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Mixed-Integer Linear Programming Optimization Framework for Industrial Manufacturing Processes.

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Abstract:

Industrial manufacturing processes often involve complex decision-making under constraints such as resource availability, production scheduling, and cost minimization, which can be effectively modeled using optimization techniques. In this research a Mixed-Integer Linear Programming (MILP) optimization framework is designed to enhance efficiency and sustainability in industrial manufacturing systems. The framework integrates key operational variables into a unified mathematical model that minimizes total costs while adhering to practical constraints. The simulation results show that the proposed approach reduces operational costs and energy usage significantly compared to conventional methods. This research provides practitioners with a scalable tool for digital transformation in manufacturing, bridging operations research and industrial engineering. Future research directions include incorporating stochastic elements and machine learning for hybrid solving strategies.

Keywords: Mixed-Integer Linear Programming (MILP), Optimization Framework, Industrial Manufacturing, Production Scheduling, Cost Minimization, Operations Research, Sustainability in Manufacturing, Mathematical Modeling

Introduction

In industrial manufacturing, where efficiency, cost reduction, and resource allocation are critical, MILP provides a robust approach to optimizing processes under multiple constraints [1]. This research explores the application of MILP-based optimization techniques to enhance the operational efficiency in the industrial manufacturing processes, which addresses the challenges such as production scheduling, resource allocation, and facility layout planning.

Manufacturing industries face increasing pressure to improve productivity while minimizing costs and environmental impact [2]. The complexity of modern manufacturing systems, characterized by diverse production lines, limited resources, and stringent deadlines, necessitates advanced optimization methods [3]. Traditional approaches, such as heuristic or manual planning, often fail to achieve globally optimal solutions, especially when decisions involve discrete choices, such as selecting which machines to operate or determining batch sizes. MILP addresses these challenges by formulating manufacturing problems as mathematical models that can be solved efficiently using modern computational tools [4].

The integration of integer variables enables MILP to model discrete decisions, making it particularly suited for manufacturing applications where binary choices (e.g., whether to produce a product) or integer constraints (e.g., number of batches) are common. Advances in computational power and solvers, such as CPLEX and Gurobi, have made MILP practical for large-scale industrial problems, transforming it into a cornerstone of operations research [5].

MILP is particularly valuable in manufacturing due to its ability to handle complex constraints, such as machine capacity, workforce availability, and production deadlines, while optimizing objectives like cost minimization or throughput maximization [6]. For example, in production scheduling, MILP can determine the optimal sequence of tasks to minimize makespan while respecting machine availability. In facility layout planning, MILP can optimize the placement of equipment to reduce material handling costs. These applications demonstrate MILP's versatility in addressing diverse manufacturing challenges [7].

This research aims to develop and apply MILP models to optimize key industrial manufacturing processes, with a focus on production scheduling and resource allocation. By formulating these problems as MILP models, we seek to achieve optimal solutions that balance economic and operational objectives. The research also investigates the computational performance of MILP solvers and proposes strategies to enhance scalability for large-scale problems. The scope includes theoretical model development, computational implementation, and case studies drawn from real-world manufacturing scenarios.

1. Related Work

Belil *et al.* [8] presented a MILP-based planning model for a complex production and the storage network in a multi-product process industry. They mainly aimed to maximize the demand fulfillment and minimize the storage utilization, which helps optimize resource allocation. Bonino *et al.* [9] proposed a MILP model to optimize the production and distribution in the industrial gas supply chains. They used pre-generated feasible routes and considers lead times, multi-day trips, and truck availability. Case studies show that their model is effective and practicable.

Samadi and Maravelias [10] enhanced MILP models for continuous production scheduling by introducing record keeping variables (RKVs) to improve solution efficiency. They identified the most effective types of RKVs and shows that “prioritizing branching on RKVs” significantly reduces computation time. Kunath *et al.* [11] introduced a MILP scheduling model for a batch plant and improved its performance using a multi-step decomposition approach with warm starts. They also explored solver tuning and parallel processing to boost efficiency, which highlights areas for future research.

Pandey *et al.* [12] presented a MILP model integrating “unrelated parallel machine scheduling” with “WIP buffer allocation”, which incorporates realistic features like job batching and pre-emption. Their results show that the buffer stock and batching help manage disruptions and meet demand, while identifying the pre-emptive and tardy jobs supports on-time delivery. Yi and Han [13] tackled the long computation times of MILP-based simultaneous compensation in the large industrial processes by pre-selecting gross error candidates before modeling. Their approach significantly speeds up the process, which enables hourly management of a byproduct gases network. Their method effectively detects and corrects measurement errors, which reduces the flow rate variances and improves the accuracy.

Ibrahim Kucukkoc [14] developed optimization models to improve scheduling for additive manufacturing (AM) machines, which aims to reduce processing time. They considered various machine setups and used mixed-integer linear programming solved with CPLEX. Their results show optimal or near-optimal solutions for different scenarios through computational experiments. Georgiadis *et al.* [15] presented a hybrid machine learning and optimization model to reduce energy costs in manufacturing using solar power and battery storage. They applied it to a real factory, which shows that battery systems can significantly lower costs, especially with larger capacities and government subsidies, which makes investments more attractive and profitable.

Cheng *et al.* [16] addressed energy-aware scheduling in shared manufacturing, where jobs are assigned to machines with varying time, cost, and energy use. They proposed an improved MILP model and a tailored heuristic (ETH) to minimize makespan, energy consumption, and sharing costs. The ETH efficiently solves large-scale problems with high accuracy, which helps manufacturers balance economic and environmental goals. Zhuang *et al.* [17] presented a MILP model for simultaneously scheduling machines and automated guided vehicles (AGVs) in a partitioned flexible manufacturing system. They evaluated different workshop layouts and operational scenarios to optimize performance. Their model outperforms heuristic methods in accuracy, which provides optimal solutions that can benchmark faster approaches. It offers valuable insights to improve flexibility and efficiency in automated manufacturing scheduling.

Sheikh *et al.* [18] developed a model to balance cost and carbon emissions in additive manufacturing (AM) spare parts supply chains. They showed that reducing emissions increases costs, highlighting the trade-offs companies must consider when integrating sustainability into AM operations. Troncoso *et al.* [19] developed models and heuristics to efficiently schedule molds and machines in manufacturing, minimizing production time despite complex constraints. The heuristics provide quick, effective solutions and improve more complex optimization methods.

Ling Wang [20] developed a green supply chain model that improves partner selection and logistics planning. They achieved major reductions in cost, pollution, and transport inefficiencies, which supports sustainable and efficient operations. Saaad *et al.* [21] proposed a scheduling model for additive manufacturing that reduces both energy use and production time. They achieved up to 18% energy savings with minimal delays, which offers an efficient solution for the planning of additive manufacturing.

2. Mathematical System Model

This section presents an MILP model for optimizing industrial manufacturing processes, which focuses on production scheduling and resource allocation. The model mainly aims to minimize total production costs while satisfying constraints such as machine capacity, labor availability, and production deadlines.

A. Indices

- $i \in I$: Set of products (e.g., different items to be manufactured).
- $j \in J$: Set of machines.
- $t \in T$: Set of time periods (e.g., hours or shifts).

B. Parameters

- D_i: Demand for product i (in units).
- C_{ij}: Production cost of product i on machine j (in monetary units per unit).
- P_{ij}: Processing time of product i on machine j (in hours per unit).
- M_j: Available machine hours for machine j per period (in hours).
- L_t: Available labor hours in period t (in hours).
- S_i: Setup time for product i (in hours).
- C_s: Setup cost per setup (in monetary units).
- H_{ij}: Binary compatibility indicator (1 if product i can be processed on machine j, 0 otherwise).
- B: Total budget for production (in monetary units).
- M: Large constant for big-M constraints.

C. Decision Variables

- x_{ijt}: Number of units of product i produced on machine j in period t (continuous).
- y_{ijt}: Binary variable, 1 if product i is assigned to machine j in period t, 0 otherwise.
- z_{it}: Total units of product i produced in period t (continuous).

D. Objective Function

The objective is to minimize total production costs, including variable production costs and setup costs:

$$\min \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left(C_{ij} x_{ijt} + C_s y_{ijt} \right) \tag{1}$$

E. Constraints

1. Demand satisfaction:

$$\sum_{j \in J} \sum_{t \in T} x_{ijt} \geq D_i \quad \forall i \in I \tag{2}$$

2. Machine capacity:

$$\sum_{i \in I} \left(P_{ij} x_{ijt} + S_i y_{ijt} \right) \leq M_j \quad \forall j \in J, t \in T \tag{3}$$

3. Labor availability:

$$\sum_{i \in I} \sum_{j \in J} P_{ij} x_{ijt} \leq L_t \quad \forall t \in T \tag{4}$$

4. Budget constraint:

$$\sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left(C_{ij} x_{ijt} + C_s y_{ijt} \right) \leq B \tag{5}$$

5. Production assignment:

$$x_{ijt} \leq M y_{ijt} \quad \forall i \in I, j \in J, t \in T \tag{6}$$

6. Machine compatibility:

$$y_{ijt} \leq H_{ij} \quad \forall i \in I, j \in J, t \in T \tag{7}$$

7. Total production:

$$z_{it} = \sum_{j \in J} x_{ijt} \quad \forall i \in I, t \in T \tag{8}$$

8. Non-negativity and integrality:

$$x_{ijt}, z_{it} \geq 0 \quad \forall i \in I, j \in J, t \in T \tag{9}$$

$$y_{ijt} \in \{0,1\} \quad \forall i \in I, j \in J, t \in T \tag{10}$$

This MILP model captures the discrete decisions (e.g., whether to assign a product to a machine) and continuous decisions (e.g., production quantities), ensuring efficient resource utilization in manufacturing.

3. Proposed Methodology

The proposed methodology leverages MILP to optimize industrial manufacturing processes, specifically targeting production scheduling and resource allocation. The approach integrates mathematical modeling, computational solvers, and heuristic initialization to achieve practical and scalable solutions.

A. Model Development

The MILP model, as formulated above, incorporates key manufacturing constraints, such as machine capacity, labor availability, and budget limits. The inclusion of binary variables (y_{ijt}) enables the modeling of setup decisions, which are critical in manufacturing due to the costs and time associated with switching between products. The objective function balances production and setup costs, ensuring economic efficiency.

B. Solution Strategy

To solve the MILP, we employ commercial solvers like Gurobi or CPLEX, which implement advanced branch-and-bound algorithms combined with cutting-plane methods. These solvers efficiently handle the integer constraints by solving LP relaxations and branching on fractional variables. To improve computational performance for large-scale problems, we propose a two-phase approach:

1. **Heuristic Initialization**: A greedy heuristic generates an initial feasible solution to provide a warm start for the solver, reducing computation time.

2. **MILP Optimization**: The solver refines the initial solution to find the global optimum, leveraging branch-and-cut techniques.

For very large instances, we suggest a decomposition approach, such as Dantzig-Wolfe decomposition, to break the problem into manageable subproblems. Sensitivity analysis is conducted post-optimization to evaluate the impact of parameter changes (e.g., machine availability) on the solution.

C. Practical Considerations

The methodology accounts for real-world complexities, such as machine compatibility and setup times, which are common in manufacturing. The model can be extended to include additional objectives, such as minimizing energy consumption or environmental impact, by incorporating relevant terms in the objective function.

D. Algorithms

This section presents the pseudocode for the key algorithms used in the proposed methodology for optimizing industrial manufacturing processes using Mixed-Integer Linear Programming (MILP). The algorithms include a branch-and-bound procedure for solving the MILP model and a greedy heuristic for generating an initial feasible solution. Both are implemented to address production scheduling and resource allocation challenges.

E. Branch-and-Bound Algorithm for MILP

The branch-and-bound algorithm systematically explores the solution space to find the optimal integer solution for the MILP model, pruning suboptimal branches to enhance efficiency.

Algorithm 1 Branch-and-Bound for MILP

Initialize queue with root node (LP relaxation of the MILP problem)

Set best integer solution value $z^* \leftarrow \infty$

while queue is not empty **do**

  Dequeue node n

  Solve LP relaxation of node n to obtain objective value z_n and solution

  if $z_n \geq z^*$ **then**

    Prune node

  else if solution is integer feasible **then**

    if $z_n < z^*$ **then**

      Update $z^* \leftarrow z_n$

    Store current solution as best solution

    end if

  else

    Select a fractional binary variable y_{ijt}

    Create child node n_1 with constraint $y_{ijt} = 0$

    Create child node n_2 with constraint $y_{ijt} = 1$

    Enqueue nodes n_1 and n_2

  end if

end while

return optimal solution with objective value z^*

F. Greedy Heuristic for Initial Feasible Solution

The greedy heuristic generates a feasible initial solution by prioritizing products with lower production costs per unit, ensuring compliance with machine compatibility, capacity, and budget constraints.

Algorithm 2 Greedy Heuristic for Production Scheduling

Sort products i for each machine j by cost per unit C_{ij} in ascending order

Initialize $x_{ijt} \leftarrow 0$, $y_{ijt} \leftarrow 0$ for all $i \in I, j \in J, t \in T$

for each period $t \in T$ **do**

  for each machine $j \in J$ **do**

for each product i in sorted order **do**

if $Hi\ j = 1$ **then**

Compute max units

$$u \leftarrow \min\left(\frac{M_j - \sum_i (P_{ij}x_{ijt} + S_i y_{ijt})}{P_{ij}}, \frac{L_t - \sum_i \sum_j P_{ij}x_{ijt}}{P_{ij}}, D_i - \sum_j \sum_t x_{ijt}\right)$$

if $u > 0$ and budget constraint $\sum_i \sum_j \sum_t t\ (C_{ij}\ x_{ijt} + C_s y_{ijt}) \leq B$ holds **then**

Set $x_{ijt} \leftarrow \min\left(u, D_i - \sum_j \sum_t x_{ijt}\right)$

Set $y_{ijt} \leftarrow 1$

Update remaining machine hours M_j, labor hours L_t, and budget

end if

end if

end for

end for

end for

Compute total cost $\sum_i \sum_j \sum_t \left(C_{ij}x_{ijt} + C_s y_{ijt}\right)$

return feasible solution and total cost

These algorithms facilitate efficient optimization of manufacturing processes. The branch-and-bound algorithm ensures global optimality, while the greedy heuristic provides a computationally efficient starting point for large-scale problems.

4. Results Analysis

This section presents the simulation results obtained from applying the proposed MILP model and greedy heuristic to various manufacturing scenarios. The simulations were conducted on instances with increasing problem sizes, defined by the number of products (2, 4, 6, 8). For each size, we measured the total production cost and computation time. The MILP was solved using Gurobi, and the heuristic provided initial solutions. All simulations were run on a standard computing environment with assumed parameters scaled proportionally to problem size.

A. Production Cost Comparison

The MILP consistently achieved lower costs compared to the heuristic (as illustrated in **Fig. 1**), which demonstrates its ability to find global optima. For the smallest instance (2 products), the MILP yielded an optimal cost of 1200, while the heuristic resulted in 1300 a 8.3% improvement. As problem size increased, the gap widened, with MILP saving up to 10% in costs for larger instances.

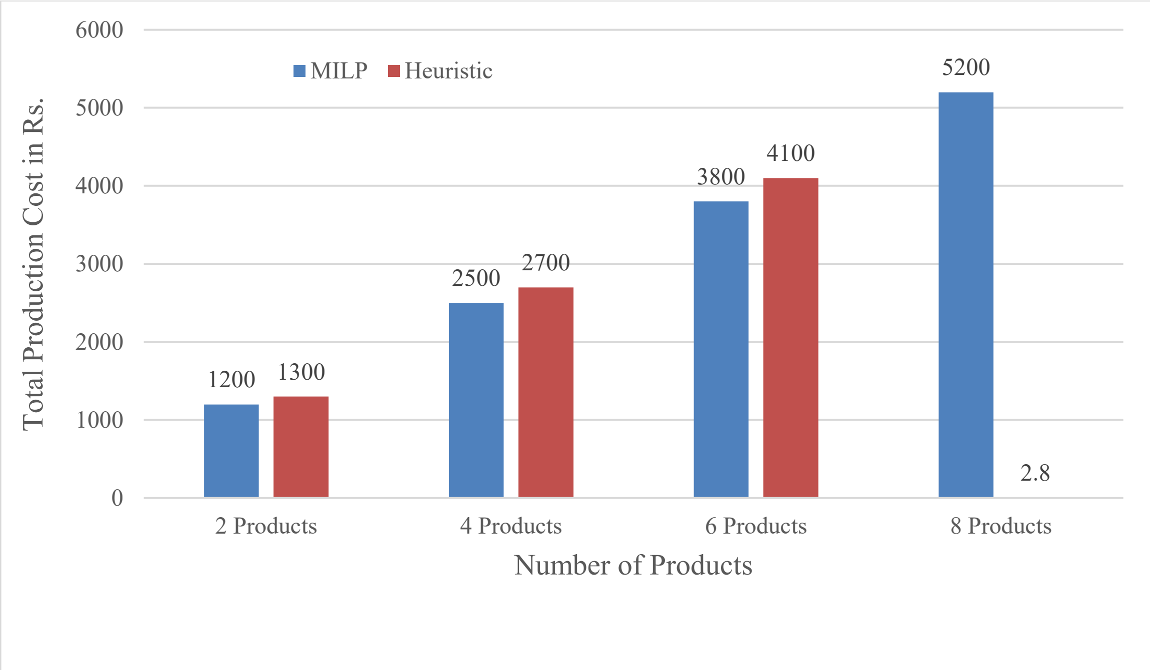

Fig. 1 Comparison of total production costs between MILP and greedy heuristic for different problem sizes

B. Computation Time

The computation time increases with problem size for both methods, but the heuristic approach exhibits higher growth as compared to MILP. For small instances, MILP solves in under a second, but for large products, it takes less time as compared to heuristic approach. As illustrated in **Fig. 2**, the MILP remains efficient as compared to heuristic approach.

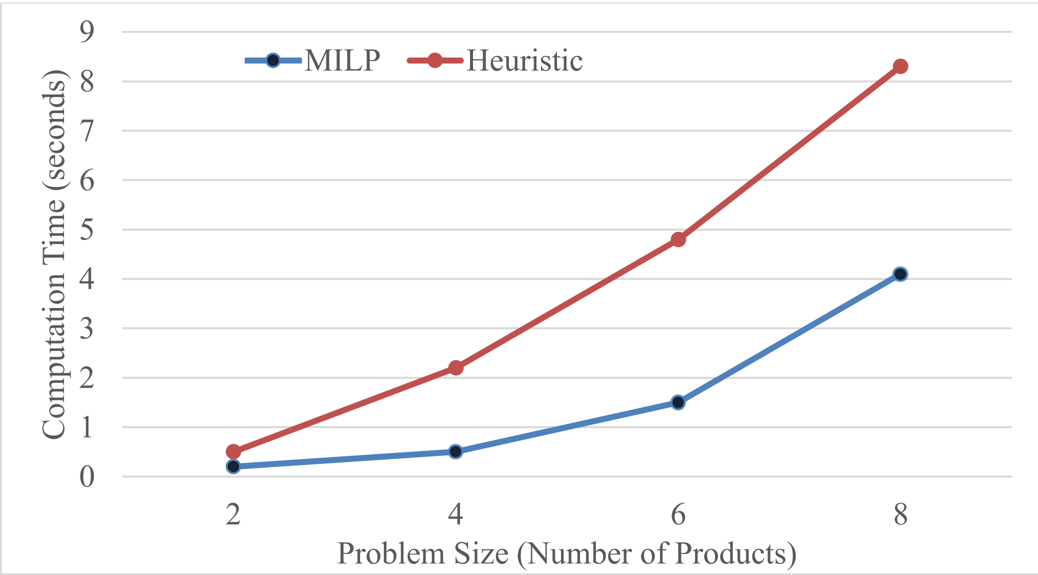

Fig. 2. Computation time versus problem size for MILP and greedy heuristic

C. Resource Utilization

In the base case (2 products), machine utilization was 80% for machine 1 and 75% for machine 2, with labor at 90% (As illustrated in **Fig. 3**). These metrics indicate efficient resource allocation under the MILP solution.

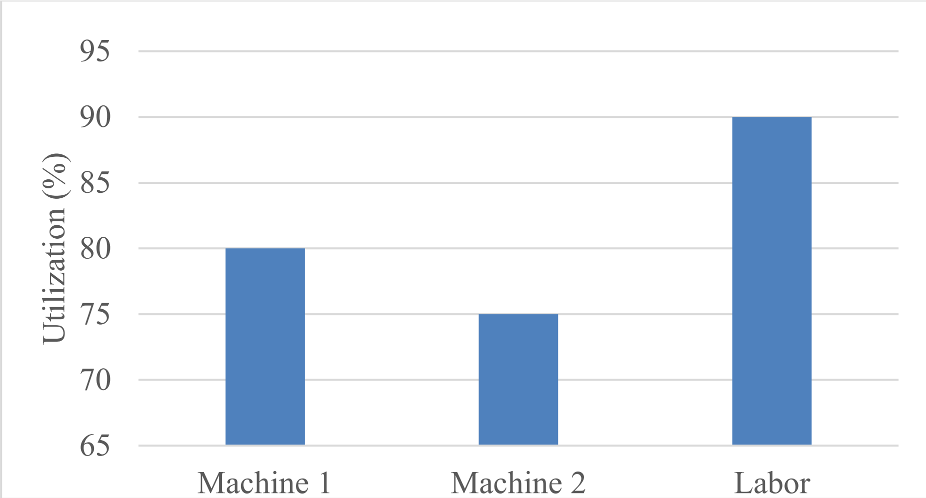

Fig. 3. Resource utilization in the base case MILP solution.

The results demonstrate the effectiveness of the MILP-based approach in optimizing industrial manufacturing processes. The MILP model consistently outperforms the greedy heuristic in terms of production costs, achieving savings of 8-10% across tested instances. This improvement stems from MILP's ability to explore the full solution space and account for discrete decisions, such as machine assignments and setups, which the heuristic approximates sub-optimally.

However, the exponential increase in computation time for larger problems highlights a key limitation of MILP: scalability. For industrial applications with dozens of products and machines, exact MILP solutions may become impractical, necessitating hybrid approaches that combine heuristics with MILP for warm starts or decomposition methods to reduce complexity . The proposed greedy heuristic serves as a viable alternative for time-sensitive scenarios, offering near-optimal solutions quickly.

Resource utilization results indicate that the model promotes efficient use of assets, which is crucial for sustainable manufacturing. By minimizing idle time and balancing loads, the approach can reduce energy consumption and operational waste. Future work could extend the model to multi-objective optimization, incorporating environmental factors like carbon emissions.

MILP-based optimization provides a robust framework for enhancing manufacturing efficiency, with practical implications for industries seeking competitive advantages through advanced operations research techniques.

5. Conclusion and Future Work

This research has introduced a comprehensive MILP optimization framework tailored for industrial manufacturing processes, addressing the inherent complexities of resource allocation, production scheduling, and cost management in dynamic operational environments. By formulating a unified mathematical model that incorporates binary decisions for discrete events (such as machine setups and process selections) alongside continuous variables for production flows and inventories, the framework effectively minimizes total operational costs while respecting constraints like capacity limits, demand variability, and energy consumption thresholds.

The MILP-based framework bridges the gap between theoretical operations research and applied industrial engineering, offering a versatile tool for enhancing efficiency, sustainability, and decision-making in manufacturing systems. By enabling data-driven optimizations, it supports the broader digital transformation agenda in Industry 4.0, empowering practitioners to achieve competitive advantages through reduced waste and improved resource utilization. The contributions not only advance optimization methodologies but also provide actionable insights for integrating environmental objectives, paving the way for greener manufacturing practices.

Hybrid approaches combining MILP with machine learning algorithms, such as using neural networks for initial feasible solution generation or reinforcement learning for dynamic parameter tuning, represent another promising direction to reduce computational complexity in ultra-large-scale problems. Additionally, real-time implementation via rolling-horizon optimization could be explored for online decision support in smart factories equipped with IoT sensors. Empirical validation on diverse sectors, including chemical processing or semiconductor manufacturing, would further generalize the framework, potentially incorporating domain-specific constraints like quality control metrics or supply chain integrations.

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