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Shifted Interpolation Methods for Efficient Resolution of Algebraic and Transcendental Equations

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Abstract: A popular method in mathematics and statistics for determining values between any two points is interpolation. For both equally and unequally spaced data sets, numerous techniques have been employed to derive practical interpolation equations. This work aims to develop a difference interpolation formula derived from Gauss's Formula and another formula where we substituted ρ for $\rho + 4$ and reused the subscript in Gauss's Forward Formula by four units. Additionally, based on the differences, we compared the generated interpolation formula with the current interpolation formulae. We discovered from the error analysis that the new formula we obtained is significantly more accurate and efficient at resolving functional values between the provided data.

Keywords: Interpolation, Central Difference, Mathematical Norm, Gauss's Formula

Introduction

A fundamental method in numerical analysis, interpolation is used to estimate values that fall between known data sets. Finding new data points within the range of a discrete set of known data points is known as interpolation. [5] It is crucial in a variety of domains, including scientific computing, computer graphics, signal processing, and geographic information systems (GIS). The type of data, the required level of precision, and processing efficiency all influence the interpolation method selection. Based on recent research, this study examines a number of widely used interpolation techniques, including their theoretical underpinnings, uses, benefits, and drawbacks. [10-13] Its primary function is to forecast unknown values for any geographically connected data points, like elevation, rainfall, noise level, and so forth. The process of interpolation involves taking a discrete data set and using it to create a basic function that passes through the given data points. This aids in identifying the data points that lie between the ones that are provided. [3] When calculating a function's value for an intermediate value of the independent function, this procedure is always required. The technique of identifying the unknown values that fall between the known data points is known as interpolation. Its primary function is to forecast unknown values for any geographically connected data points, like elevation, rainfall, noise level, and so forth.

Interpolation Methods

There are different types of interpolation methods. They are:

Nearest Neighbors Method – This method inserts the value of an interpolated point to the value of the most adjacent data point. Therefore, this method does not produce any new data points.[6]

Thin-plate Spline Method – This method consists of smooth surfaces that also extrapolate well.[7] It is only for surfaces only.

Cubic Spline Interpolation Method – This method fits a different cubic polynomial between each pair of data points for curves, or between sets of three points for surfaces.[8]

Linear Interpolation Method – This method applies a distinct linear polynomial between each pair of data points for curves, or within the sets of three points for surfaces.[9]

Shape-Preservation Method – This method is also known as Piecewise Cubic Hermite Interpolation (PCHIP). It preserves the monotonicity and the shape of the data. It is for curves only.

In this paper, we tried to develop a central difference interpolation formula which is obtained from Gauss's Formula and another formula in which we reused the subscript in Gauss's Forward Formula by four units and replacing ρ by $\rho + 4$ [1]. Also, we compared the obtained interpolation formula with the existing interpolation formulae, (Gauss's, Stirling's and Bessel's etc.) based on differences and use the concept of mathematical norm to select which method is best for evaluating functional values between data.

Das and Chakrabarty (2016) derived a formular from Lagrange's interpolation method and this was used to obtain a numerical data for total population of India. The work was extended by deriving other methods for the same purpose. [2]

Proposed And Existing Interpolation Formulae [4]

Given below are the Gauss's Central-Difference Formulae

Gauss's Forward Central-Difference Formula

$$y = y_0 + \rho\Delta y_0 + \rho(\rho - 1)\frac{\Delta^2 y_{-1}}{2!} + \rho(\rho + 1)(\rho - 1)\frac{\Delta^3 y_{-1}}{3!} + \rho(\rho - 2)(\rho - 1)(\rho + 1)\frac{\Delta^4 y_{-2}}{4!} + \cdots \quad \text{(i)}$$

Gauss's Backward Central-Difference Formula

$$y = y_0 + \rho\Delta y_{-1} + \rho(\rho+1)\frac{\Delta^2 y_{-1}}{2!} + \rho(\rho-1)(\rho+1)\frac{\Delta^3 y_{-2}}{3!} + \rho(\rho-2)(\rho-1)(\rho+1)\frac{\Delta^4 y_{-2}}{4!} + \dots \quad \text{(ii)}$$

Sterling Interpolation Formula

$$y = y_0 + \rho\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{\rho^2}{2!}\Delta^2 y_{-1} + \frac{\rho(\rho^2-1)}{3!}\left(\frac{\Delta^3 y_0 + \Delta^3 y_{-2}}{2}\right) + \frac{\rho^2(\rho^2-1)}{4!}\Delta^4 y_{-2} + \cdots \quad \text{(iii)}$$

Bessel's Interpolation Formula

$$y = y_1 + (\rho-1)\Delta y_0 + \rho(\rho-1)\frac{\Delta^2 y_{-1}}{2!} + \rho(\rho-1)(\rho-2)\frac{\Delta^3 y_{-2}}{3!} + \rho(\rho^2-1)(\rho-2)\frac{\Delta^4 y_{-2}}{4!} + \rho(\rho^2-1)(\rho-2)(\rho-3)\frac{\Delta^5 y_{-2}}{5!} + \dots \quad \text{(iv)}$$

$$y = \frac{y_0 + y_1}{2} + \left(\rho - \frac{1}{2}\right)\Delta y_0 + \frac{\rho(\rho-1)}{2!}\frac{(\Delta^2 y_{-1} + \Delta^2 y_0)}{2} + \frac{\rho(\rho-\frac{1}{2})(\rho-1)}{3!}\Delta^3 y_{-1} + \frac{\rho(\rho^2-1)(\rho-2)}{4!}\frac{(\Delta^4 y_{-2} + \Delta^4 y_{-1})}{2} + \frac{\rho(\rho-\frac{1}{2})(\rho^2-1)(\rho-1)}{5!}\Delta^5 y_{-2} + \dots \quad \text{(v)}$$

Equation (v) is derived as Bessel's Interpolation Formula.

Previously Proposed Formula

$$y = \frac{y_{-2} + y_0}{2} + \left[1 - \frac{\rho}{2}\left\{1 + \frac{\Delta y_{-1}}{\Delta y_{-2}}\right\}\right]\Delta y_{-2} + \frac{(\rho+1)}{2!}\left[\frac{(\rho+2)(\Delta^2 y_{-3}) + \rho\Delta^2 y_{-1}}{2}\right] + \frac{(\rho+1)}{3!}\left[\frac{(\rho+2)(\rho+3)\Delta^2 y_{-3} + \rho(\rho-1)\Delta^3 y_{-2}}{2}\right] + \cdots$$

New Interpolation Formula (NIF)

To derive the new formula, we retreat the subscript in Gauss's Forward Interpolation Formula by four units and replacing ρ by $\rho + 4$.
From equation (i)

$$y = y_0 + \rho\Delta y_0 + \rho(\rho-1)\frac{\Delta^2 y_{-1}}{2!} + \rho(\rho+1)(\rho-1)\frac{\Delta^3 y_{-1}}{3!} + \rho(\rho-2)(\rho-1)(\rho+1)\frac{\Delta^4 y_{-2}}{4!} + \dots \quad \text{(vi)}$$

So, we obtain,

$$y = y_{-4} + (\rho+4)\Delta y_{-4} + (\rho+4)(\rho+3)\frac{\Delta^2 y_{-5}}{2!} + (\rho+4)(\rho+5)(\rho+3)\frac{\Delta^3 y_{-5}}{3!} +$$

$$(\rho+4)(\rho+2)\,(\rho+3)(\rho+5)\ \frac{\Delta^4 y_{-6}}{4!}\ +\cdots \tag{vii}$$

Also, from equation (ii)

$$y = y_0 + \rho\Delta y_{-1} + \rho(\rho+1)\frac{\Delta^2 y_{-1}}{2!} + \rho(\rho-1)\,(\rho+1)\frac{\Delta^3 y_{-2}}{3!} + \rho(\rho-2)(\rho-1)(\rho+1)\frac{\Delta^4 y_{-2}}{4!} + \ldots \tag{viii}$$

So, we obtain,

$$y = y_{-4} + (\rho+4)\Delta y_{-5} + (\rho+4)(\rho+5)\frac{\Delta^2 y_{-5}}{2!} + (\rho+4)(\rho+3)(\rho+5)\frac{\Delta^3 y_{-6}}{3!} + (\rho+4)(\rho+2)(\rho+3)(\rho+5)\frac{\Delta^4 y_{-6}}{4!} + \ldots \tag{ix}$$

Taking the mean of equations (viii) and (ix) we get the New Interpolation Formula

$$y = y_{-4} + (\rho+4)\left(\frac{\Delta y_{-4}+\Delta y_{-5}}{2}\right) + \frac{(\rho+4)^2}{2!}\Delta^2 y_{-5} + \frac{(\rho+4)\left((\rho+4)^2-1\right)}{3!}\left(\frac{\Delta^3 y_{-4}+\Delta^3 y_{-6}}{2}\right) + \frac{(\rho+4)^2((\rho+4)^2-1)}{4!}\Delta^4 y_{-6} + \ldots \tag{x}$$

Numerical Examples

To evaluate the assess of our proposed interpolation formula, we examined several examples in comparison with established methods.

Example 1:
Consider the algebraic function $y = 3t^2 + 2t + 1$ for some equidistantly spaced value of t.
Here we take $t = 6, h = 1, t_0 = 2$ then $\rho = \frac{t-t_0}{h} = \frac{6-2}{1} = 4$.
Actual value of $y(6) = 121$.

Table 1: Difference Table for Example 1.

t	y	Δy	Δ^2y	Δ^3y	Δ^4y
-2	9	-7			
-1	2	-1	6		
0	1	5	6	0	
1	6	11	6	0	0
2	17	17	6	0	
3	34	23	6		
4	57	29			
5	86				

Table 2: Comparison of Different Interpolation Techniques along with Error.

Sr. No.	Methods	Approximate Value of y at $t = 6$	Error	% Error
1.	Gauss's Forward	**121**	**0**	**0**
2.	Gauss's Backward	**121**	**0**	**0**
3.	**Stirling's Method**	**121**	**0**	**0**
4.	**Bessel's Method**	**121**	**0**	**0**
5.	**Previously proposed**	**121**	**0**	**0**
6.	**NIF**	**121**	**0**	**0**

Example 2:
Consider the algebraic function $y = 4t^2 + 3t + 2$ for some equidistantly spaced value of t.
Here we take $t = 4.5$ $t_0 = 0, h = 1$, and $\rho = \frac{t-t_0}{h} = \frac{4.5-0}{1} = 4.5$.
Actual value of $y(4.5) = 96.5$.

Table 3: Difference Table for Example 2

t	y	Δy	Δ^2y	Δ^3y	Δ^4y
-5	87	-33			
-4	54	-25	8		
-3	29	-17	8	0	
-2	12	-9	8	0	0
-1	3	-1	8	0	0
0	2	7	8	0	0
1	9	15	8	0	
2	24	23	8		
3	47	31			
4	78				

Table 4: Comparison of Different Interpolation Techniques along with Error.

Sr. No.	Methods	Approximate Value of y at $t = 4.5$	Error	% Error
1.	Gauss's Forward	96.5	**0**	**0**
2.	Gauss's Backward	96.5	**0**	**0**
3.	**Stirling's Method**	96.5	**0**	**0**

4.	**Bessel's Method**	96.5	**0**	**0**
5.	**Previously proposed**	96.5	**0**	**0**
6.	**NIF**	96.5	**0**	**0**

Example 3:
Consider the transcendental function $y = cos\theta$ for some equidistantly spaced. Here we have to calculate value of y at θ=52^0.
Actual value of $y(52^0) = 0.615661.$

Table 5: Difference Table for Example 3.

θ (°)	**y**	**Δy**	**Δ^2y**	**Δ^3y**	**Δ^4y**	**Δ^5y**	**Δ^6y**
0	1.000000	–0.01519	–0.02992	+0.001367	+0.000874	–0.001115	–0.001071
10	0.984807	–0.04511	–0.02855	+0.001989	–0.000241	–0.002186	+0.000752
20	0.939693	–0.07366	–0.02631	+0.001748	–0.002427	–0.001434	+0.003978
30	0.866025	–0.09998	–0.02381	–0.000679	–0.003861	+0.002544	
40	0.766044	–0.12325	–0.01953	–0.004540	–0.001317		
50	0.642788	–0.14278	–0.01574	–0.005857			
60	0.500000	–0.15798	-0.010392				
70	0.342020	–0.16837					
80	0.173648						

Table 6: Comparison of Different Interpolation Techniques along with Error.

Sr. No	**Methods**	**Approximate Value of y at $\theta = 52^0$**	**Error**	**% Error**
1.	Gauss's Forward	0.615661475	**0.000609**	**0.09891807**
2.	Gauss's Backward	0.615858611	**0.000197**	**0.03209737**
3.	**Stirling's Method**	0.615661475	**0.00000047**	**0.00007715**
4.	**Bessel's Method**	0.61556142	**0.00009958**	**0.01617448**
5.	**Previously proposed**	0.61556153	**0.00009947**	**0.01615662**

6.	**NIF**	**0.615053**	**0.000608**	**0.09875565**

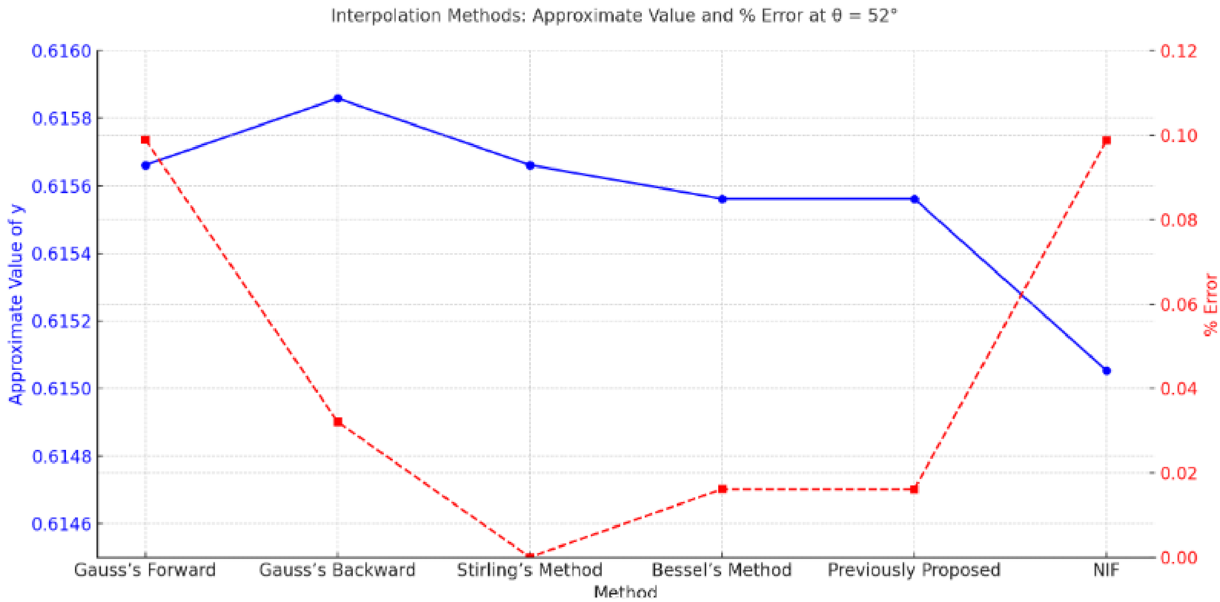

Figure 1 - Graphical Representation for Problem

Example 4:

Consider the exponential function $y = e^t$ for some equidistantly spaced values of t. In this problem we have to find out the value of y at $t = 0.74$.

Actual value of $y(0.74) = 2.09532$.

Table 7: Difference Table for Example 4.

t	y	Δy	Δ^2y	Δ^3y	Δ^4y
0.64	1.896481	0.019060	0.000192	0.000002	-0.00000001
0.65	1.915541	0.019252	0.000193	0.000002	0.00000004
0.66	1.934792	0.019445	0.000195	0.000002	0.00000002
0.67	1.954237	0.019640	0.000197	0.000002	0.00000000
0.68	1.973878	0.019838	0.000199	0.000002	0.00000004
0.69	1.993716	0.020037	0.000201	0.000002	0.00000002
0.70	2.013753	0.020239	0.000203	0.000002	0.00000000
0.71	2.033991	0.020442	0.000205	0.000002	0.00000006
0.72	2.054433	0.020647	0.000208	0.000002	-0.00000003
0.73	2.075081	0.020855	0.000210	0.000002	
0.74	2.095936	0.021065	0.000212		—
0.75	2.117000	0.021276	—	—	—
0.76	2.138276	—	—	—	—

Table 8: Comparison of Different Interpolation Techniques along with Error.

Sr. No.	Methods	Approximate Value of y at $t = 0.74$	Error	% Error
1.	Gauss's Forward	2.09532	**0**	**0**
2.	Gauss's Backward	2.09532	**0**	**0**
3.	**Stirling's Method**	2.09532	**0**	**0**
4.	**Bessel's Method**	2.07422	**0.0211**	**1.00700**
5.	**Previously proposed**	2.017533	**0.077787**	**3.7124162**
6.	**NIF**	2.09585	**0.00053**	**0.025294**

Table 9: Analysis of all Problems.

Examples	**Gauss's Forward Formula**	**Gauss's Backward Formula**	**Stirling's Formula**	**Bessel's formula**	**Previously Proposed Formula**	**New Interpolation formula**	**True Value**
Example 1	121	121	121	121	121	121	121
Example 2	96.5	96.5	96.5	96.5	96.5	96.5	96.5
Example 3	0.6156615	0.61585867	0.6156615	0.6156615	0.6155615	**0.615053**	0.6156615
Example 4	2.09532	2.09532	2.09532	2.07422	2.017533	2.09585	2.09532

Conclusion

This paper presents a new interpolation formula and compares it with existing interpolation models using the concept of mathematical norms. The proposed model, given in Equation (viii), is designed for cases where the point to be interpolated lies near the center of the given data. It was derived by modifying Gauss's Forward Interpolation Formula specifically, by shifting the subscript back by four units and replacing the variable p with p+4. This modified expression was then averaged with Gauss's Backward Interpolation Formula to produce the new model. The performance of this proposed formula was evaluated and compared with established methods, including Gauss's Forward Interpolation, Gauss's Backward Interpolation, Bessel's Interpolation and Stirling's Interpolation. From all the tables above, we have shown that how different techniques are applied on different problems along with error. So as a result, we have found that the proposed interpolation technique

NIF shows better results in comparison to previously defined interpolation techniques. So, the NIF stands as a robust and reliable alternative for central interpolation tasks in numerical analysis.

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