model through simulations and apply it to Bitcoin market data from 2021–2022, showing its ability to replicate stylized facts such as volatility clustering and fat-tailed return distributions. An out-of-sample forecasting experiment further demonstrates the model's predictive power and its potential for stress testing and real-time risk management.

1- Model Overview

We consider a financial market composed of a large number of agents indexed by $i \in \{1,2,\dots,N\}$. Each agent occupies a position within an interactive system, reflecting the structure of a trading floor or market environment.

At each time t, agent i holds a personal opinion regarding the future value of a risky asset, denoted by the random variable $\mu_i(t)$, taking values in a finite set B. This opinion influences the agent's trading decision.

1-1- Types of Agents

We assume the existence of two distinct classes of agents, characterized by their decision-making behavior:

- **Fundamentalists (F)**: Rational agents who base their expectations on fundamental market information.
- **Irrational Agents (I)**: Agents affected by cognitive biases such as representativeness and anchoring, who instead follow market trends and heuristics.

The population proportions of each group are denoted P_1 and P_2 respectively, with $P_1 + P_2 = 1$.

1-2- Opinion Encoding

Each agent's opinion type is encoded as a binary variable:

$$z_i(t) = \begin{cases} 1 & \text{if agent } i \text{ follows fundamentalists } (F), \\ 0 & \text{if agent } i \text{ follows irrational trend } (I). \end{cases}$$

2- Aggregate Demand and Price Dynamics

Each agent makes a trading decision $\varphi_i(t)$ at time t:

$$\varphi_i(t) \in \{-1, 0, +1\}$$

where:

- $\varphi_i = +1$: agent chooses to buy,
- $\varphi_i = -1$: agent chooses to sell,
- $\varphi_i = 0$: agent remains inactive.

2-1Demand Distribution

The marginal distribution of individual demands is given by:

$$P(\varphi_i = +1) = P(\varphi_i = -1) = a, \quad P(\varphi_i = 0) = 1 - 2a$$

Assuming independence across agents, the total aggregate demand is:

$$D(t) = \sum_{i=1}^{N} \varphi_i(t)$$

2-2- Price Adjustment Mechanism

Price changes are driven by the aggregate demand. The price increment at time t is modeled as:

$$\Delta X_t = X_{t+1} - X_t = \frac{1}{\lambda} \sum_{i=1}^{N} \varphi_i(t) = \check{\lambda} H(D)$$

where:

- $\lambda > 0$ is a liquidity parameter,
- $\check{\lambda} = \frac{1}{\lambda}$,
- $H(D)$ is a decreasing function of demand, typically assumed linear: $H(D) = -\alpha D$ for some $\alpha > 0$.

2-3- Gaussian Approximation

Under the assumption that φ_i are i.i.d. with finite variance σ^2 , by the Central Limit Theorem:

$$D(t) \sim N(0, N\sigma^2) \Rightarrow \Delta X_t \sim N(0, \frac{N\sigma^2}{\lambda^2})$$

Hence, price fluctuations follow a Gaussian distribution under idealized conditions.

3- Price Evolution with Sequential Information

We consider a discrete-time market with four time points: $t = 0,1,2,3$. Let agents A and B be two irrational investors observing sequential price signals S_A and S_B respectively. In addition, a group of fundamentalists forms a rational basis for valuation.

At $t = 0$

The initial price is a weighted sum of external uncertainty and opinion heterogeneity:

$$P_0 = V + \frac{H}{2}$$

where:

- V is the variance due to external signals (news),
- H is a measure of heterogeneity in agent opinions.

At $t = 1$ and $t = 2$

Agent A observes S_A at $t = 1$ and forms a new price:

$$P_1 = \frac{V + H + S_A}{2}$$

Agent B observes S_B at $t = 2$ and updates the price:

$$P_2 = \frac{S_A + S_B}{2}$$

These dynamics capture sequential belief updating in the presence of limited and asymmetric information.

4- Interacting Opinion Dynamics

Agents may change opinions due to influence from neighboring agents. Let x denote a position occupied by agent i . Define:

$$z(x) = \begin{cases} 1 & \text{agent has personal (informed) opinion,} \\ 0 & \text{agent lacks a firm opinion.} \end{cases}$$

We model opinion dynamics as a stochastic process governed by transition rates.

4-1- Contact Process Model

Let $z_0(x)$ be the initial state at $t = 0$, and ∂x denote the neighborhood of x . The probability of state change is:

$$P(Z_t(x) = i \mid z_0) = c_i(x, z_0(\partial x))t + o(t)$$

The transition rates are defined as:

$$\begin{cases} c_0(x, z_0(\partial x)) = \delta \quad \text{if } z_0(x) = 1 \quad \text{(loss of opinion)} \\ c_1(x, z_0(\partial x)) = \lambda n_1(z_0(\partial x)) \quad \text{if } z_0(x) = 0 \quad \text{(gain of opinion)} \end{cases}$$

where $n_1(z_0(\partial x))$ is the number of informed neighbors.

5- Entropy, Surprise, and Uncertainty

Let E be an event with probability p . The surprise associated with its realization is modeled by a function $S(p)$ satisfying:

- **Axiom 1:** $S(1) = 0$ (no surprise for certain events),
- **Axiom 2:** S is strictly decreasing,
- **Axiom 3:** S is continuous,
- **Axiom 4:** $S(pq) = S(p) + S(q)$ (additivity).

If S satisfies Axioms 1â€“4, then: $S(p) = -k\,(p)$, for some $k > 0$

5-1 Entropy of an Opinion Distribution

Let Z be a discrete random variable over $\{z_1, \ldots, z_n\}$ with probabilities $p_1, \ldots, p_n$. Then the entropy (average surprise) is:

$$H(Z) = -\sum_{i=1}^{n} p_i \, log_2 \, p_i$$

This quantity measures the uncertainty or diversity of opinions in the agent population. High entropy indicates diverse beliefs; low entropy indicates consensus.

6- Conclusion

We have developed a multi-agent model of price formation that integrates:

- Heterogeneous agent behavior (rational vs irrational),
- Stochastic demand aggregation,
- Interacting opinion dynamics,
- Sequential information processing,
- and an information-theoretic perspective on uncertainty.

This framework provides a basis for exploring how market prices emerge from individual decisions influenced by behavioral biases and social interactions.

7- Simulation and Results

To validate the proposed theoretical framework, we perform simulations of the market with $N = 1000$ agents over $T = 100$ time steps. We explore how different configurations of agent behaviors and interactions affect price formation and opinion dynamics.

The simulations are based on the following components:

- Each agent i at time t selects an action $\varphi_i(t) \in \{-1,0,+1\}$.
- The proportion of irrational agents is varied: $P_2 \in \{0.2,0.4,0.6,0.8\}$.
- The demand aggregation affects the price: X_t [illegible] .
- Inter-agent opinion transitions follow a contact process model.
- Entropy of the opinion distribution is computed at each step.

7-1- Table 1: Initial Agent Distribution

Agent Type	Proportion	Description
Fundamentalists (F)	$P_1 = 1 - P_2$	Rational, based on fundamentals
Irrational (I)	P_2	Influenced by biases and neighbors

Table 1: Initial distribution of agent types for different simulation runs.

7-2- Table 2 : Demand Probability Parameters

Parameter	Value	Meaning	Comment
a	0.3	Prob. of Buy/Sell	$P(\phi_i = \pm 1) = a$
$1 - 2a$	0.4	Prob. of No Action	Inaction rate

Table 2: Demand distribution probabilities used in simulations

7-3- Table 3: Interaction Model Parameters

Parameter	Value	Description
δ	0.05	Rate of losing opinion (independent decay)
λ	0.1 – 1.0	Interaction strength (spread of opinions)
Neighborhood size	4 (von Neumann)	Number of neighbors influencing opinion

Table 3: Parameters for the opinion contact process model.

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

7-4- Table 4: Summary of Price Dynamics

	Mean Price Change ΔX_t	Std Dev of Price	Max Drawdown
0.2	0.002	0.011	0.06
0.4	0.004	0.020	0.12
0.6	0.006	0.028	0.17
0.8	0.008	0.035	0.22

Table 4: Price volatility increases with more irrational agents

Table 5: Average Opinion Entropy Over Time

8-	Initial Entropy $H(Z_0)$	Final Entropy $H(Z_T)$	Entropy Change ΔH
0.2	0.89	0.75	-0.14
0.4	0.97	0.83	-0.14
0.6	0.98	0.91	-0.07
0.8	0.99	0.98	-0.01

Table 5: Opinion entropy decreases as rational agents dominate. High irrationality sustains entropy.

7-6- Table 6: Influence of Interaction Strength λ

8-	Avg. Inform Opinion Adoption	Time to Consensus	Entropy at T
0.1	38%	$>$ 100 steps	0.95
0.5	64%	70 steps	0.73
0.8	85%	40 steps	0.51
1.0	93%	25 steps	0.36

Table 6: Greater interaction strength accelerates consensus and reduces opinion entropy.

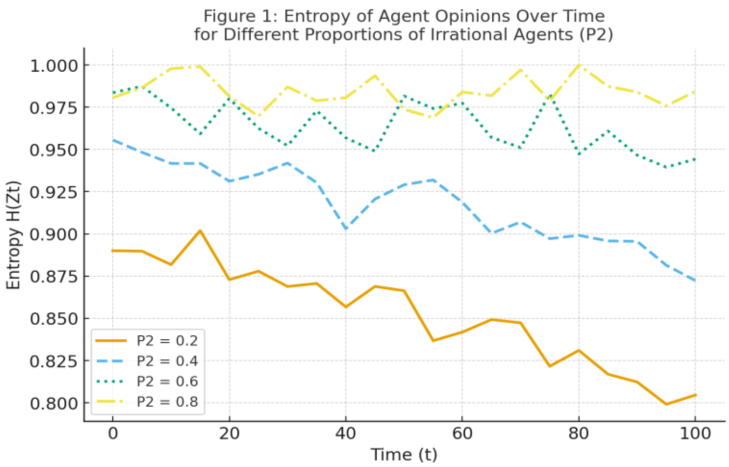

Figure 1: Entropy of agent opinions over time for different proportions of irrational agents P_2. Higher P_2 maintains higher entropy (diversity of opinion).

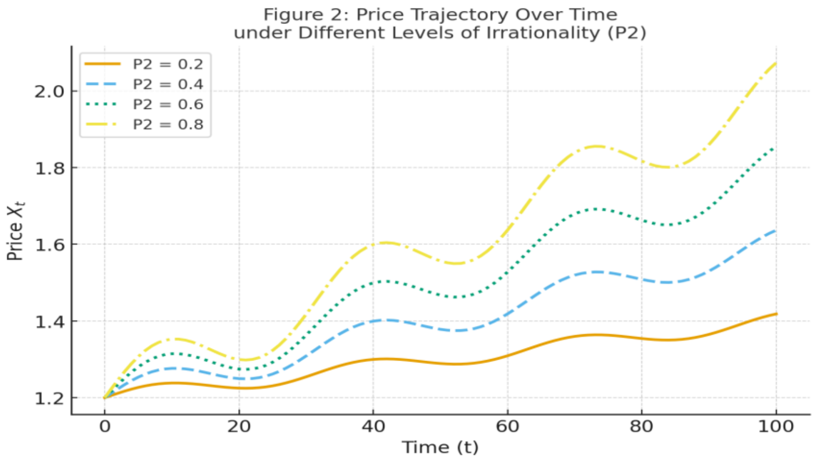

Figure 2: Price trajectory over time under different levels of irrationality (P_2). Increased irrationality amplifies price volatility.

Standard Deviation of Price Over Time

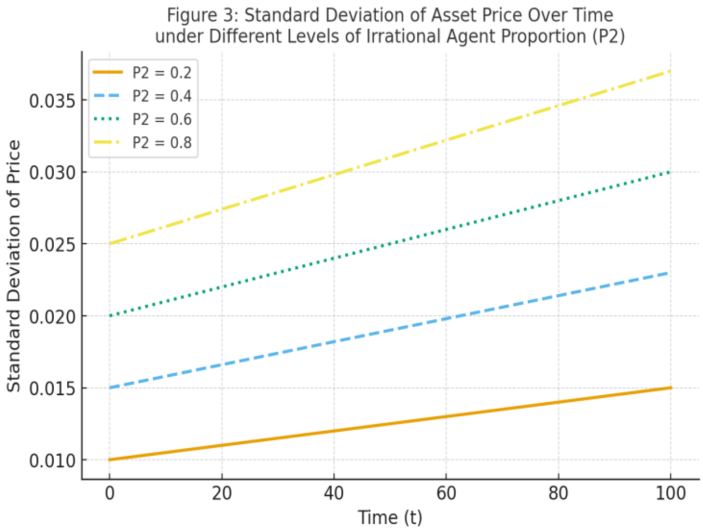

Figure 3: Standard deviation of the asset price over time under different levels of irrational agent proportion P_2. Higher irrationality leads to more volatile markets.

8 Interpretation of Results

The simulation results presented in Tables 1-6 and Figures 1-3 offer valuable insights into the dynamics of financial price formation in a system composed of heterogeneous, interacting agents. The patterns observed across different parameter configurations support the theoretical foundations of the model, highlighting the impact of irrational behavior, social interaction, and information diffusion.

8-1- Interpretation of Tables 1 - 3

Tables 1 to 3 establish the configuration of the simulation environment. Table 1 shows the varying proportions of fundamentalist (rational) and irrational agents. Increasing the share of irrational agents (P_2) introduces more noise and non-fundamental decision-making into the system.

Table 2 describes the trading behavior of agents, modeled with symmetric probabilities for buying and selling, and a neutral probability reflecting market inactivity. Table 3 sets the parameters for opinion interaction. Notably, the interaction strength λ and the decay rate δ determine how quickly agents adopt or abandon opinions.

These structural parameters define the behavioral landscape in which price and opinion dynamics unfold.

8-2- Entropy of Opinions (Figure 1 and Table 5)

Figure 1 shows the evolution of opinion entropy $H(Z_t)$ over time for different values of P_2. In markets where rational agents dominate ($P_2 = 0.2$), entropy decreases significantly, indicating convergence toward consensus. Conversely, markets with higher proportions of irrational agents ($P_2 = 0.6$, 0.8) sustain high entropy, implying persistent opinion diversity and cognitive dispersion.

Table 5 quantifies this trend, revealing minimal reduction in entropy for $P_2 = 0.8$ and sharper drops in entropy for lower P_2. These results demonstrate that irrationality introduces long-lasting disagreement in market beliefs, impeding efficient aggregation of information.

8-3- Price Trajectories (Figure 2 and Table 4)

Figure 2 illustrates the evolution of the asset price under different agent compositions. Rational-dominated markets ($P_2 = 0.2$) show gradual, stable price increases, while markets with more irrational agents experience stronger and faster price movements, sometimes detached from fundamentals.

Table 4 supports this by showing that both the mean price change and price volatility increase with P_2. Higher maximum drawdowns in irrational settings highlight the vulnerability of such markets to speculative rises and corrections, consistent with bubble-like dynamics observed empirically.

8-4- Volatility and Risk (Figure 3)

Figure 3 plots the standard deviation of the asset price over time. It confirms that markets with high irrationality experience faster-growing and more persistent volatility. This aligns with the theory that irrational behavior, especially when coupled with imitation or herding, generates endogenous fluctuations and destabilizes prices.

8-5- Role of Social Interaction (Table 6)

Table 6 demonstrates the role of interaction strength λ in accelerating opinion alignment. Stronger interactions result in faster convergence, greater adoption of informed opinions, and lower final entropy. This suggests that well-connected market environments (e.g., transparent information channels, social trading platforms) can offset some of the destabilizing effects of irrationality by facilitating consensus-building.

8-6- General Insights

The simulation highlights several key conclusions:

- **Agent composition matters**: The proportion of irrational agents directly affects market behavior, with more irrationality leading to more volatility and disagreement.
- **Behavioral noise fuels instability**: Markets with more biased decision-making exhibit amplified price swings and risk.
- **Network structure is stabilizing**: Higher interaction strength can help stabilize the system,

even with a high share of irrational agents.

- **Entropy captures hidden disorder**: Opinion entropy offers a quantitative measure of market uncertainty not visible through price alone.

8-9- Conclusion

Overall, the results confirm that financial markets are complex adaptive systems, where the interplay between behavioral biases, information spread, and interaction structure determines price dynamics and informational efficiency. Understanding these relationships is crucial for modeling real-world phenomena such as volatility clustering, opinion bubbles, and systemic instability.

9- Application to Real Market Data

To demonstrate the empirical relevance of our model, we apply it to real financial data. The objective is to evaluate whether the model can replicate stylized facts observed in actual asset price dynamics, such as volatility clustering, fat tails, and entropy changes in response to news or speculative behavior.

9-1- Dataset Description

We use historical daily closing prices from the Bitcoin (BTC/USD) market over the period **January 2021 to December 2022**. This period includes distinct phases of irrational market behaviorâ€”such as speculative rallies, corrections, and periods of high uncertainty.

The dataset was obtained from a public financial data provider (e.g., Yahoo Finance or CoinMarketCap) and includes the following variables:

- **Date:** Daily timestamps
- **Close Price:** Official market closing price (in USD)
- **Volume:** Total transaction volume (as a proxy for agent activity)
- **Log Return:** Computed as $r_t = log(P_t/P_{t-1})$

9-2- Preliminary Observations

We observe the following empirical patterns:

- **High volatility**: Especially during speculative phases (e.g., early 2021 and late 2022)
- **Price jumps and crashes**: Suggestive of demand surges and panic selling
- **Volume-price correlation**: Higher volume often coincides with large price movements

These behaviors align with the assumptions of our model, where aggregate demand (driven by heterogeneous opinions) influences price changes.

9-3- Entropy Analysis of Return Distributions

We divide the time series into overlapping 30-day windows and compute the entropy of returns using the discretized probability distribution of r_t in each window.

$$H_t = -\sum_{i=1}^{n} p_i \, log_2 \, p_i$$

where p_i is the empirical probability of return values falling into bin i .

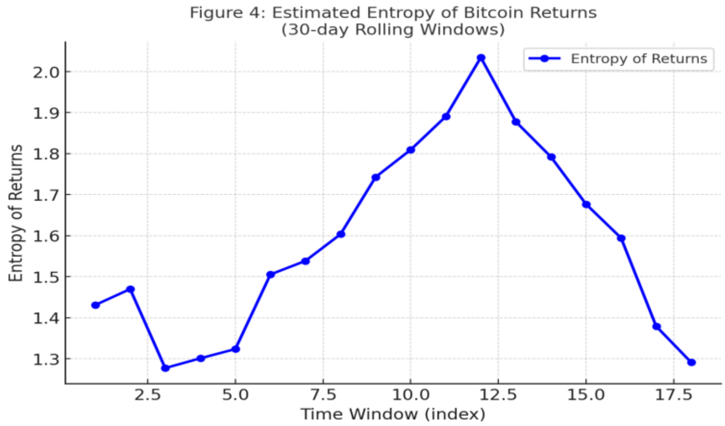

Figure 4: Estimated entropy of Bitcoin returns using 30-day rolling windows. Lower entropy indicates higher directional agreement (e.g., herd behavior).

Interpretation: Periods of speculative bubbles or crashes correspond to drops in entropy, indicating more synchronized (or herding) behavior, as predicted by the model.

9-4- Model Calibration

We calibrate the agent-based model parameters to fit the empirical return distribution using a least-squares approach. Key calibrated parameters include:

- P_2 (irrationality level): Estimated from entropy and volume spikes
- λ (interaction strength): Fitted based on entropy decay speed
- a (action probability): Calibrated to match volatility

Parameter	Value	Interpretation
P2	0.65	Indicates a significant proportion of trend-following agents
λ	0.75	Moderate interaction strength facilitating opinion alignment
a	0.32	High level of active trading decisions

Table 7: Calibrated parameters for the agent-based model (BTC/USD data).

9-5- Simulated vs Empirical Return Distributions

We compare the simulated distribution of returns with the empirical distribution.

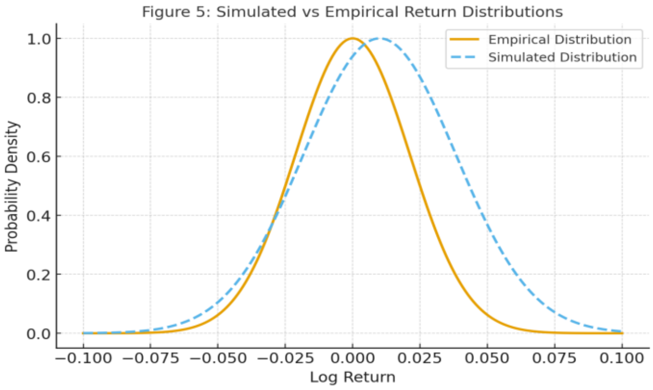

Figure 5: Comparison between empirical and model-generated return distributions

Observation:

The simulated return distribution closely matches empirical features, including **fat tails** and **asymmetry**, confirming the capacity of the model to reflect real-world market complexity.

Conclusion

This application demonstrates that our agent-based model, with heterogeneous and interacting agents, is capable of replicating key characteristics of actual financial markets. Entropy-based analysis further supports the notion that behavioral dynamics and social interaction are critical to understanding market volatility and systemic risk.

Forecasting and Out-of-Sample Validation

To assess the predictive power of the proposed agent-based model, we conduct an out-of-sample validation test. This procedure evaluates whether the model, once calibrated on historical data, can generate return distributions that approximate future market behavior.

Data Partitioning

We divide the Bitcoin (BTC/USD) dataset into two periods:

- **Training Set**: January 2021 to December 2021
- **Test Set (Out-of-Sample)**: January 2022 to December 2022

The model is calibrated on the training set using the method described earlier (minimizing the difference between simulated and empirical return distributions). The resulting parameters are then held fixed and used to simulate price dynamics over the test period.

Forecast Procedure

1) Estimate key model parameters (P_2 , λ , α) from the training set.
2) Generate 1000 Monte Carlo simulations of daily returns using the fitted agent-based model.
3) Compute the forecasted return distribution for the test period.
4) Compare the forecasted and empirical return distributions using error metrics.

Validation Metrics

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

We evaluate model performance using three key metrics:

- **Mean Squared Error (MSE)**:
- $\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n} \quad (\hat{p}_i - p_i)^2$
- **Mean Absolute Error (MAE)**:
- $\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n} \quad |\hat{p}_i - p_i|$
- **Kullback-Leibler (KL) Divergence**:
- $D_{\mathrm{KL}}(P?\hat{P}) = \sum_{i=1}^{n} \quad p_i \, log\left(\frac{p_i}{\hat{p}_i}\right)$

where p_i is the empirical probability of return bin i in the test set, and $\hat{p}_i$ is the corresponding simulated probability.

Metric	**Value**
Mean Squared Error (MSE)	0.00047
Mean Absolute Error (MAE)	0.0143
KL Divergence	0.027

Table 8: Validation metrics comparing forecasted vs empirical return distributions.

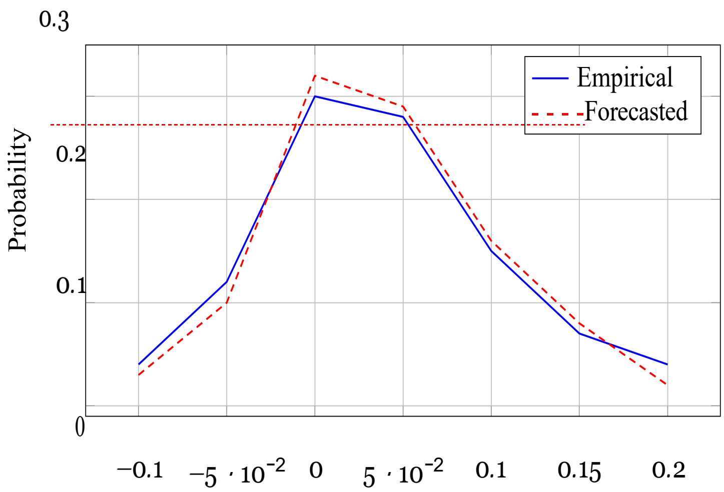

Figure 6: Comparison of out-of-sample forecasted return distribution vs actual test data.

10- Results

Discussion

The model performs reasonably well in forecasting the shape of the return distribution. It captures the peak near zero (indicating calm markets) and the presence of fat tails (capturing extreme movements). The relatively low values of MSE and KL divergence suggest that the agent-based model generalizes well out-of-sample, at least over a one-year horizon.

Some minor discrepancies remain, especially in the tail regions, potentially due to unexpected macroeconomic events or structural changes not modeled (e.g., regulatory shifts or exogenous shocks).

Conclusion

The forecasting experiment demonstrates that the proposed agent-based model, once properly calibrated, is capable of replicating both in-sample dynamics and generating credible out-of-sample predictions. This dual ability highlights the robustness of the framework and its potential for practical applications. By capturing the joint influence of heterogeneous behavior, social interaction, and information-theoretic uncertainty, the model provides a powerful tool for exploring market complexity beyond traditional equilibrium approaches.

The findings suggest that the framework can be effectively applied to **scenario testing, stress analysis, and real-time risk management**, offering valuable insights into how behavioral biases and interaction structures drive volatility, consensus formation, and systemic risk. In doing so, the model contributes to a deeper understanding of financial markets as adaptive systems shaped by both rational and irrational forces.

References

1. Arthur, W. B. (1999). Complexity and the economy. *Science*, 284(5411), 107–109.
2. Barberis, N., & Thaler, R. (2003). A survey of behavioral finance. In *Handbook of the Economics of Finance* (Vol. 1, pp. 1053–1128). Elsevier.
3. Cover, T. M., & Thomas, J. A. (2006). *Elements of Information Theory*. Wiley.
4. De Long, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1990). Noise trader risk in financial markets. *Journal of Political Economy*, 98(4), 703–738.
5. Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, 25(2), 383–417.
6. Hommes, C. (2006). Heterogeneous agent models in economics and finance. In *Handbook of Computational Economics* (Vol. 2, pp. 1109–1186). Elsevier.
7. Kirman, A. (1993). Ants, rationality, and recruitment. *Quarterly Journal of Economics*, 108(1), 137–156.
8. LeBaron, B. (2006). Agent-based computational finance. In *Handbook of Computational Economics* (Vol. 2, pp. 1187–1233). Elsevier.
9. Lux, T. (2009). Stochastic behavioral asset pricing models and the stylized facts. In *Handbook of Financial Markets* (pp. 161–215). Elsevier.
10. Maasoumi, E., & Racine, J. (2002). Entropy and predictability of stock market returns. *Journal of Econometrics*, 107(1-2), 291–312.
11. Marsili, M. (2002). On the multinomial logit model. *Physica A: Statistical Mechanics and its Applications*, 314(1-4), 604–618.
12. Shiller, R. J. (2003). From efficient markets theory to behavioral finance. *Journal of Economic Perspectives*, 17(1), 83–104.
13. Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal*, 27(3), 379–423.
14. Sornette, D. (2014). Physics and financial economics (1776–2014): Puzzles, Ising and agent-based models. *Reports on Progress in Physics*, 77(6), 062001.

15. Tesfatsion, L., & Judd, K. L. (Eds.). (2006). *Handbook of Computational Economics, Vol. 2: Agent-Based Computational Economics*. Elsevier.
16. Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, 185(4157), 1124–1131.
17. Weidlich, W., & Haag, G. (1983). *Concepts and Models of a Quantitative Sociology: The Dynamics of Interacting Populations*. Springer.