Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

Effect of Linearly Distributed Sources in Normal Loading of a Dual-Phase-Lag Thermoelastic Diffusion with Non-Local Formulation

Rajneesh Kumar[1], Pawan Kumari[2], Kavita Rani[3], Seetu Rana[4]

[1]Department of Mathematics, Kurukshetra University, Kurukshetra 136119, India
email: rajneeshkuk@rediffmail.com
[2] Associate Professor, Department of Mathematics, Govt. College, Hisar, Haryana,125001 India
email: pawankumarigawarya@gmail.com
[3]Associate Professor, Department of Mathematics, CMG GCW, Bhodia Khera, Fatehabad, Haryana,125050, India
email: karya4@gmail.com
[4] Assistant Professor, Department of Mathematics, Govt. College, Hisar, Haryana,125001 India
email: rana.seetu@gmail.com

Article History:

Received: 04-10-2025

Revised: 23-11-2025

Accepted: 14-12-2025

Abstract:

The present study aims at exploring the effects of non-locality and dual-phase-lags in a thermoelastic half-space subjected to normal loading. The governing equations are solved using harmonic vibration analysis coupled with Fourier transform techniques. Linearly distributed sources are considered to demonstrate practical relevance for mechanical excitations. Numerical simulations are performed to evaluate displacements, stresses, temperature variations and chemical potential, with results presented graphically. The study is applicable to predicting the mechanical responses in electronics devices, aerospace components, and energy systems subjected to time-harmonic loads.

Keywords: Thermoelasticity, Non-local theory, Dual-Phase-Lag, Fourier transform, Time Harmonic Sources.

1. Introduction

In non-local elasticity, the stress at a point is affected by both the local strain and the strain distribution throughout the body. This is different from classical (local) elasticity, which says that the stress at a point is only affected by the strain at that point. Researchers like Edelen and Laws [1], Eringen and Edelen [2], and others built on these ideas to find the basics of non-local elasticity. Later, Eringen [3-8], McCay and Narsimhan [9], and Narsimhan and McCay [10] added to these ideas. Eringen's monograph [11] goes into a lot of detail about the subject.

Non-local response and lagging response are related ideas. Non-local response is about effects in space and lagging response is about delays in time. The results were compared with the models of Cao and Guo [13] and Guo and Hou [14] by Tzou [12]. He used the non-local formulation and single-phase-lag heat conduction. Later, Tzou and Gao [15] built on this idea by adding non-local elasticity to the dual-phase-lag model that Tzou [16,17] had first suggested. This created a single framework that included both effects. At the same time, Sherief et al. [18] created a thermoelastic diffusion theory with a single relaxation time, and Sherief and Saleh [19] looked at the half-space problem in this context.

Later additions to these theories made them more general. In his work [20,21], Sharma looked into boundary-value problems in generalized thermodiffusive elastic media and wave reflection in thermodiffusive elastic half-spaces with holes. Sharma et al. [22] looked at how viscosity affects wave propagation in Green–Naghdi type-II and type-III thermoelastic solids that are not all the same. Sharma and Marin [23] studied the problem of reflection and transmission of plane waves from imperfect boundary between two heat conducting micropolar thermoelastic solids.

Abouelregal and Zenkour [24] looked at phase-lag effects in functionally graded thermoelastic microbeams that were heated in a ramp-type way. Sharma and Sharma [25] looked at how heat sources and relaxation time affect the way temperatures are distributed in living things. Marin et al. [26] looked at the Saint-Venant principle in micropolar thermoelastic diffusion when it is relaxed. Yu et al. [27] suggested a size-dependent thermoelastic model for more complex materials. This model takes into account size effects in both heat conduction and elasticity. It does this by using generalized free energy and extended irreversible thermodynamics.

There are also studies by Kumar and Abbas [28], who looked into how thermomechanical sources can cause disturbances in poro-thermoelastic media, and Kumar et al. [29], who looked into how deformation happens in a modified couple stress thermoelastic rotating medium when Hall current and magnetic field influences happen, using the Lord-Shulman and Green-Lindsay theories with a ramp-type thermal source.

A study by Kumar et al. [30] looked at a non-local microstretch thermoelastic thick circular plate that was subject to phase lags. Kumar et al. [31] looked at how thermal and chemical potential sources affected a thin beam in a modified couple stress thermoelasticity (MCT) model with a three-phase-lag diffusion model in a different study.

Marin et al. [32] used the Moore-Gibson-Thompson (MGT) heat equation to come up with a basic solution and Green's function for a semi-infinite orthotropic photo-thermoelastic medium whose properties change with temperature. They used the operator theory and the superposition principle to come up with a general solution using harmonic functions. This solution was then used to find specific solutions for sources of steady point heat at the medium's surface and inside.

Sharma et al. [33] looked into how the Moore-Gibson-Thompson equation controls thermomechanical deformation in a micropolar thermoviscoelastic solid. They found that the two-temperature effects were not local and were hyperbolic. In their study [34], Abouelregal et al. created a thermoviscoelastic model by combining non-local elasticity with the Kelvin-Voigt viscoelastic framework and a Klein–Gordon-type non-local elasticity formulation. They then looked at a one-dimensional half-space problem that was affected by an instantaneous in-line heat source.

We look at how non-locality and phase lags affect thermoelastic diffusion when sources are spread out in a time-harmonic way in this study. The integral transform method is used to write the problem which takes into account different kinds of excitations such as normal load, thermal sources and chemical potential sources.

2. Basic Equations

Based on the work by Tzou & Gao [15], Sherief et al. [18], and Yu et al. [27], the equations that describe thermoelastic diffusion with dual-phase-lag and non-local effects are :

(i) Constitutive Relations

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}\,[\lambda_o e_{kk} - \gamma_1 T - \gamma_2 P] \tag{1}$$

(ii) Equation of motion

$$(\lambda_o + \mu)\,\nabla(\nabla.\mathbf{u}) + \mu\Delta\mathbf{u} - \gamma_1\nabla T - \gamma_2\nabla P = \rho(1-\xi^2\Delta)\frac{\partial^2\boldsymbol{u}}{\partial t^2} \tag{2}$$

(iii) Equation of heat conduction

$$\left(1-\zeta^2\Delta + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)(\gamma_1 T_o\dot{e} + l_1 T_o\dot{T} + T_o\dot{P}\,\mathrm{d}) = K\left(1+\tau_t\frac{\partial}{\partial t}\right)\Delta T \tag{3}$$

(iv) Equation of mass diffusion

$$\left(1-\varsigma^2\Delta + \tau_u\frac{\partial}{\partial t} + \frac{1}{2}\tau_u^2\frac{\partial^2}{\partial t^2}\right)(\gamma_2\dot{e} + d\dot{T} + n\dot{P}) = D\left(1+\tau_p\frac{\partial}{\partial t}\right)\Delta P \tag{4}$$

Where:

$$\lambda_o = \lambda - \beta_2^2/b, \quad \gamma_1 = \beta_1 + \frac{a}{b}\beta_2, \quad \gamma_2 = \frac{\beta_2}{b}, \quad l_1 = \rho\, C_e/T_O + a^2/b, \quad d = \frac{a}{b}, \quad n = \frac{1}{b} \tag{5}$$

where u is the displacement vector, T is the temperature, P is the chemical potential, e_{ij} is the strain tensor, t_{ij} is the stress tensor, and μ , λ_0 , γ_1 , γ_2, l_1 , d, n, K, D, τ_q, τ_t, τ_u, τ_p, ξ, ζ, ς are material and model parameters defined as per the cited references.

ξ, ζ, ς -are non-local parameters in equations (1)-(4). τ_q & τ_t are the thermal relaxation times with $\tau_q, \tau_t \geq 0$ and τ_u & τ_p are the diffusion relaxation times with $\tau_u, \tau_p \geq 0$. $\beta_1 = (3\lambda+2\mu)\alpha_t$, $\beta_2 = (3\lambda+2\mu)\alpha_c$. In this case, α_t , α_c are the coefficient of linear thermal expansion and diffusion expansion respectively. Δ is the Laplacian operator, ∇ is nabla operator which is also known as the Laplacian operator. Other signs mean what they normally do.

3. Problem Description

We look at a half-space that is uniform, isotropic, non-local, thermoelastic, and diffusive. This half-space has dual-phase-lag effects and is in the region $x_3 \geq 0$. A rectangular set of Cartesian coordinate system is chosen with (x_1, x_2, x_3) as its points of reference. The origin is placed on the boundary plane with $x_3 = 0$. The study only looks at deformations in a plane, and all the field variables are functions of (x_1, x_3, t). The half-space is loaded mechanically with a normal force, heated up, and given a chemical potential source at the boundary plane $x_3 = 0$. We assume that the variations are limited to the x_1 - x_3 plane for two-dimensional modeling. $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ can be written in terms of $\varphi(x_1, x_3, t)$ and $\psi\ (x_1, x_3, t)$ in a form that doesn't depend on the dimensions.

For two dimensional problems, we take:

$$u = (u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t)),\ T\ (x_1, x_3, t),\ P\ (x_1, x_3, t) \tag{6}$$

The following dimensionless quantities are introduced for normalization:

$$\xi' = \frac{\omega*}{c_1}\xi, \quad \zeta' = \frac{\omega*}{c_1}\zeta, \quad \varsigma' = \frac{\omega*}{c_1}\varsigma, \quad x_i' = \frac{\omega*}{c_1}x_i, \quad u_i' = \frac{\omega*}{c_1}u_i, \quad t' = \omega* t, \quad \tau_t' = \omega*\tau_t,$$
$$\tau_q' = \omega*\tau_q, \quad \tau_u' = \omega*\tau_u, \quad \tau_p' = \omega*\tau_p, \quad t_{ij}' = \frac{1}{\gamma_1 T_0}t_{ij}, \quad m_{ij}' = \frac{1}{\gamma_1 T_0}\frac{\omega*}{c_1}m_{ij}, \tag{7}$$
$$T' = \frac{\gamma_1}{\rho {c_1}^2}T, \quad P' = \frac{1}{b\gamma_2}P$$

where:

$$\omega* = \frac{\rho C_e c_1^2}{K}, \quad {c_1}^2 = \frac{\lambda_o + 2\mu}{\rho}$$

Here ω* denotes the characteristic frequency and c_1 is the longitudinal wave velocity in the medium.

Using equations (2)-(4) along with (6)- (7) and suppressing the primes, the system reduces to the following dimensionless form

$$\frac{(\lambda_o + \mu)}{\rho {c_1}^2}\frac{\partial e}{\partial x_1} + \frac{\mu}{\rho {c_1}^2}\Delta u_1 - \frac{\partial T}{\partial x_1} - \frac{b{\gamma_2}^2}{\rho {c_1}^2}\frac{\partial P}{\partial x_1} = (1 - \xi^2\Delta)\frac{\partial^2 u_1}{\partial t^2}, \tag{8}$$

$$\frac{(\lambda_o + \mu)}{\rho {c_1}^2}\frac{\partial e}{\partial x_3} + \frac{\mu}{\rho {c_1}^2}\Delta u_3 - \frac{\partial T}{\partial x_3} - \frac{b{\gamma_2}^2}{\rho {c_1}^2}\frac{\partial P}{\partial x_3} = (1 - \xi^2\Delta)\frac{\partial^2 u_3}{\partial t^2}, \tag{9}$$

$$(1 - \zeta^2\Delta + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})\left(\frac{{\gamma_1}^2}{K\rho\omega*}T_o\dot{e} + \frac{l_1{c_1}^2}{K\omega*}T_o\dot{T} + \frac{b\gamma_2\gamma_1}{K\rho\omega*}T_o\dot{P}\ d\right) = (1 + \tau_t\frac{\partial}{\partial t})\ \Delta T, \tag{10}$$

$$(1 - \varsigma^2\Delta + \tau_u\frac{\partial}{\partial t} + \frac{1}{2}\tau_u^2\frac{\partial^2}{\partial t^2})\left(\frac{{c_1}^2}{Db\omega*}\dot{e} + \frac{{c_1}^4 d\rho}{Db\omega*\gamma_1\gamma_2}\dot{T} + \frac{n{c_1}^2}{D\omega*}\dot{P}\right) = (1 + \tau_p\frac{\partial}{\partial t})\ \Delta P \tag{11}$$

Where:

$$\Delta = \frac{\partial^2}{\partial {x_1}^2} + \frac{\partial^2}{\partial {x_3}^2}, \quad e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$$

4. Solution Procedure

The displacement components $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ can be expressed in terms of the scalar potentials $\varphi(x_1, x_3, t)$ and $\psi\ (x_1, x_3, t)$ in dimensionless form as

$$u_1 = \frac{\partial\varphi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \qquad u_3 = \frac{\partial\varphi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{12}$$

Substituting (12) into equations (8)-(11) the governing system leads to the following set of coupled equations:

$$(b_1 + b_2)\,\Delta\varphi - T - b_3 P - (1 - \xi^2\Delta)\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{13}$$

$$b_2\Delta\psi - (1 - \xi^2\Delta)\frac{\partial^2\psi}{\partial t^2} = 0, \tag{14}$$

$$(1 - \zeta^2\Delta + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})\,(b_4\Delta\dot{\varphi} + b_5\dot{T} + b_6\dot{P}) = (1 + \tau_t\frac{\partial}{\partial t})\,\Delta T, \tag{15}$$

$$(1 - \varsigma^2\Delta + \tau_u\frac{\partial}{\partial t} + \frac{1}{2}\tau_u^2\frac{\partial^2}{\partial t^2})(b_7\Delta\dot{\varphi} + b_8\dot{T} + b_9\dot{P}) = (1 + \tau_p\frac{\partial}{\partial t})\,\Delta P \tag{16}$$

where:

$$b_1 = \frac{(\lambda_o + \mu)}{\rho {c_1}^2}, \quad b_2 = \frac{\mu}{\rho {c_1}^2} \quad b_3 = \frac{b{\gamma_2}^2}{\rho {c_1}^2}, \quad b_4 = \frac{{\gamma_1}^2}{K\rho\omega*}T_o, \quad b_5 = \frac{l_1 {c_1}^2}{K\omega*}T_o,$$
$$b_6 = \frac{b\gamma_2\gamma_1}{K\rho\omega*}T_o d, \quad b_7 = \frac{{c_1}^2}{Db\omega*}, \quad b_8 = \frac{{c_1}^4 d\rho}{Db\omega*\gamma_1\gamma_2}, \quad b_9 = \frac{n{c_1}^2}{D\omega*} \tag{17}$$

We assume

$$(\bar{\varphi}, \bar{\psi}, \bar{T}, \bar{P}) = (\varphi, \psi, T, P)e^{i\omega t} \tag{18}$$

We define Fourier transform as:

$$\hat{f}\,(\xi_1, x_3, \omega) = \int_{-\infty}^{\infty} \bar{f}(x_1, x_3, \omega)e^{i\xi_1 x_1}\,dx_1 \tag{19}$$

By substituting equations (18) and (19) into (13)-(16) and simplifying, the system reduces to

$$(K_1 D_1^6 + K_2 D_1^4 + K_3 D_1^2 + K_4)(\hat{\varphi}, \hat{T}, \hat{P}) = 0 \tag{20}$$
$$(D_1^2 - m_4^2)\,\hat{\psi} = 0 \tag{21}$$

Where:

$$K_1 = K_{11}K_{13}K_{15} - K_{33}K_{35}\zeta^2\varsigma^2K_{11} + K_{31}\zeta^2K_{15} - K_{33}K_{34}\zeta^2\varsigma^2 - K_{31}K_{35}\zeta^2\varsigma^2b_4 + K_{34}b_4K_{13}\varsigma^2,$$
$$K_2 = K_{01} - 3{\xi_1}^2K_1,$$
$$K_3 = 3{\xi_1}^4K_1 - 2{\xi_1}^2K_{01} + K_{02},$$
$$K_4 = K_{01}{\xi_1}^4 - {\xi_1}^6K_1 - {\xi_1}^2K_{02} - K_{03},$$

$$m_4^2 = \xi_1^2 - \frac{\omega^2}{b_2 - \xi^2\omega^2}$$

$$D_1 = \frac{d}{dx_3}$$

and

$$K_{01} = -K_{11}K_{22}K_{15} - K_{11}K_{13}K_{26} + K_{13}K_{15}\omega^2 + K_{33}K_{11}\zeta^2K_{25} + K_{35}\varsigma^2K_{23}K_{11} - K_{33}K_{35}\zeta^2\varsigma^2\omega^2 - K_{31}\zeta^2K_{26} - K_{21}K_{15} + K_{33}\zeta^2K_{24} + K_{34}\varsigma^2K_{23} + K_{31}\zeta^2K_{24}b_3 + K_{35}\varsigma^2K_{21}b_3 - K_{13}K_{24}b_3 - b_3K_{34}\varsigma^2K_{22},$$

$K_{02} = K_{11}K_{22}K_{26} - K_{13}K_{26}\omega^2 - K_{22}K_{15}\omega^2 - K_{11}K_{23}K_{25} + K_{33}\zeta^2K_{25}\omega^2 + K_{35}\varsigma^2K_{23}\omega^2 + K_{21}K_{26} - K_{23}K_{24} - b_3K_{21}K_{25} + b_3K_{22}K_{24}$,

$K_{03} = \omega^2K_{26}K_{22} - \omega^2K_{23}K_{25}$,

$K_{11} = b_1 + b_2 - \xi^2\omega^2$, $K_{12} = 1 + i\omega\tau_q - \frac{\omega^2}{2}\tau_q^2$, $K_{13} = b_5 i\omega\zeta^2 + 1 + i\omega\tau_t$, $K_{14} = 1 + i\omega\tau_u - \frac{\omega^2}{2}\tau_u^2$, $K_{15} = 1 + i\omega\tau_p + i\omega\zeta^2 b_9$, $K_{21} = K_{31}K_{12}$, $K_{22} = K_{32}K_{12}$, $K_{23} = K_{33}K_{12}$,

$K_{24} = K_{34}K_{14}$, $K_{25} = K_{35}K_{14}$, $K_{26} = K_{36}K_{14}$, $K_{31} = b_4 i\omega$, $K_{32} = b_5 i\omega$, $K_{33} = b_6 i\omega$,

$K_{34} = b_7 i\omega$, $K_{35} = b_8 i\omega$, $K_{36} = b_9 i\omega$

To solve equations (20) and (21) with $\hat{\varphi}, \widehat{\psi}, \widehat{T}$ and $\hat{P}$ disappear as x_3 approaches infinity, we write

$$(\hat{\varphi}, \widehat{T}, \widehat{P})(x_3, \xi_1, s) = \sum_{i=1}^{3}(1, R_i^*, S_i^*)\, A_i e^{-m_i x_3} \tag{22}$$

$$\hat{\psi}\,(x_3, \xi_1, s) = A_4 e^{-m_4 x_3} \tag{23}$$

The roots of the characteristic equations (20), (21) are m_i (i=1,2,3,4)

and A_i(i=1,2,3,4) *are* the corresponding amplitude coefficients determined from boundary conditions and R_i^* and S_i^* are derived as follows:

$$R_i^* = \frac{\begin{gathered}({m_i}^2 - {\xi_1}^2)^3(K_{33}K_{34}\zeta^2\varsigma^2 - K_{31}\zeta^2K_{15}) + ({m_i}^2 - {\xi_1}^2)^2(K_{31}\zeta^2K_{26} + K_{21}K_{15} \\ -K_{33}\zeta^2K_{24} - K_{35}\varsigma^2K_{21}) + ({m_i}^2 - {\xi_1}^2)(K_{21}K_{26} - K_{23}K_{24})\end{gathered}}{\begin{gathered}({m_i}^2 - {\xi_1}^2)^2\,(K_{13}K_{15} - K_{33}K_{35}\zeta^2\varsigma^2) + ({m_i}^2 - {\xi_1}^2)(-K_{22}K_{15} - K_{13}K_{26} + K_{33}\zeta^2K_{25} \\ +K_{35}\varsigma^2K_{23}) + (K_{22}K_{26} - K_{23}K_{25})\end{gathered}}$$

$$S_i^* = \frac{\begin{gathered}({m_i}^2 - {\xi_1}^2)^3(K_{31}K_{35}\zeta^2\varsigma^2 - K_{34}K_{13}\varsigma^2) + ({m_i}^2 - {\xi_1}^2)^2(K_{13}K_{24} + K_{34}\varsigma^2K_{22} - K_{31}\zeta^2K_{25} \\ -K_{35}\varsigma^2K_{21}) + ({m_i}^2 - {\xi_1}^2)(K_{21}K_{25} - K_{22}K_{24})\end{gathered}}{\begin{gathered}({m_i}^2 - {\xi_1}^2)^2\,(K_{13}K_{15} - K_{33}K_{35}\zeta^2\varsigma^2) + ({m_i}^2 - {\xi_1}^2)(-K_{22}K_{15} - K_{13}K_{26} + K_{33}\zeta^2K_{25} \\ +K_{35}\varsigma^2K_{23}) + (K_{22}K_{26} - K_{23}K_{25})\end{gathered}}$$

i=1,2,3

5. Boundary Conditions

At the point where $x_3 = 0$, on the plane boundary, the half-space is excited from the outside by a normal mechanical load, a thermal input, and a chemical potential source. So, the boundary conditions at $x_3 = 0$ are set up to show these three types of applied fields.

$$t_{33} = -F_1(x_1)e^{i\omega t}, \quad t_{31} = 0, \quad T = F_2(x_1)e^{i\omega t}, \quad P = F_3(x_1)e^{i\omega t} \tag{24}$$

The non dimensional stress components are expressed by

$$t_{33} = \frac{2\mu}{\gamma_1 T_0}\left(\frac{\partial u_3}{\partial x_3}\right) + \frac{\lambda_0}{\gamma_1 T_0}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) - \frac{\rho {c_1}^2}{\gamma_1 T_0} T - \frac{{\gamma_2}^2 b}{\gamma_1 T_0} \mathrm{P} \qquad (25)$$

$$t_{31} = \frac{\mu}{\gamma_1 T_0}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) \qquad (26)$$

Applying Fourier transform on (25) and (26),we get

$$\hat{t}_{33} = -\hat{F}_1\,(\xi_1)e^{i\omega t},\ \ \hat{t}_{31} = 0,\ \ \hat{\mathrm{T}} = \hat{F}_2\,(\xi_1)e^{i\omega t},\ \ \hat{\mathrm{P}} = \hat{F}_3(\xi_1)\,e^{i\omega t} \qquad (27)$$

Displacement, stress, temperature variation, and chemical potential are determined by applying equations (22) and (23) to the boundary conditions (27), taking into account equations (12), (22) and (23), as follows:

$$\hat{u}_1 = -\frac{i\xi_1}{\Delta}\sum_{i=1}^{3}\Delta_\mathrm{i} e^{-m_\mathrm{i} x_3} + \frac{\Delta_4 m_4 e^{-m_4 x_3}}{\Delta} \qquad (28)$$

$$\hat{u}_3 = -\frac{1}{\Delta}\sum_{i=1}^{3} m_\mathrm{i}\Delta_\mathrm{i} e^{-m_\mathrm{i} x_3} - \frac{i\xi_1\Delta_4 e^{-m_4 x_3}}{\Delta} \qquad (29)$$

$$\hat{t}_{33} = \frac{1}{\Delta}\sum_{i=1}^{3} b_{1i}\,\Delta_\mathrm{i} e^{-m_\mathrm{i} x_3} + \frac{\Delta_4 b_{14} e^{-m_4 x_3}}{\Delta} \qquad (30)$$

$$\hat{t}_{31} = \frac{1}{\Delta}\sum_{i=1}^{3} b_{2i}\,\Delta_\mathrm{i} e^{-m_\mathrm{i} x_3} + \frac{\Delta_4 b_{24} e^{-m_4 x_3}}{\Delta} \qquad (31)$$

$$\hat{\mathrm{T}} = \frac{1}{\Delta}\sum_{i=1}^{3} R_i^*\,\Delta_\mathrm{i} e^{-m_\mathrm{i} x_3} \qquad (32)$$

$$\hat{\mathrm{P}} = \frac{1}{\Delta}\sum_{i=1}^{3} S_i^*\,\Delta_\mathrm{i} e^{-m_\mathrm{i} x_3} \qquad (33)$$

Here

$\Delta = (S_2^* R_1^* - S_1^* R_2^*)n_1 + (S_1^* R_3^* - S_3^* R_1^*)n_2 + (S_3^* R_2^* - S_2^* R_3^*)n_3,$

$n_1 = b_{13}b_{24} - b_{14}b_{23},$

$n_2 = b_{12}b_{24} - b_{14}b_{22},$

$n_3 = b_{11}b_{24} - b_{14}b_{21},$

and

$b_{1i} = (2r_1 + r_2){m_i}^2 - {\xi_1}^2 r_2 - r_3 R_i^* - r_4 S_i^*,\ \ b_{14} = 2i\xi_1 r_1 m_4,$

$b_{2i} = 2i\xi_1 r_1 m_i,\ \ b_{24} = -r_1(m_4^2 + {\xi_1}^2),$

$r_1 = \frac{\mu}{\gamma_1 T_0},\ r_2 = \frac{\lambda_0}{\gamma_1 T_0},\ r_3 = \frac{\rho {c_1}^2}{\gamma_1 T_0},\ r_4 = \frac{{\gamma_2}^2 b}{\gamma_1 T_0},$ i=1, 2, 3

$\Delta_\mathrm{i}\ (=\ 1,2,3,4)$ are determined by changing the first, second, third, and fourth columns of Δ to $[-\hat{F}_1(\xi_1)e^{i\omega t}, 0, \hat{F}_2(\xi_1)e^{i\omega t}, \hat{F}_3(\xi_1)\,e^{i\omega t}]^{\mathrm{Tr}}$

6. Applications

Linearly distributed source

The linearly varying source is defined as:

$$[F_1(x_1), F_2(x_1), F_3(x_1)] = \begin{cases} 1 - \frac{|x_1|}{a_0} & if\ |x_1| \le a_0 \\ 0 & if\ |x_1| > a_0 \end{cases}$$

Applying the Fourier transform at the plane boundary $x_3 = 0$ in dimensionless form gives

$$[\hat{F}_1(\xi_1), \hat{F}_2(\xi_1), \hat{F}_3(\xi_1)] = \frac{2[1-\cos(\xi_1 a_0)]}{{\xi_1}^2 a_0} \quad (34)$$

Here, a_0 represents the dimensionless width of the source strip. Substituting these expressions into equations (28)-(33) yields the corresponding solutions for the field variables.

7. Validation

Setting $F_2 = F_3 = 0$ in equations (28)-(33) yields the field quantities corresponding to a normal force.

Special Cases

- Setting $\xi = \zeta = \varsigma = 0$ in equations (28)-(33) invoke the case of thermoelastic diffusion described by the dual-phase-lag model.
- Setting $\tau_t = \tau_q = \tau_u = \tau_p = 0$ in equations (28)-(33) invoke the case of non-local thermoelastic diffusion.

8. Inversion of the Transformation

The transforms in equations (28)-(33) are inverted following the procedure outlined in [29].

9. Numerical Implementation and Explanation

In the numerical analysis, copper is employed as the representative thermoelastic diffusion material, in line with the procedure outlined in [19].

$\lambda = 7.76\times 10^{10}\,\mathrm{Kg}m^{-1}s^{-2}$, $\mu = 3.86 \times 10^{10}\ \mathrm{Kg}m^{-1}s^{-2}$, $T_0 = 0.293\times 10^3 K$, $C_e = 0.3891\times 10^3\ \mathrm{J}Kg^{-1}K^{-1}$, $\alpha_t = 1.78\times 10^{-5}K^{-1}$, $\alpha_c = 1.98\times 10^{-4}m^3Kg^{-1}$, $a = 1.02 \times 10^4 m^2s^{-2}K^{-1}$, $b = 9 \times 10^5 Kg^{-1}m^5s^{-2}$, $D = 0.85 \times 10^{-8}\mathrm{Kgs}m^{-3}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$, $K = 0.386\times 10^3 Wm^{-1}K^{-1}$, $t = 0.01s$, $t_0 = 0.2\mathrm{s}$, $\tau_t = 0.6s$, $\tau_q = 0.7\mathrm{s}$, $\tau_p = 0.8s$, $\tau_u = 0.9\mathrm{s}$, $\xi = 0.395 \times 10^{-9}\mathrm{m}$, $\zeta = 0.2 \times 10^{-9}m$, $\varsigma = 0.15 \times 10^{-9}\mathrm{m}$

We use MATLAB (R2016a) to carry out numerical simulations that evaluate normal stress, tangential stress, tangential couple stress, temperature changes, and chemical potential under the following conditions:

1. Thermoelastic diffusion incorporating both non-local effects and dual-phase-lag.

2. Dual-phase-lag thermoelastic diffusion without non-local effects.
3. Non-local thermoelastic diffusion without diffusion phase-lags.
4. Non-local thermoelastic diffusion without thermal phase-lags.
5. Non-local thermoelastic diffusion with no phase-lags.

In all figures:

- Solid line (——) represents thermoelastic diffusion with non-local effects and dual-phase-lag (TNP).
- Small dashed line (- - -) denotes thermoelastic diffusion with dual-phase-lag but without non-local effects (WNTP).
- Large dashed line (— —) corresponds to non-local thermoelastic diffusion without diffusion phase-lag (TNWDP).
- Solid line with central marker (—*—) indicates non-local thermoelastic diffusion without thermal phase-lag (TNWTP).
- Large dashed line with central marker (—o—) represents non-local thermoelastic diffusion without any phase-lags (TN).

Figure 1 displays the distribution of t_{33} when the boundary is subjected to a normal load. Near the loading zone, t_{33} decreases in magnitude, reflecting stress release in the immediate vicinity of the applied force. Beyond this region, the stress response becomes oscillatory for all the cases considered. Interestingly, the oscillation patterns are nearly identical in amplitude and frequency across all cases. This indicates a strong dominance of the applied mechanical source over model-specific parameters.

The variation of shear stress t_{31} , illustrated in Figure 2, exhibits alternating increasing and decreasing trends within the bounded region close to the source. This indicates that the shear response induced by normal loading oscillates locally before gradually stabilizing further from the boundary. The similarity of t_{31} across all cases shows that, like the normal stress response, shear stress is primarily governed by the direct mechanical loading and is less sensitive to the differences in non-locality or phase-lag formulations.

Figure 3 describe T with x_1. For all the considered cases, the temperature decreases monotonically with distance from the boundary, showing a consistent thermal diffusion process. The rate of decrease is almost identical across all models, suggesting that under normal loading, the thermal response is influenced by the presence or absence of non-local or phase-lag effects.

The chemical potential P, shown in Figure 4, also decreases steadily with increasing x_1 for all cases. The trend is smooth and nearly identical in each case, reflecting uniform diffusion of chemical potential away from the loaded boundary. The fact that the decrement is the same across all models indicates that, similar to the thermal field, chemical potential diffusion under normal loading is affected by variations in non-local elasticity and phase-lag formulations.

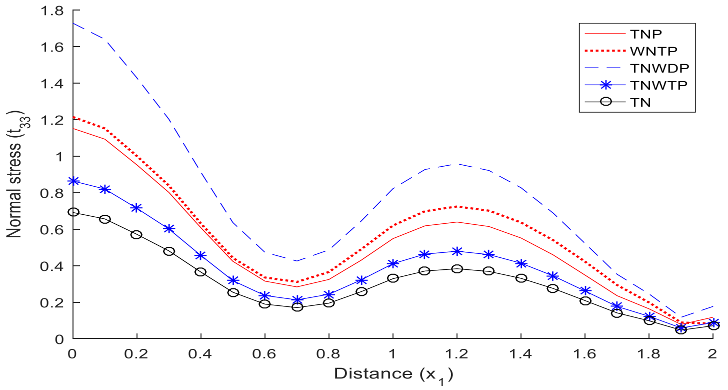

Fig.1: Distance-dependent normal stress distribution.

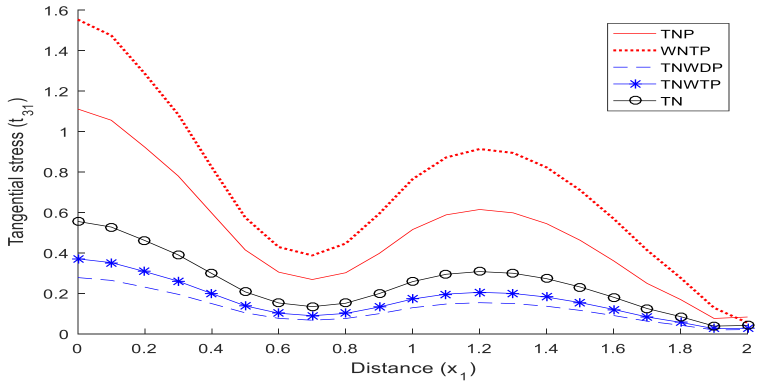

Fig. 2: Distance-dependent tangential stress distribution.

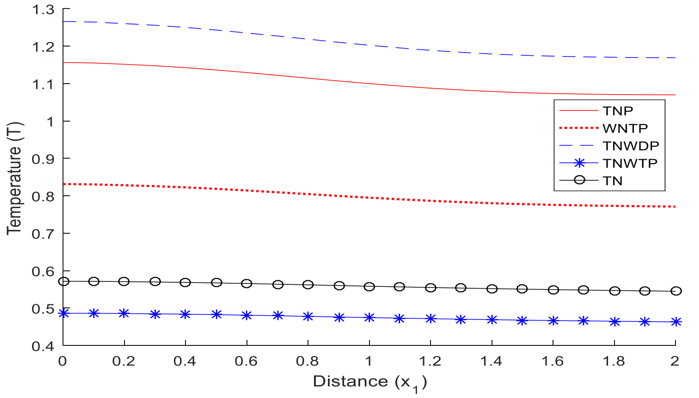

Fig.3: Distance-dependent temperature change distribution.

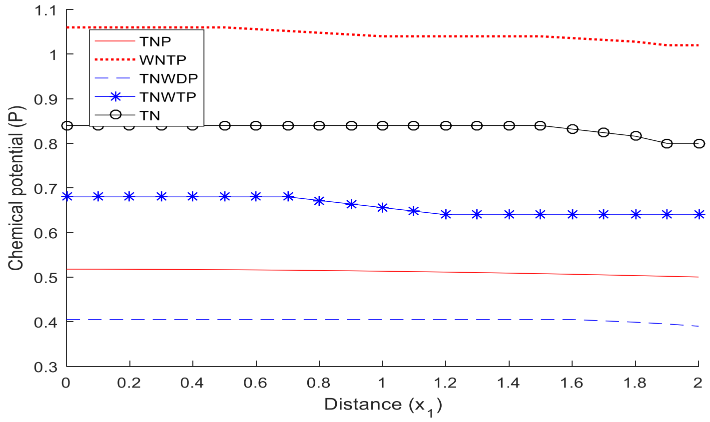

Fig4: Distance-dependent chemical potential distribution.

10.Conclusion

This study investigates the combined effects of non-locality and phase-lag phenomena in a modified couple stress thermoelastic half-space exposed to mechanical loading. The mathematical model was developed and solved using integral transform methods.

For linearly distributed normal loading, the normal stress and chemical potential decreased in magnitude, while other stress components and the temperature field displayed oscillatory fluctuations.

The problem find application in semiconductor technologies, thermal barrier coatings, and composite materials, due to coupled thermo-diffusive loading. Furthermore, the results are useful in biomechanics and nanotechnology.

There is no conflict of interest involved in this study.

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