

An Edge Detection Algorithm based on Second Order Difference

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Abstract:

Edge detection is an important technique in image processing. Mathematics is the foundation of the edge detection algorithm. In this article, the relation of mathematics and edge detection algorithms is discussed. Also, the different edge detection algorithms like Canny, Sobel, Prewitt, Roberts etc. are discussed with their mathematical foundations.

In this article, a new edge detection algorithm is proposed based on second order forward difference with the equidistance points. The second order forward difference of the image intensity of each pixel is calculated. It is calculated in horizontal direction (x-direction) and vertical direction (y-direction) say M_x and M_y . The resultant kernel (M) is obtained by

combining both horizontal and vertical values at each pixel by $M = \sqrt{M_x^2 + M_y^2}$. This algorithm uses M as a mask for grey scale image. The effectiveness of the algorithm is tested for different images and compared with the other existing edge detection algorithms.

Keywords: Image Processing, Signal Processing, Edge Detection, Mask, Operator

1 Introduction:

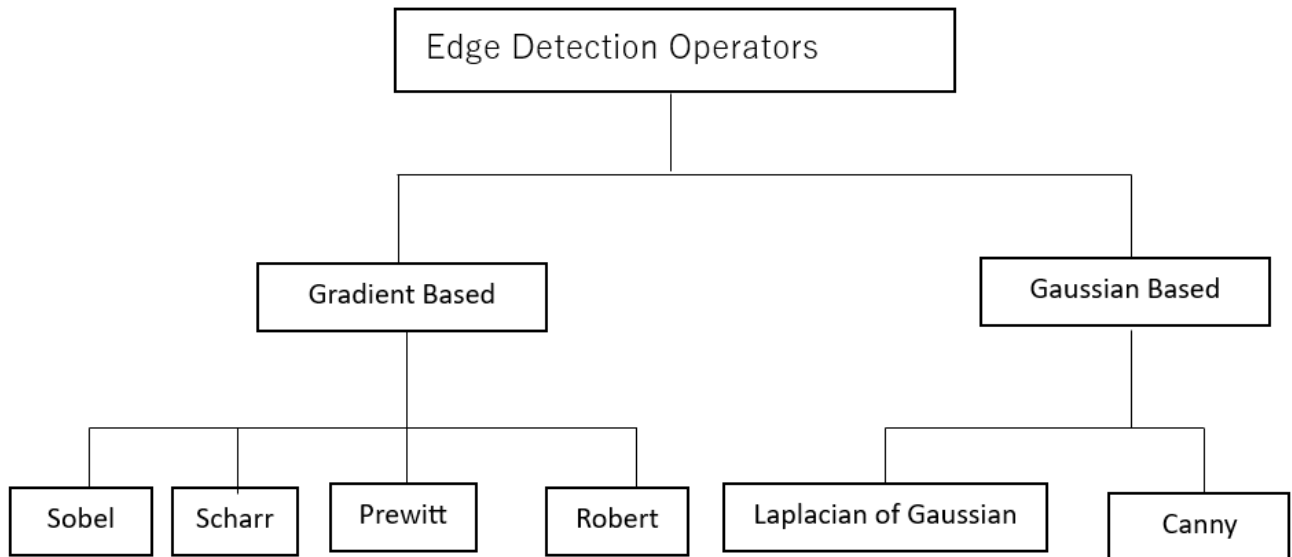
Image processing plays an important role in extracting the features from the images. These features are the useful information for different applications. Edge detection is very popular area in image processing. Edge detection has many applications in industry, like finding the cracks in pipeline, structures, finding hairline fractures in medical images, finding the boundaries in images, finding the shapes of objects in images are few to list. These edge detection algorithms require a good foundation of mathematics.

The connection of mathematics and edge detection algorithms is discussed in this article. The different edge detection algorithms like Sobel, Roberts, Canny, Prewitt, etc. are discussed with their mathematical foundations. These edge detection algorithms are useful in computer vision, image processing, signal processing and many applications. These algorithms use the mathematical concepts like Numerical Analysis, Approximations, and Linear Algebra.

Our emphasize is on exploring the mathematical background of these algorithms. In this article, the edges are detected using the above algorithms and the results are compared.

Literature Survey :

In this section, the different edge detection algorithms are discussed with their mathematical form. Many researchers have used various these operators [7, 11, 14], including Laplacian of Gaussian (LoG), Prewitt, Scharr, Sobel, Robert, and Canny for edge detection. The classification of these edge detection algorithms is based on Gradient and Gaussian. This classification is as follows:



Edges [1] are local features of an image i.e. local regions with spatial features and characteristics. Edges are pixels that have significant local variations in intensity. So edges are having large gradient of the intensity. Edges occur at the boundaries between two different intensities of the image. These edge detection techniques perform important role of finding Edges, lines, curves etc. These edge detection algorithms are discussed in [3, 4, 5].

Steps for edges detection are:

1. Smoothing: minimizes noise while protecting real edges to the greatest extent.
2. Enhancement: applies a filter on the picture to improve the quality of the edges which sharpens edges.
3. Detection: using thresholding the pixels of edges are chosen and the noise is removed.
4. Localization: determines the exact positioning of an edge. This step usually involves thinning and joining the edges.

In mathematics, derivatives are used to describe continuous changes in function. In 2D image, partial derivatives represent changes in image intensity.

The pixel on edge are detected through detection of

1. the local minima or maxima of the derivative of the first order and
2. the zero-crossing of the derivative of the second order.

Edge Detection in an image:

Consider the 3x3 neighbourhood of the pixel $f(x, y)$ defined as

$$f_{3 \times 3}(x, y) = \begin{bmatrix} f(x-1, y-1) & f(x-1, y) & f(x-1, y+1) \\ f(x, y-1) & f(x, y) & f(x, y+1) \\ f(x+1, y-1) & f(x+1, y) & f(x+1, y+1) \end{bmatrix}$$

and the 3x3 mask as *i. e.* $M = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}$

After applying the mask M to the image, the resultant pixel at $f(x, y)$ is given as

$$g(x, y) = f_{3 \times 3}(x, y) \cdot M$$

$$= f(x-1, y-1) \times w_1 + f(x-1, y) \times w_2 + f(x-1, y+1) \times w_3 + f(x, y-1) \times w_4 + f(x, y) \times w_5 + \dots + f(x+1, y+1) \times w_9$$

Consider a 8x8 pseudo image which is represented as

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 20 & 20 & 20 & 20 & 20 & 20 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \end{bmatrix}$$

After applying the Sobel Operator Mask $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ in X-direction,

The resultant image becomes

$$M_x * I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 60 & 60 & 40 & 60 & 80 & 60 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -60 & -60 & -40 & -60 & -80 & -60 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using threshold value 0, all the values less than 0 become 0. After making negative number as zero we get horizontal edge as

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 60 & 60 & 40 & 60 & 80 & 60 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Similarly, one can find the vertical edge using the mask $M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ in Y-direction.

Here we proposed 2 categories of edge detection operators. The general structure of the edge detection

operator as $M_x = \begin{bmatrix} a & 0 & -a \\ b & 0 & -b \\ c & 0 & -c \end{bmatrix}$ in X-direction and $M_y = (M_x)^t = \begin{bmatrix} a & b & c \\ 0 & 0 & 0 \\ -a & -b & -c \end{bmatrix}$ in Y-direction is

one of the category. We define this category of operators as ‘‘Transpose Operators’’

The following special cases can be derived from above form as:

Case 1: Let $a = -1$, $b = -1$ and $c = -1$ then

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } M_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Which is also known as Prewitt operator [1, 9]

Case 2: Let $a = -1$, $b = -2$ and $c = -1$ then

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Which is also known as Sobel Operator [1, 6, 12]

The proposed second category of the edge detection operators is defined as $M_x = \begin{bmatrix} a & 0 & -a \\ b & 0 & -b \\ c & 0 & -c \end{bmatrix}$ in

X-direction and as $M_y = -(M_x)^t = \begin{bmatrix} -a & -b & -c \\ 0 & 0 & 0 \\ a & b & c \end{bmatrix}$ in Y-direction. We define this category of operators as “Skew-Transpose Operators.”

The following special cases can be derived from above form as:

Case 1: Let $a = -\frac{3}{32}$, $b = -\frac{10}{32}$ and $c = -\frac{3}{32}$ then

$$M_x = \frac{1}{32} \begin{bmatrix} -3 & 0 & 3 \\ -10 & 0 & 10 \\ -3 & 0 & 3 \end{bmatrix} \text{ and } M_y = \frac{1}{32} \begin{bmatrix} 3 & 10 & 3 \\ 0 & 0 & 0 \\ -3 & -10 & -3 \end{bmatrix}$$

Which is also known as Scharr Operator [18]

Case 2: Let $a = -1$, $b = -2$ and $c = -1$ then

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Which is also known as Sobel Operator [17]

Proposed operator:

Let $f(x)$ be the function of x . and Δ be the forward difference operator. The forward difference of $f(x)$ is defined as $\Delta f(x) = f(x+h) - f(x)$, where h is the difference between two consecutive observations.

$$\begin{aligned} \text{Now we have, } \Delta^2 f(x) &= \Delta(\Delta f(x)) \\ &= \Delta(f(x+h) - f(x)) \\ &= \Delta f(x+h) - \Delta f(x) \\ &= (f(x+2h) - f(x+h)) - (f(x+h) - f(x)) \\ &= f(x+2h) - f(x+h) - f(x+h) + f(x) \\ \Delta^2 f(x) &= f(x+2h) - 2f(x+h) + f(x) \end{aligned}$$

Considering the coefficients in the expansion of $\Delta^2 f(x)$, we get the triad as (1, -2, 1).

Proposed Mask 1: Based on the above triad, the new mask of ‘Transpose Operator’ category is defined as below. Let $a = 1$, $b = -2$, and $c = 1$, we have the mask as

$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix} \text{ and } M_y = (M_x)^t = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \end{bmatrix}$$

Proposed Mask 2: Based on the above triad, the new mask of ‘Skew Transpose Operator’ category is defined as below. Let $a = 1$, $b = -2$, and $c = 1$, we have the mask as




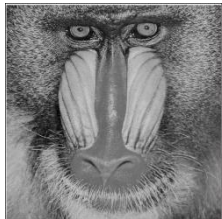
















$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix} \text{ and } M_y = -(M_x)^t = \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

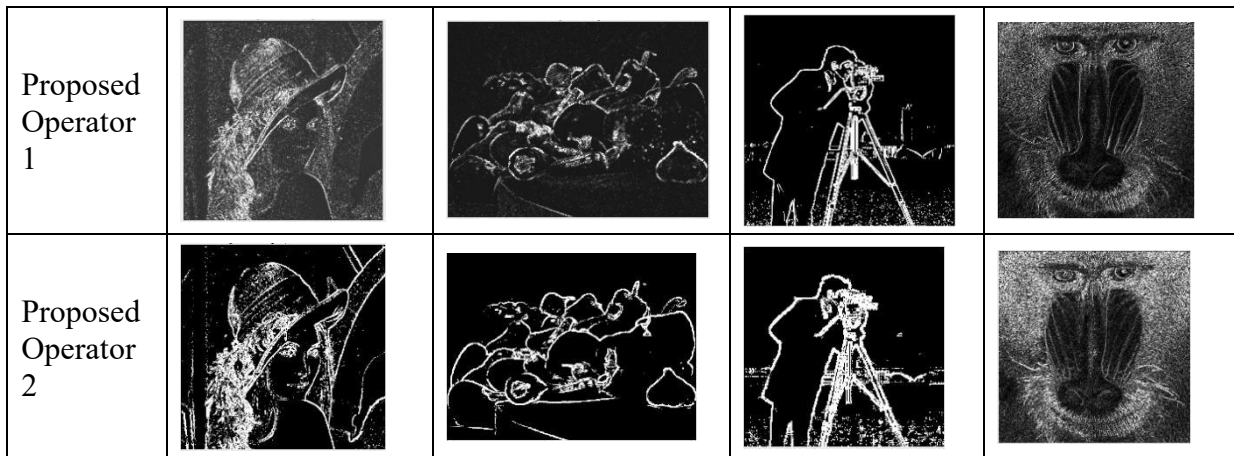
The proposed marks arte implemented with the help of MATLAB software. The results are as shown in the Table 1. First row of the Table 1 shows the names of the images used for experiments. Second

row of the Table 1 shows the original images used for experiments. The third row shows the edges detected using Roberts Operator for Lena, Peppers, Cameraman, and Mandril images. The fourth row shows the edges detected using Sobel Operator for Lena, Peppers, Cameraman, and Mandril images. The fifth row shows the edges detected using Prewitt Operator for Lena, Peppers, Cameraman, and Mandril images. The sixth row shows the edges detected using Scharr Operator for Lena, Peppers, Cameraman, and Mandril images.

The seventh row shows the edges detected using Proposed Operator-I for Lena, Peppers, Cameraman, and Mandril images. The last row shows the edges detected using Proposed Operator-II for Lena, Peppers, Cameraman, and Mandril images.

Experimental Results:

Edge Detection Operators	Images			
	Lena	Peppers	Cameraman	Mandril
				
Robert				
Sobel				
Prewitt				
Scharr				



Conclusion:

In this article, the general categories of 3×3 operators are proposed as Transpose Operators and Skew-Transpose Operators. Here 2 new edge detection operators are proposed and implemented successfully. These operators are based on the second order forward differences with the equidistance points. Proposed Operator-I is of the Transpose Category and Proposed Operator-II is of the Skew-Transpose Category. The image is a discrete signal and all pixels are equidistance hence implementation of the proposed masks becomes interesting. After applying the proposed masks, the experimental results are shown and compared in the form of images. It can be seen that the proposed operators perform better than the existing operators.

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