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Analysis of Three-Dimensional Non-Homogeneous Fractional order Thermoelastic Problem of Thick Rectangular Plate with Internal Heat Generation

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Abstract: This paper investigates the thermal behaviour of a three-dimensional thick rectangular plate governed by a fractional-order derivative. The plate, occupying the region $D = \{(x, y, z) \in R^3: 0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c\}$, is initially at an arbitrary temperature distribution $f(x, y, z)$. For $t > 0$, the plate experiences internal heat generation represented by $g(x, y, z, t)$ Btu/hr ft^3, while all boundary surfaces are maintained at zero temperature. The governing model is formulated using the Caputo fractional derivative, and analytical solutions for temperature, displacement, and thermal stresses are derived through the integral transform technique. Numerical simulations are carried out using PTC Mathcad software, and the results are illustrated graphically. The study highlights the influence of the fractional-order parameter on heat conduction, displacement, and stress distribution, demonstrating the effectiveness of fractional-order thermoelastic models in capturing nonlocal thermal effects.

Introduction: The study of thermoelastic behaviour in solid materials has gained significant importance due to its wide applications in aerospace, mechanical, and civil engineering. Classical thermoelastic theories often assume instantaneous heat propagation, which contradicts physical reality. To overcome this limitation, fractional-order thermoelasticity has been introduced as an effective approach to model heat conduction with memory and nonlocal effects. In this paper, a three-dimensional non-homogeneous thick rectangular plate with internal heat generation is analysed using the Caputo fractional derivative. The objective is to investigate how the fractional-order parameter (α) influences temperature, displacement, and stress distributions. The proposed model provides a more generalized framework that includes both the classical and wave-type heat conduction as special cases, thus offering a realistic representation of transient thermal behaviour in thick plates.

Objectives:

1. To investigate the thermal behaviour of a three-dimensional thick rectangular plate subjected to internal heat generation.
2. To analyse the effect of the fractional-order parameter (α) on temperature, displacement, and thermal stresses.
3. To demonstrate the usefulness of fractional-order thermoelastic models in capturing nonlocal and memory effects in heat conduction.

Methods: The governing equations are derived using the Caputo fractional derivative for time-fractional heat conduction. Integral transform techniques are employed to obtain analytical solutions for temperature, displacement, and stress fields. Mittag-

Leffler functions are used in the solutions to describe the fractional behaviour. Numerical simulations are performed in PTC Mathcad Prime for a thick rectangular plate using physical constants.

Results: The temperature distribution decreases with increasing fractional order α and is maximum at the centre of the plate. Displacement components show symmetrical behaviour, increasing from the edges and vanishing at the centre. Stress components are compressive, peaking at the middle of the plate and zero at the edges. The fractional order parameter α significantly affects heat transfer. Graphs demonstrate clear variation between classical ($\alpha = 1$) and fractional-order ($\alpha \neq 1$) thermoelastic behaviour.

Conclusions: For $\alpha = 1$, the model reduces to the classical heat diffusion equation, while $\alpha = 2$ corresponds to the wave equation. The fractional order $0 < \alpha < 1$, $\alpha = 1$, and $1 < \alpha < 2$ represent weak, normal, and strong conductivity, respectively. Stress distributions are tensile/compressive and follow normal curves in all directions, directly proportional to α. The fractional order α governs nonlocal heat transfer, influencing both response time and temperature overshoot. The model effectively describes transient thermoelastic behavior in thick plates and can be applied to other nonhomogeneous materials.

Keywords: Thermal Behavior, Internal Heat Source, Caputo Fractional Derivative, Thick Rectangular Plate, Mittag-Leffler Functions.

1. Introduction

Biot [1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. The generalised thermoelastic problem of a thick plate under axi-symmetric heat supply was resolved by Sherief and Hamza [2]. The generalised dynamical theory of thermoelasticity with one relaxation period, for the isotropic body, was developed by Lord and Shulman [3]. In their study of the theoretical analysis of a rectangular plate in a compressive stress field and associated thermal stresses was resolved by Tanigawa and Komatsubara [4].

The solution to the plane thermoelasticity problem for a rectangular domain is discussed in Vihak et al., [5]. A laminated rectangular plate subjected to a thermal shock was the focus of an investigation by Adam and Best [6]. Tanigawa et al. [7 8]. investigate the thermal analysis of thermal buckling caused by uniform heat supply in an orthotropic nonhomogeneous rectangular plate. The transient thermoelastic deformations of a thick functionally graded plate were investigated by Qian and Batra [9]. The thermal behaviour of thermoelastic thick plates under lateral loads was presented by Sharma et. al. [10]. Based on the equation of heat conduction in 1D and 2D with a time-fractional derivative and related thermal stresses, Povstenko [11] resolved a few thermoelastic issues. Some contribution of thermoelastic problems and fractional order thermoelastic problems have been discussed in [12-34].

2. Preliminaries

In this section we give some definitions, notations and facts that we need in this article.

- **The Riemann-Liouville Fractional Differential Operator**

The definition of the Riemann-Liouville fractional differential operator given by [37]

Suppose that $\alpha > 0$, t > a, α, a, t ∈ R. Then

$$D^{\alpha}f(t)=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\dfrac{d^n}{dt^n}\displaystyle\int_0^t\dfrac{f(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau\ , & n-1<\alpha<n\in N\\ \dfrac{d^nf(t)}{dt^n}, & \alpha=n\in N\end{cases} \tag{2.1}$$

is called the Riemann-Liouville fractional derivative or the Riemann-Liouville fractional differential operator of order α.

- **The Caputo Fractional Differential Operator**

The definition of the Caputo fractional differential operator given by [37]

Suppose that $\alpha > 0$, t > a, α, a, t ∈ R. Then

$$D^{\alpha}f(t)=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\dfrac{d^n}{dt^n}\displaystyle\int_0^t\dfrac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau\ , & n-1<\alpha<n\in N\\ \dfrac{d^nf(t)}{dt^n}, & \alpha=n\in N\end{cases} \tag{2.2}$$

is called the Caputo fractional derivative or the Caputo fractional differential operator of order α.

- **The Mittag-Leffler Function**

The Mittag-Leffler function is a generalization of the exponential function, first introduced as a one-parameter function by the series

$$E_{\alpha}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)},\quad \alpha>0,\ \alpha\ \epsilon\ R, z\ \in\ C \tag{2.3}$$

Later, the two-parameter generalization Mittag-Leffler function as:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},\quad \alpha,\beta>0,\ \alpha,\beta\ \in\ R, z\ \in\ C \tag{2.4}$$

3. Problem formulation

Consider a thick rectangular plate with length a, width b and thickness c occupying the space $D=\{(x,y,z)\in R^3: 0\le x\le a, 0\le y\le b, 0\le z\le c\}$. Initially the rectangular plate is at arbitrary temperature $f(x,y,z)$. For time $t>0$, heat is generated within the plate at a rate of $g(x,y,z,t)$ Btu/hr ft^3, while the remaining boundaries are kept at zero temperature. An equation of the Caputo type time fractional differential equation of order α is used to create a mathematical model that consider a thick rectangular plate with internal heat generation.

The temperature $T(x,y,z,t)$ of the thick rectangular plate at time t satisfying the time fractional heat conduction equation

$$\frac{\partial^2T}{\partial x^2}+\frac{\partial T}{\partial y^2}+\frac{\partial^2T}{\partial z^2}+\frac{g(x,y,z,t)}{k_t}=\frac{1}{k}\frac{\partial^{\alpha}T}{\partial t^{\alpha}}, \tag{3.1}$$

where k_t and k are thermal conductivity and thermal diffusivity of the material of the plate.

with the boundary conditions,

$$T=0,\qquad \text{at all boundary surfaces for } t>0, \tag{3.2}$$

and initial conditions,

$$T = f(x, y, z), \qquad \text{at } t = 0,\ 0 < \alpha \le 1\ , \tag{3.3}$$

$$\frac{\partial T}{\partial t} = 0, \qquad \text{at } t = 0,\ 1 < \alpha \le 2 \tag{3.4}$$

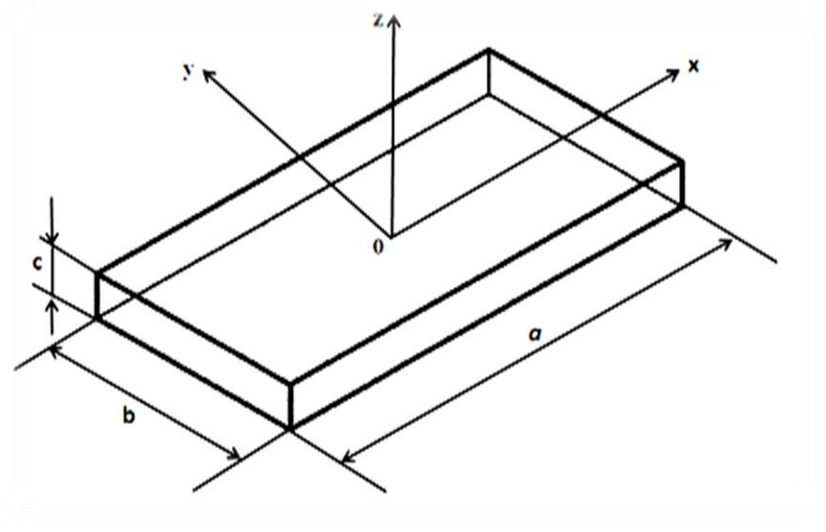

Fig. 1. Geometrical representation of the plate.

4. Analysis

Here the plate is assumed sufficiently thick and considered free from traction. Since the plate is in a plane stress state without bending. Airy stress function method is applicable to the analytical development of the thermoelastic field. Airy stress function $U(x, y, z, t)$ which satisfy the following relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 U = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T \tag{4.1}$$

where λ and E are linear coefficient of the thermal expansion, Youngs modulus elasticity of the material of the plate.

The displacement components u_x, u_y and u_z in the X, Y and Z direction are represented in the integral form as

$$u_x = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2}\right) + \lambda T\right] dx \tag{4.2}$$

$$u_y = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial y^2}\right) + \lambda T\right] dy \tag{4.3}$$

$$u_z = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2}\right) + \lambda T\right] dz \tag{4.4}$$

where ν is the poissons ratio of the material of the plate.

The stress components are:

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) \tag{4.5}$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2}\right) \tag{4.6}$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) \tag{4.7}$$

Equations [3.1] to [4.7] constitute the mathematical formulation of the problem under consideration.

5. Determination of Temperature Field

To obtain the expression for temperature function $T(x, y, z, t)$; we introduce the ‘‘triple integral transform’’and its corresponding ‘triple-inversion formula’ as defined in [35] respectively as

$$\begin{aligned}\overline{T}(\beta_m, \nu_n, \eta_p, t) = \quad & \int_{x'=0}^{a}\int_{y'=0}^{b}\int_{z'=0}^{c} K\,(\beta_m, x').K(\nu_n, y').K(\eta_p, z') \\ & \times T(x', y', z', t)dx'dy'dz'\end{aligned} \tag{5.1}$$

$$T(x, y, z, t) = \quad \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty} K\,(\beta_m, x).K(\nu_n, y).K(\eta_p, z)\overline{T}(\beta_m, \nu_n, \eta_p, t) \tag{5.2}$$

where the kernels

$$K(\beta_m, x) = \sqrt{\frac{2}{a}}\sin(\beta_m x) \tag{5.3}$$

$$K(\nu_n, y) = \sqrt{\frac{2}{b}}\sin(\nu_n y) \tag{5.4}$$

$$K(\eta_p, z) = \sqrt{\frac{2}{c}}\sin(\eta_p z) \tag{5.5}$$

and eigenvalues are

β_m is m^{th} root of transcendental equation

$$\sin(\beta_m.a) = 0$$

i.e.

$$\beta_m = \frac{m\pi}{a}, \qquad m = 1,2,3,\ldots \tag{5.6}$$

ν_n is n^{th} root of transcendental equation

$$\sin(\nu_n.b) = 0$$

i.e.

$$\nu_n = \frac{n\pi}{b}, \qquad n = 1,2,3,\ldots \tag{5.7}$$

η_p is p^{th} root of transcendental equation

$$\sin(\eta_p.c) = 0$$

i.e.

$$\eta_p = \frac{p\pi}{c}, \qquad p = 1,2,3,\ldots \tag{5.7}$$

On applying triple-integral transform defined in equation [5.1] to equations [3.1]–[3.4] and then using their inversions defined in equation [4.2], one obtains the expressions of the temperature as:

$$\begin{aligned} T(x,y,z,t) = {} & \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty} K\,(\beta_m,x).K(\nu_n,y).K(\eta_p,z)\frac{1}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)} \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\left[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t}\right. \\ & \left.\times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\right] \end{aligned} \tag{5.9}$$

where $E_{\alpha,\alpha}(.)$ - two-parameter Mittag-Leffler function,

$$\overline{f}\left(\beta_m,\nu_n,\eta_p\right) = \int_{x'=0}^{a}\int_{y'=0}^{b}\int_{z'=0}^{c}\sin\,(\beta_m x')\sin(\nu_n y')\sin\left(\eta_p z'\right)f\,(x',y',z')dx'dy'dz'$$

$$\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right) = \int_{x'=0}^{a}\int_{y'=0}^{b}\int_{z'=0}^{c}\sin\,(\beta_m x')\sin(\nu_n y')\sin\left(\eta_p z'\right)g(x',y',z',t')dx'dy'dz'$$

6. Distributions of Displacement and Stresses

Using equation [5.9] in [4.1], one obtains

$$\begin{aligned} U = {} & \frac{8E}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sin\,(\beta_m x)\sin(\nu_n y)\sin\left(\eta_p z\right)\frac{1}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2} \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\left[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t}\right. \\ & \left.\times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\right] \end{aligned} \tag{6.1}$$

Now using equations [5.9] and [6.1] in equations [4.2] to [4.4], one obtains the expressions for displacement as

$$\begin{aligned} u_x = {} & \frac{-8}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\left(\frac{\cos(\beta_m x)}{\beta_m}\right)\sin(\nu_n y)\sin\left(\eta_p z\right)\left[\frac{(\nu-1)\beta_m^2-2\left(\nu_n^2+\eta_p^2\right)}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2}\right] \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\left[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t}\right. \\ & \left.\times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\right] \end{aligned} \tag{6.2}$$

$$\begin{aligned} u_y = {} & \frac{-8}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sin\,(\beta_m x)\left(\frac{\cos(\nu_n y)}{\nu_n}\right)\sin\left(\eta_p z\right)\left[\frac{(\nu-1)\nu_n^2-2\left(\beta_m^2+\eta_p^2\right)}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2}\right] \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\left[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t}\right. \\ & \left.\times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\right] \end{aligned} \tag{6.3}$$

$$\begin{aligned} u_z = \; & \frac{-8}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sin(\beta_m x)\sin(\nu_n y)\left(\frac{\cos(\eta_p z)}{\eta_p}\right)\left[\frac{(\nu-1)\eta_p^2-2(\beta_m^2+\nu_n^2)}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2}\right] \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\Bigg[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t} \\ & \times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\Bigg] \end{aligned} \tag{6.4}$$

Now using equations [5.9] and [6.1] in equations [4.5] to [4.7], one obtains expressions for thermal stresses as

$$\begin{aligned} \sigma_{xx} = \; & \frac{-8E}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sin(\beta_m x)\sin(\nu_n y)\sin(\eta_p z)\frac{\left(\nu_n^2+\eta_p^2\right)}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2} \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\Bigg[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t} \\ & \times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\Bigg] \end{aligned} \tag{6.5}$$

$$\begin{aligned} \sigma_{yy} = \; & \frac{-8E}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sin(\beta_m x)\sin(\nu_n y)\sin(\eta_p z)\frac{\left(\beta_m^2+\eta_p^2\right)}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2} \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\Bigg[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t} \\ & \times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\Bigg] \end{aligned} \tag{6.6}$$

$$\begin{aligned} \sigma_{zz} = \; & \frac{-8E}{abc}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sin(\beta_m x)\sin(\nu_n y)\sin(\eta_p z)\frac{\left(\beta_m^2+\nu_n^2\right)}{k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)^2} \\ & \times\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\Bigg[\overline{f}\left(\beta_m,\nu_n,\eta_p\right)+\frac{k}{k_t} \\ & \times\int_{t'=0}^{t}\left[t^{\alpha-1}E_{\alpha,\alpha}\left(-k\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t^{\alpha}\right)\right]\overline{g}\left(\beta_m,\nu_n,\eta_p,t'\right)\Bigg] \end{aligned} \tag{6.7}$$

7. Numerical Calculations and Discussion

Setting

$f(x,y,z)=e^{t}(a-x)(1-e^{x})(b-y)(1-e^{y})(c-z)(1-e^{z})$,

$g(x,y,z,t)=g_i\delta(x-x_1)\delta(y-y_1)\delta(z-z_1)\delta(t-\tau)$ where δ is the Derac-delta function and $g(x,y,z,t)$ is an instantaneous line heat source situated at the center of the rectangular plates.

Dimension

Length of rectangular plate a = 3 ft
Breadth of rectangular plate b = 2 ft
Height of rectangular plate c = 1 ft
Central length of rectangular plate x_1 =1.5 ft

Central breadth of rectangular plate y_1 = 1 ft
Central 1eight of rectangular plate z_1 = 0.5 ft
For the purpose of doing a numerical calculation for a thick rectangular plate, the material aluminium (Pure) was selected. PTC Mathcad Prime, a computational mathematical software, was used to create the numerical calculations and graphics.
The aluminium (pure) material was chosen for purposes of numerical evaluations. The parameters of the problem are given in FPS units, as illustrated in Table 1.

TABLE -1:- MATERIAL CONSTANT

Physical constant	**Value**
Thermal diffusivity (k)	3.33 ft^2/hr
Thermal conductivity (k_t)	117 Btu/(hr.ft.^{0}F)
Density (ρ)	169 lb/ft^3
Specific heat (c_p)	0.208 Btu/(lb.^{0}F)
Coefficient of linear thermal expansion (a_t)	12.84×10^{-6} 1/F
Poisson ratio (ν)	0.35
Lame's constants (μ)	26.67
Young's modulus (E)	70 GPa

In this section, we analysed a fractional heat conduction problem for a thick rectangular plate. We have discussed the variation of temperature, displacement and thermal stresses for different values of fractional-order parameter $\alpha = 0.5,1,1.5,2$. The graphs are plotted for different fractional-order parameter $\alpha = 0.5,1,1.5,2$ at time $t = 1$ hr.

Figure 2 shows the temperature distribution for different values of fractional-order parameter $\alpha = 0.5,1,1.5,2$ in X-direction. It is observed that, the magnitude of the temperature increases with decrease the value of fractional-order parameter α and it attains its peak at $x = 1.5$. Also, it is zero at both edges ($x = 0$ and $x = 3$) of the rectangular plate along the X-direction and it shows the normal curve.

Figures 3–5 show the displacement distribution functions u_x, u_y and u_z in X, Y and Z-directions for different values of α. It is clear that, the displacement functions increase from initial edge towards the extreme edge and it becomes zero at the middle part for different values of α.

Figure 6-8 shows the stress distribution σ_{xx}, σ_{yy} and σ_{zz} for different values of α in X, Y and Z-direction respectively. We observe that, the magnitude of both the stresses is maximum at the middle part with increasing the value of fractional-order parameter α and becomes zero at the initial and extreme edges in the X, Y and Z-direction respectively. Also, the stress components σ_{xx}, σ_{yy} and σ_{zz} are compressive throughout the plate and shows the normal curve.

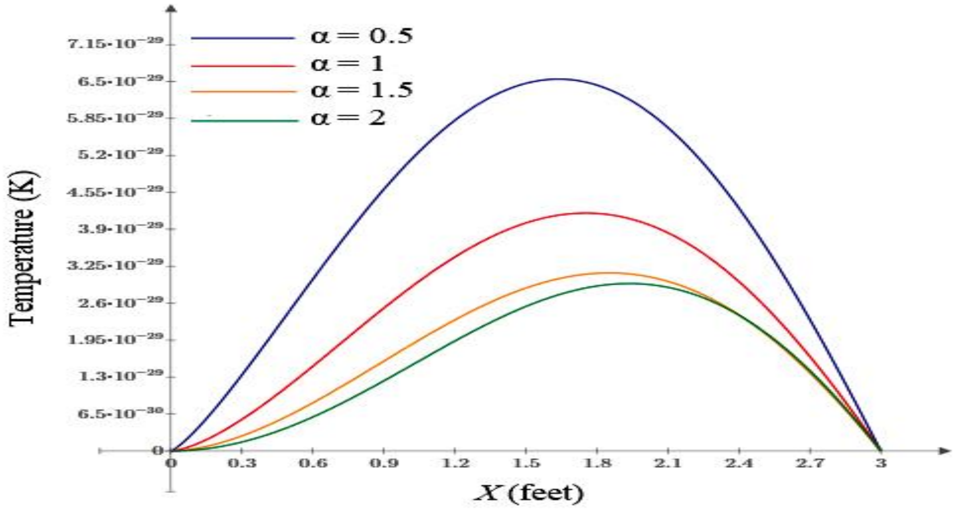

Fig. 2. Temperature distribution for different values of α in X-direction.

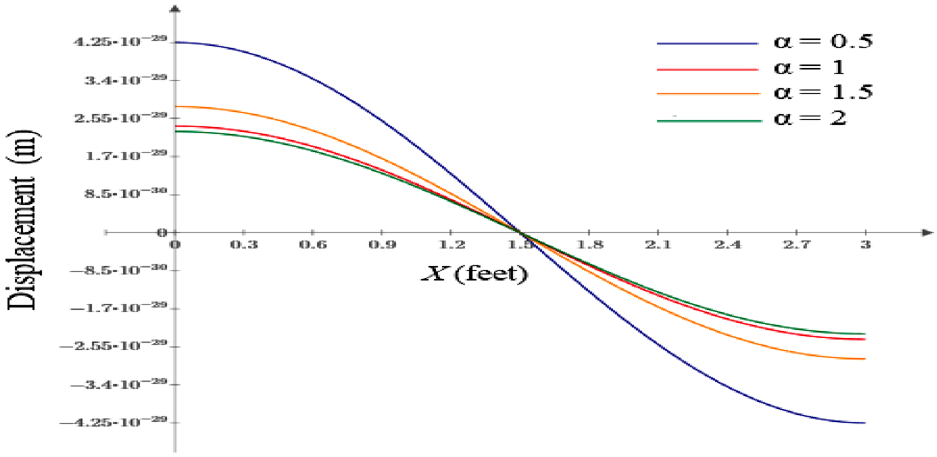

Fig. 3. Displacement distribution u_x for different values of α in X-direction.

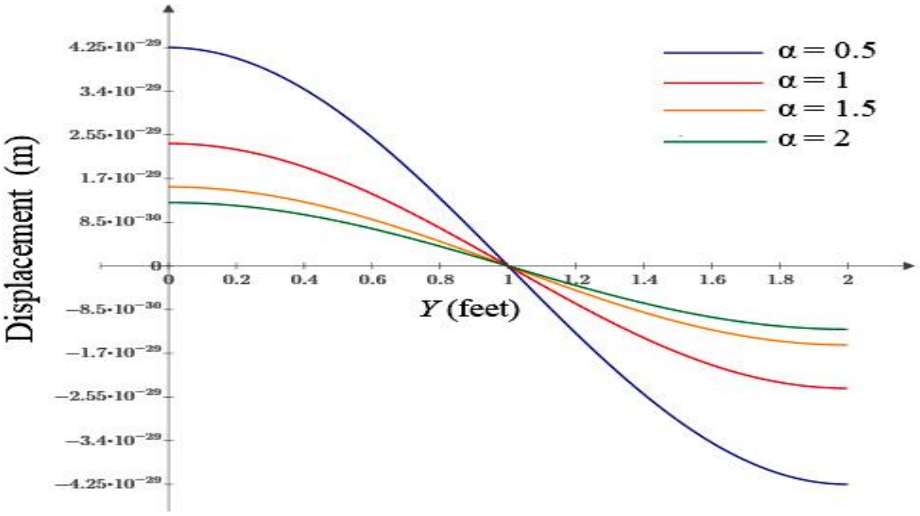

Fig. 4. Displacement distribution u_y for different values of α in Y-direction.

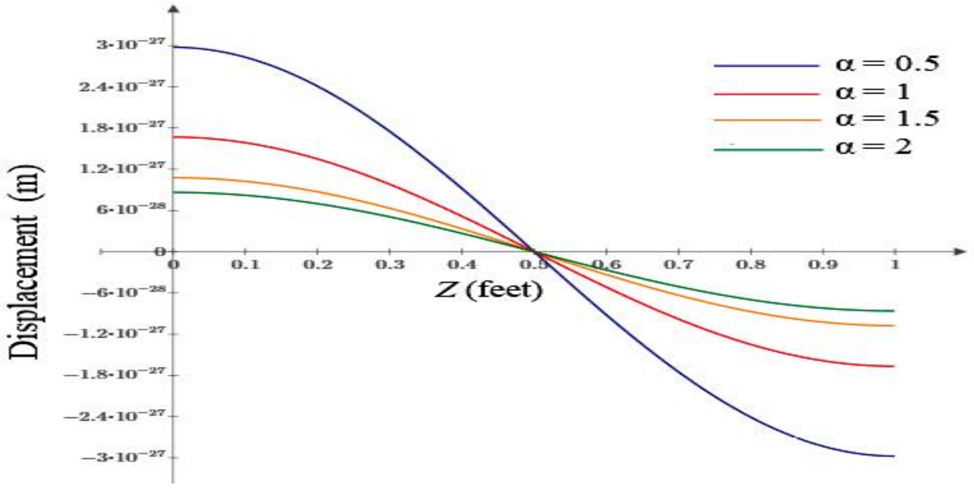

Fig. 5. Displacement distribution u_z for different values of α in Z-direction.

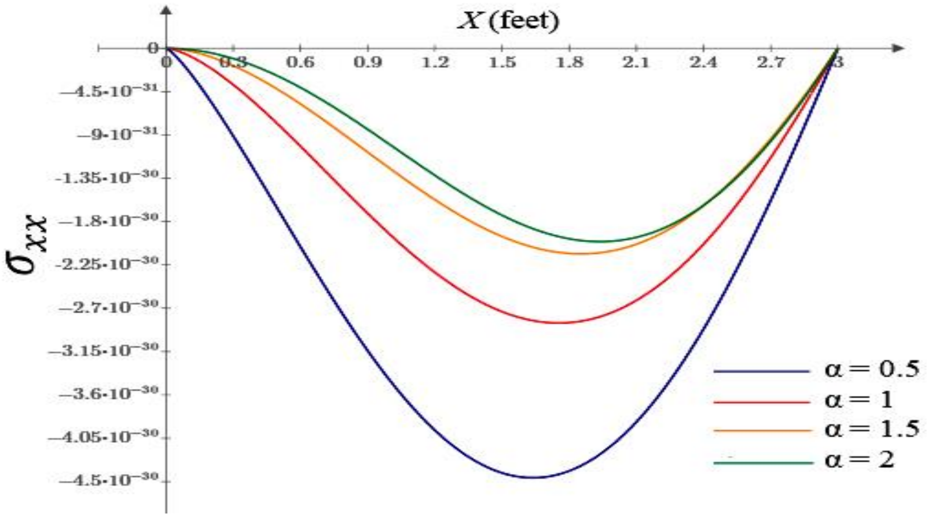

Fig. 6. Stress distribution σ_{xx} for different values of α in X-direction.

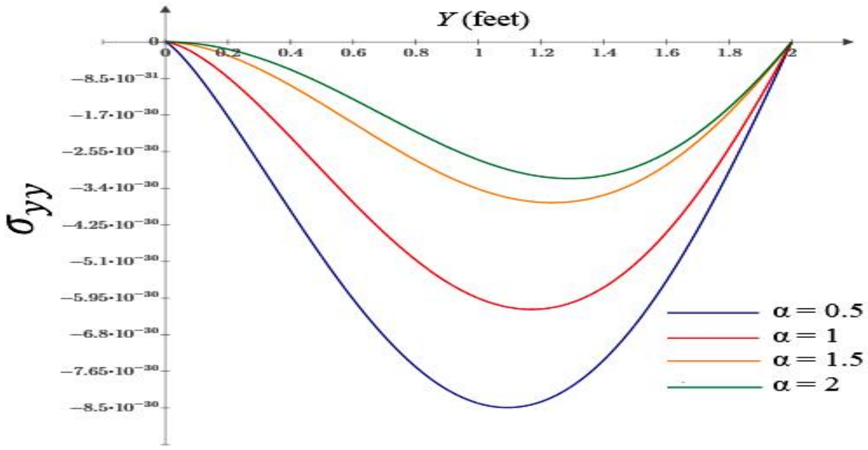

Fig. 7. Stress distribution σ_{yy} for different values of α in Y-direction.

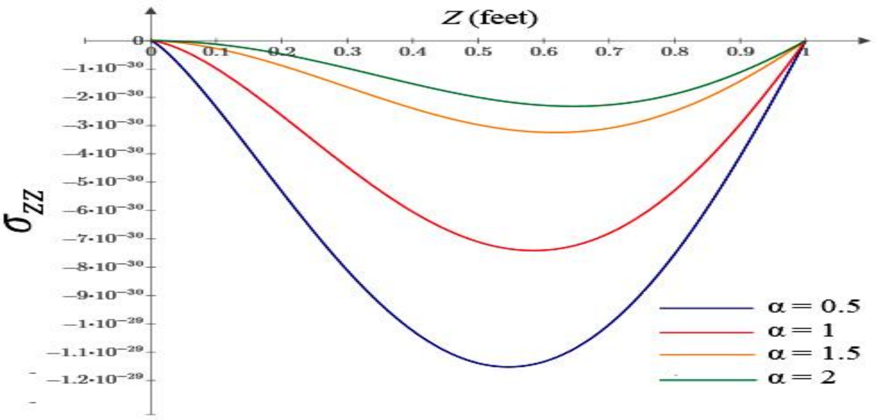

Fig. 8. Stress distribution σ_{zz} for different values of α in Z-direction.

8. Conclusion

In this article, we proposed the time-fractional heat conduction equation with zero initial conditions. Integral transform technique is adopted for computing the temperature of heat conduction equation. The results are obtained in the form of two-parameter Mittag-Leffler function. Figures 2-8 shows the behavior of temperature, displacement and thermal stresses along X, Y and Z directions for the different values of the fractional-order parameter $\alpha = 0.5,1,1.5,2$ and shows the variation between classical and fractional-order thermoelasticity.

The following concluding remakes can be considered according to the results of the present study.

1. When $\alpha = 1$, all the graphical representation satisfied the heat diffusion equation, whereas $\alpha = 2$, gives the representation of wave equations.

2. It is observed that, for the different values of the fractional-order parameter α in the X, Y and Z directions the effect temperature, displacement and thermal stresses represents the weak, normal and strong conductivity, within the range of $0 < \alpha < 1, \alpha = 1$ and $1 < \alpha < 2$ respectively.

3. The stress distribution functions σ_{xx}, σ_{yy} and σ_{zz} are tensile and they form normal curve along the X, Y and Z direction respectively. Also, they are directly proportional to the different values of α in the X, Y and Z direction respectively.

4. Fractional order α governs the non-local behavior of heat transfer process, and then alters the response time and overshooting occurrence of the temperature. Larger the parameter α, the slower the velocity of heat waves-like. Its effect is limited to short times and disappears for long time.

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