

A Study of a Modified Equation of State In (2+1)-Dimensional Holographic Cosmology

S. G. Ghodmare¹, K. P. Pande²

¹Department of Mathematics, Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur, Maharashtra, 440033, India

²Department of Mathematics, VMV Commerce, JMT Arts and JJP Science College, Nagpur, Maharashtra, 440008, India

¹ E-mail: sushghodmare26@gmail.com

² E-mail: kalpanappande@gmail.com

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Abstract:

Objectives: This study examines the validity of the cosmic holographic principle in closed (2+1)-dimensional FRW models using a modified equation of state $p = \omega\rho + \Lambda$ with $\Lambda = \alpha\rho$. It aims to determine whether such modifications can ensure holographic compliance. Ultimately, the work highlights how spatial curvature and global topology critically influence the applicability of holographic bounds.

Methods: Using a modified equation of state $p = \omega\rho + \Lambda$, with $\Lambda = \alpha\rho$, we investigate a closed (2+1)-dimensional FRW cosmological model under the Susskind-Fischler holographic principle. We compare the results with flat and open models, highlighting the influence of topology and spatial curvature, to determine whether this density-dependent term is consistent with holographic bounds for closed geometries.

Results: Consistent with (3+1)-dimensional results, our analysis demonstrates that cosmic holographic bounds are satisfied by flat and open (2+1)-dimensional FRW universes. Even with a density-dependent cosmological constant ($\Lambda = \alpha\rho$), closed geometries defy these limits. This persistent violation identifies closed models from flat or open ones, indicating that holographic validity is determined by spatial curvature and global topology rather than matter content or parameterization.

Conclusion: Our analysis demonstrates that, even in the presence of a density-dependent cosmological constant, the cosmic holographic principle fails in closed geometries but is valid for flat and open (2+1)-dimensional FRW universes. This persistent violation shows that holographic applicability is determined by global topology and spatial curvature rather than by content or parameterization. The findings highlight the inherent limitations imposed by topological closure, thereby reaffirming the essential function of geometry in holographic descriptions.

Keywords: Dark energy, the holographic principle, (2+1)-dimensional gravity, Cosmology, Modified FRW cosmology

1. Introduction

As is well known, two liquid cosmological models in [1], the Saez-Ballester scalar-tensor theory of gravity in (2 + 1) dimensions is examined with matter and radiating source. The CMB radiation is reflected by a single fluid in the two-fold fluid model, since the universe's structure composition is demonstrated by different fluid. The difficult nature of two-dimensional models enables them complicated to function with. It is only possible for two-dimensional models efficiently work out in separable-denominator form when there are separated into two sub-models. The separable-denominator form must meet conditions of the lowest rank-decomposition in order to decompose the two-dimensional model into two sub-models. Sans employing separable-denominator form or minimal rank-decomposition needs, this research paper signifies a changed shift that generate a two-dimensional decoupled model form from

the initial two-dimensional general model, which consequently divides into two cascaded one-dimensional models, or two one-dimensional sub-models [2].

We understand that the formula $S = A / 4$ is largely employed to state a black hole's energy as a function of the event horizon's area. This inseparable correlation could be formed as signalling that a black hole's perimeter stores all of its degrees of freedom. The understanding that a physical system's comprehensive structures can be found onto its border in quantum gravity under unique conditions has long been pursued in an endeavour to expand this interconnectedness to more solutions in gravity. We refer to this as the holographic principle [3,4]. A lot of interest has largely been generated in the research of gravitational theories other than the four. Although there are many other factors for this, quantum gravity, grand unified theory, and string theory facilitate them to sustain the main driving forces. For example, the Weyl curvature is exactly zero, the weak field limit of the three-space-time general theory of relativity does not match Newtonian gravity in (2+1)-dimensions, the absence of black holes and gravitational waves in the absence of a negative cosmological constant, and many other special simplifying characteristics. The structure of universal relativistic gravity in (2+1)-dimensional spacetime has been investigated in a number of research [5–7]. (2+1)-gravity is an especially intriguing instance to explore because of the unique characteristics of Einstein's field equations in two space and one time dimension. As signified by quantum field theory, lower-dimensional systems are always researched in physical systems, and applying this method to gravity produces highly qualitative and insightful results. It is predicated that (2+1)-gravity will produce unique outlook for understanding the physical importance of (3+1)-gravity [8]. [9] achieved a particular extension of the holographic principle to cosmology.

Numerous endeavours have been made to apply various holographic principle formulations to different cosmological models [10–13]. Fischler and Susskind proposed a particular realization. Their proposal's unique feature is that the principle of holography holds true for both open and flat universes whose state equation satisfies the requirements $0 \leq p \leq \rho$. The principle is separated for the closed universe, though. understanding the cosmos with a negative cosmic constant makes the issue more tougher and complicated [13]. Irrespective of matter whether the cosmos is even, open, or closed, the holographic principle does not succeed in that scenario. The pre-big bang theory was studied employing the holographic consideration [14], and relied on this investigation, it was stated that the theory solves the entropy problem in pre-big bang theory, which is in divergent with the findings of [12]. A lot of work has been made on extended gravity over the last ten years [15-20], incorporating constantly changing the equation of state or adding the so-called dark energy to illustrate the studied cosmic acceleration. The generally validated equation of state for dark energy is $p = \omega\rho$, where $\omega < 0$ is hydrodynamically not constant. A more general form of a linear equation of state, $p = \alpha(\rho - \rho_0)$, where α and ρ_0 are constants, is proposed by [21] to get around the problem of hydrodynamic instability. Both hydrodynamically stable ($\alpha > 0$) and unstable ($\alpha < 0$) fluids are described by this non-homogeneous linear equation of state. The linear model in question is decreased to the $\omega = \text{constant}$ linear perfect fluid model in the particular solution of $\rho_0 = 0$.

The goal of this study is to re-examine earlier research on cosmological models in (2+1) dimensions, where the mathematical simplicity could aid in a deeper comprehension of the physics. We are further motivated to study (2+1)-dimensional models since the issue of the microscopic interpretation of black hole entropy was resolved by the first successful application of AdS/CFT (Anti-de Sitter/Conformal Field Theory) duality appeared in (2+1)-dimensional models, offering more clues in the case of higher-dimensional black holes [15]. It is obligatory for us to make an endeavour to understand the fundamental foundation of cosmic holography and state that the principle of holography is met for open and even cosmos with broad equations of state. The harmonic principle in a (2+1)-dimensional closed universal, where $\lambda = 2\pi\omega^2 G\rho_0/\omega + 1$, cannot be accomplished for $\lambda < 0$, according to the technique signified by [19,24]. Moreover, λ functions as a negative cosmological constant for $\rho_0 < 0$. We have more substantial validation to look for cosmological harmonic principle reformation in a cosmos where the cosmological constant is negative.

2. Metric and Field Equations

We must begin by exploring the homogeneous expansion equations in (2+1)-dimensions. Concerning the element of the Robertson-Walker line in (2+1) dimensions

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 \right) \quad (1)$$

In (2+1)-dimension, the Einstein field equation is given as,

$$R_{\mu\vartheta} - \frac{1}{2} g_{\mu\vartheta} = -2\pi G T_{\mu\vartheta} \quad (2)$$

We use the matter with energy momentum tensor $T_{\mu\vartheta} = (\rho, -p, -p)$ to keep things simple. Einstein field equation (2) for model (1) becomes

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = 2\pi G\rho, \quad (3)$$

$$\frac{\ddot{a}}{a} = -2\pi Gp, \quad (4)$$

$$\dot{\rho} = -2H(\rho + p) \quad (5)$$

where $H = \frac{\dot{a}}{a}$ represents the Hubble parameter, wherein p and ρ stand for pressure and energy density, respectively. Moreover, a variable's time derivative is reflected by a dot over it. We'll assume that the modified equations of state have the following form:

$$p = \omega\rho + \Lambda \tag{6}$$

here we present $\Lambda = \alpha\rho$, $p = (\omega + \alpha)\rho$

where ω and α are two parameters.

3. Solution of Field Equations

Here, $p = 0$ ($\omega = -\alpha$) for the dust-filled universe, and energy conservation (5) results in

$$\rho a^2 = constant = \rho_0 a_0^2 \tag{7}$$

$$\text{from (3) we get, } \dot{a} = \sqrt{2M_0G - k} \tag{8}$$

where $M_0 = \pi\rho_0 a_0^2$. In order to solve this problem, $M_0 > k / 2G$. Therefore, the answer to (8) gives linearly expanding scale factor as

$$a(t) = a_0 + \sqrt{2M_0G - k}t \tag{9}$$

Regardless of the choice of k , this equation suggest that the universe is continuously expanding and full with dust. [19] has already looked into a similar solution.

The definition of the horizon's comoving distance r_H is

$$r_H = \frac{1}{\sqrt{2M_0G - k}} [\ln(a_0 + \sqrt{2M_0G - kt}) - \ln a_0] \tag{10}$$

whereas the equivalent physical distance is denoted as $L_H = a(t)r_H$. The ratio entropy/area is given by

$$\frac{S}{A} \sim \frac{1}{a} \ln\left(\frac{a}{a_0}\right) \tag{11}$$

where the comoving entropy density is denoted by σ and satisfies $\sigma \sim a_0^2$ initially. With the expansion of the cosmos throughout time, this bound becomes even more strongly satisfied. This finding is in consonance with the results for the open and flat universes that were solved in (3+1)-dimensional scenarios.

Now let's understand at a universe that is asserted by radiation, specifically $\rho = 2p$. For the particular case $\rho_0 = 0$ and $\omega = \frac{1}{2} - \alpha$ the conservation (5) offers

$$\rho a_0^3 = \text{constant} = \rho_0 a_0^3 \quad (12)$$

$$\dot{a}^2 = \frac{2M_0 G}{a} - k \quad (13)$$

$$\ddot{a} = -\frac{M_0 G}{a^2} \quad (14)$$

The answer to (13) is simple for a flat universe ($k = 0$)

$$a(t) = \left[\frac{3}{2} (2M_0 G)^{\frac{1}{2}} \right]^{2/3} t^{2/3} \quad (15)$$

Here, $a(t) \propto t^{2/3}$ matching standard (3+1)-dimension radiation-dominated cosmology, the comoving horizon r_H is given by

$$r_H = \frac{3}{\left[\frac{3}{2} (2M_0 G)^{\frac{1}{2}} \right]^{1/3}} t^{1/3} \quad (16)$$

The comoving entropy area relation acts as follows for big t :

$$\frac{S}{A} = \sigma \frac{r_H}{a(t)} \quad (17)$$

$$\frac{S}{A} \sim t^{-1/3} \quad (18)$$

Resultantly, the S/A ratio is not in tandem with time. The holographic bound will strengthen as the cosmos grows, which is consonance with the (3+1)-dimensional results, provided that the initial holographic constraint, $\sigma/a_0^2 \leq 1$, was satisfied at the starting point. The nature of the flat universe is suited similarly to the open universe ($k = -1$). The closed universe model ($k = 1$) in the radiation-dominated phase is worth talking about. First, equation (13) shows that \dot{a} disappears at $a = 2M_0 G$, after which it turns negative and the universe collapses. This occurs a limited amount of time after the expansion starts.

Let's now examine a flat universe when $\lambda < 0$. In this part, we will talk about how to solve the field equation. Assuming that $\lambda < 0$, we use a modified equation of state to study the holographic principle for $\rho_0 < 0$. Using the modified equation of state (6) and conservation (5), we obtain

$$\rho = \frac{\rho_0}{a^{2(\omega+1)}} - B_0 \tag{19}$$

Where, $\frac{c}{\omega+1} = \rho_0$ and $\frac{\Lambda}{\omega+1} = B_0$

In this case, field equations (3) and (4) can be expressed differently as

$$\left(\frac{\dot{a}}{a}\right)^2 = 2\pi G \left(-B_0 + \frac{\rho_0}{a^{2(\omega+1)}} - \frac{k}{a^2}\right) \tag{20}$$

$$\ddot{a} = -2\pi G \omega \left(-B_0 \omega + \frac{\rho_0}{a^{2(\omega+1)}}\right) \tag{21}$$

For $k = 0$, (20) becomes

$$\dot{a}^2 = -\lambda a^2 + \frac{2M_0 G}{a^{2\omega}} \tag{22}$$

where, $M_0 = \pi\rho_0$. The scaling factor has the following form and can be computed from (22).

$$a(t) = \left(\frac{2M_0 G}{\lambda}\right)^{1/2(\omega+1)} [\sin(\omega + 1)\sqrt{\lambda}t]^{1/\omega+1} \tag{23}$$

For our purposes, this particular solution is not required. Initially, we observe that \dot{a} disappears at $a = \left(\frac{2M_0G}{\lambda}\right)^{1/2(\omega+1)}$ afterwards, \dot{a} turns into a negative collapse. This occurs a limited amount of time after the expansion starts. The value L_H at the turning point can be seen employing (22), which is the definition of the particle horizon.

$$L_H(\text{turning}) = \frac{1}{2(\omega+1)\lambda^{1/2}} \beta\left(\frac{\omega}{2(\omega+1)}, \frac{1}{2}\right) \quad (24)$$

where the Euler beta function is demonstrated by $\beta(p, q)$. Altogether, including all these formulas unfold that in the beginning

$$\frac{S}{A} \sim \sigma \lambda^{1/(\omega/2)} \quad (25)$$

up to a factor of order unity. For $\omega \geq 1$, at the turning point, the ratio S / A is very tiny since the power of λ is positive. As the scale factor becomes closer to the Planck scale, we may study what happens near the last stage of collapse. Through symmetry $L_H \sim \frac{2a_0}{a(\text{turning})} L_H \sim \lambda^{1/(\omega/2)}$ presently. At the plank time, the scale factor $t = 1$ is $a(t = 1) = \left(\frac{2M_0G}{\lambda}\right)^{1/2(\omega+1)} \left[\sin((\omega + 1)\sqrt{\lambda})\right]^{1/\omega+1}$. This ratio is $S/A \sim \lambda^{1/(\omega/2)}$. Hence for small λ the ratio $S/A \sim \lambda^{1/(\omega/2)} \gg 1$ for $\omega \leq 2$. Consequently, we determine the ratio S/A simultaneously achieves oneness following the pivotal moment and the harmonic bounds become violated consequently, as long as the cosmos is still very big, it stays in a classical phase [10].

Conclusion:

We have examined the holographic principle in a (2+1)-dimensional cosmological model by taking into account the cosmology of a perfect fluid with a modified equation of a state. We have found that the holographic principle is satisfied in all models of the flat and open universes for a certain value of $(\omega + 1)$. This finding is in sync with research on (3+1)-dimensional cosmologies. Although, in a (2+1)-dimensional closed universe, the holographic principle does not succeed to achieve. The study has been made to investigate the basic assumptions of the holographic principle in a flat cosmological model during the radiation-dominated phase and addressed the (2+1)-dimensional Friedman equations for a general equation of state, it was found that the Fischler-Susskind bound does not succeed in (2+1)-dimensional cosmological models with a negative cosmological constant, independent of k, as in the (2+1)-dimensional closed universe. Constraints are also found to be valid prior to the point of maximal expansion. Nevertheless, the holographic bound can no longer be upheld after the ratio surpasses unity. This outcome is in line with research in cosmologies with three dimensions.

The (3+1)-dimensional Einstein equations have largely compounded global cosmological solutions that are probably dominated by non-integrability. However, the theory's structure in (2+1)-dimensional spacetime facilitates tangible advancements in the pursuit of general solutions in a number of intriguing circumstances. The investigative study of Einstein's theory in (2+1)-dimensions has been intended by this as well as the largely ingrained held belief that quantum field theory is much better suited to three dimensions than four.

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