

## Fuzzy Soft Set Applications Explored in W-Algebras

M. Indhumathi<sup>1</sup>, K. Jeya Lekshmi<sup>2</sup>, A. Ibrahim<sup>3</sup>

<sup>1</sup> Department of Mathematics, RVS College of Arts and Science, Sulur, Affiliated to Bharathiar University, Coimbatore, Tamil Nadu, India. Email: indumathi@rvsgroup.com

<sup>2</sup> Department of Mathematics RVS College of Arts and Science, Sulur, Affiliated to Bharathiar University, Coimbatore, Tamil Nadu, India. Email: jeyalekshmi@rvsgroup.com

<sup>3</sup> P.G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkottai, Affiliated to Bharathidasan University, Tiruchirappalli, Tamil Nadu, India.

Email: ibrahimaadhil@yahoo.com, dribrahimaadhil@gmail.com

---

### Article History:

**Received:** 12-04-2024

**Revised:** 24-05-2024

**Accepted:** 06-06-2024

### Abstract

Fuzzy soft set theory in Wajsberg algebras (W-algebras) is presented in this paper and its application to a problem of decision making is illustrated. We first introduce the notion of fuzzy soft ideals (fs-ideals) and then analyse some of the associated characteristics.

**Keywords:** W- algebras, soft set, fs-set, fs-ideals, SOFT ENGINE.

**2010 MSC:** 03B20, 03B52, 18B20

---

## 1. INTRODUCTION

Research on the soft set theory is now progressing at a rapid pace. The application of soft set theory to a decision-making issue was covered by Maji et al. [12]. Maji and [13] additionally investigated some techniques related to soft set theory. A new definition of soft set parametrization reduction was developed out by Chen et al. [4], who also compared it to the same attribute reduction idea in rough set theory. The algebraic structure of uncertainty-aware set theories has been studied by a few authors. The best theory for dealing with uncertainty is Zadeh's notion of fuzzy sets. M. Wajsberg introduced the ideas of W-algebra [15]. LPW-Algebras were first presented by Ceterchi Rodica [2]. A generalisation of regular soft sets, known as fs-sets, is presented in [11]. The use of fs-sets to a decision-making scenario is then demonstrated. In this work, we use the notion of fs-ideals and fs-W-algebras and then derive their basic characteristics. In the future, this paper's approach will be expanded upon to analyse many kinds of ideals in W-algebras.

## 2. BASIC RESULTS ON SOFT SETS

The following is a description of Molodtsov's [14] definition of the soft set. Suppose that  $\mathcal{M}$  is an initial universe set and that  $\mathcal{K}$  is a set of parameters. Let  $I \subset \mathcal{K}$  and  $\mathcal{T}(\mathcal{M})$  represent the power set of  $\mathcal{M}$  and  $I \subset \mathcal{K}$ .

**Definition 2.1.** [14] [17] A pair  $(\mathcal{S}, I)$  is called a soft set over  $\mathcal{M}$ , where  $\mathcal{S}$  is a mapping given by  $\mathcal{S}: I \rightarrow \mathcal{T}(\mathcal{M})$ .

A soft set over  $\mathcal{M}$ , stated differently, defines a parameterized family of universe  $\mathcal{M}$  subsets. It is possible to consider elements of the soft set  $(\mathcal{S}, I)$  that are  $\alpha$ -approximate if  $\alpha \in I$ . It is evident that a soft set is not a set.

**Definition 2.2.** [11] Let  $\mathcal{K}$  be a set of parameters and let  $\mathcal{M}$  be the initial universe set.  $\mathcal{T}(\mathcal{M})$  denote the set of fuzzy sets within  $\mathcal{M}$ . Where  $I \subseteq \mathcal{K}$  and  $\check{\mathcal{P}} : I \rightarrow \mathcal{T}(\mathcal{M})$ .

$\check{\mathcal{P}}[a]$  is often referred to as the fuzzy value set of parameter  $a$ . It is a fuzzy set in  $\mathcal{M}$  for every  $a \in I$ .  $(\check{\mathcal{P}}, I)$  degenerates to the usual soft set. If  $\check{\mathcal{P}}[a]$  is a crisp subset of  $\mathcal{M}$  for every  $a$  in  $I$ , fs-sets are therefore a generalization of standard soft sets due to the above concept.

### 3. MAIN RESULT (FUZZY SOFT WAJSBERG-ALGEBRAS)

Unless otherwise noted, let  $\mathcal{K}$  be a collection of the following text's parameters. The expression "Soft engine" will be used, referring to the fact that it generates a W-algebras.

**Definition 3.1.** Let  $(\check{\mathcal{P}}, I)$  be a fs-set over a W-algebra  $\mathcal{X}$ , because  $I$  is a subset of  $\mathcal{K}$ . We say that  $(\check{\mathcal{P}}, I)$  is a fs-set set on  $\mathcal{M}$  over  $\mathcal{X}$ , if there exist  $m \in I$  such that  $\check{\mathcal{P}}(m)$  is a fuzzy W-algebra in  $\mathcal{X}$ . It may be stated that  $(\check{\mathcal{P}}, I)$  is a fs-W-algebra over  $\mathcal{X}$ . If  $(\check{\mathcal{P}}, I)$  is based on parameter  $m$  over  $\mathcal{X}$  for all  $m \in I$ .

**Example 3.2.** Assuming the universe  $\mathcal{M}$  has five colours, that is

$$\mathcal{M} = \{pink, gray, red, olive, tan\}.$$

Assume the role of  $\odot$  a Soft engine that combines two colours in the prescribed order to get the desired results.

$$pink \odot \varrho = pink \text{ for all } \varrho \in \mathcal{M}$$

$$gray \odot \varsigma = \left\{ \begin{array}{l} pink : \varsigma \in \{gray, olive, tan\} \\ gray : \varsigma \in \{pink, red\} \end{array} \right\}$$

$$red \odot z = \left\{ \begin{array}{l} pink : z \in \{red, tan\} \\ gray : z \in \{pink, gray, olive\} \end{array} \right\}$$

$$olive \odot u = \left\{ \begin{array}{l} pink : u \in \{olive, tan\} \\ olive : u \in \{pink, gray, red\} \end{array} \right\}$$

$$tan \odot v = \left\{ \begin{array}{l} pink : v = tan \\ red : v = olive \\ olive : v = red \\ tan : v = \{pink, gray\} \end{array} \right\}$$

Then  $(\mathcal{M}, \odot, pink)$  is a W-algebra.

(i) Let  $(\check{\mathcal{P}}, \mathcal{K})$  be a fs-set over  $\mathcal{M}$ , then  $\check{\mathcal{P}}[glorious]$ ,  $\check{\mathcal{P}}[elegant]$  &  $\check{\mathcal{P}}[modest]$  are fuzzy sets in  $\mathcal{M}$ . Here is how we define them:

$\check{\mathcal{P}}$	<i>pink</i>	<i>gray</i>	<i>red</i>	<i>olive</i>	<i>tan</i>
<i>glorious</i>	0.7	0.7	0.7	0.4	0.4
<i>elegant</i>	0.8	0.7	0.4	0.6	0.4
<i>modest</i>	0.6	0.2	0.5	0.2	0.2

Then, fs-sets W-algebras  $\check{\mathcal{P}}[glorious]$ ,  $\check{\mathcal{P}}[elegant]$  and  $\check{\mathcal{P}}[modest]$  are based on parameters “glorious”, “elegant” and “modest” over  $\mathcal{M}$ , respectively. Thus,  $(\check{\mathcal{P}}, \mathcal{K})$  is a fs-W-algebras over  $\mathcal{M}$ .

(ii)  $\check{\mathcal{Q}}[glorious]$ ,  $\check{\mathcal{Q}}[elegant]$  and  $\check{\mathcal{Q}}[modest]$  are fuzzy sets in  $\mathcal{M}$ . Let  $(\check{\mathcal{Q}}, \mathcal{K})$  be fs-set over  $\mathcal{M}$ . We define them as listed below,

$\check{\mathcal{Q}}$	<i>pink</i>	<i>gray</i>	<i>red</i>	<i>olive</i>	<i>tan</i>
<i>glorious</i>	0.8	0.8	0.8	0.8	0.8
<i>elegant</i>	0.4	0.6	0.5	0.5	0.3
<i>modest</i>	0.9	0.1	0.7	0.1	0.1

$(\check{\mathcal{Q}}, \mathcal{K})$  is not a fs-W-algebras over  $\mathcal{M}$ . As  $(\check{\mathcal{Q}}, \mathcal{K})$  is not a fs-W-algebras. It is based on a parameter “elegant” over  $\mathcal{M}$ .

$$\begin{aligned} \text{Infact } \check{\mathcal{Q}}[elegant] (\text{gray} \odot \text{olive}) &= \check{\mathcal{Q}}[elegant] (\text{pink}) = 0.4 \neq 0.5 \\ &= \min \{ \check{\mathcal{Q}}[elegant](\text{gray}), \check{\mathcal{Q}}[elegant] (\text{olive}) \} \end{aligned}$$

A fs-W-algebras based on both a parameter “glorious” and a parameter “modest” over  $\mathcal{M}$ , we are able to confirm that  $(\check{\mathcal{Q}}, \mathcal{K})$ .

**Example 3.3:** Think about the universe  $\mathcal{M} = \{pink, gray, red, olive, tan\}$  and examine the soft engine  $\Delta$  that generates the products listed below

$$pink \Delta q = \left\{ \begin{array}{l} pink : q \in \{pink, gray, red\} \\ olive : q \in \{olive, tan\} \end{array} \right\}$$

$$gray \Delta \varsigma = \left\{ \begin{array}{l} pink : \varsigma = gray \\ red : \varsigma = \{pink, red\} \\ olive : \varsigma = tan \\ tan : \varsigma = olive \end{array} \right\}$$

$$red \Delta z = \left\{ \begin{array}{l} pink : z = red \\ red : z \in \{pink, gray\} \\ olive : z \in \{olive, tan\} \end{array} \right\}$$

$$olive \Delta u = \left\{ \begin{array}{l} pink : u \in \{olive, tan\} \\ olive : u \in \{pink, gray, red\} \end{array} \right\}$$

$$tan \Delta v = \left\{ \begin{array}{l} pink : v = tan \\ gray : v = olive \\ olive : v = gray \\ tan : v = \{pink, red\} \end{array} \right\}$$

Then  $(\mathcal{M}, \Delta, pink)$  is a W-algebra.

$\mathcal{K} = \{glorious, elegant, modest\}$ . Let  $(\check{\mathcal{P}}, \mathcal{K})$  be a fs-set over  $\mathcal{M}$ , then  $\check{\mathcal{P}}[glorious]$ ,  $\check{\mathcal{P}}[elegant]$  &  $\check{\mathcal{P}}[modest]$  are fuzzy sets in  $\mathcal{M}$ . We define,

$\check{\mathcal{P}}$	pink	gray	red	olive	tan
glorious	0.9	0.6	0.2	0.1	0.1
elegant	0.6	0.4	0.6	0.5	0.4
modest	0.7	0.1	0.5	0.4	0.1

Then  $(\check{\mathcal{P}}, \mathcal{K})$  is a fs-W-algebras over  $\mathcal{M}$ .

**Proposition 3.4:** A fs-W-algebras  $(\check{\mathcal{P}}, I)$  over  $\mathcal{X}$  is defined as  $(\check{\mathcal{P}}(m)(0) \geq \check{\mathcal{P}}(m)(q)$  for every  $q \in \mathcal{X}$  where  $m$  is any parameter in  $I$ .

Proof: Let  $q \in \mathcal{X}$  and  $m \in I$ , then  $\check{\mathcal{P}}(m)(0) = \check{\mathcal{P}}(m)(q \rightarrow q)$   
 $\geq \min \{ \check{\mathcal{P}}(m)(q), \check{\mathcal{P}}(m)(q) \} = \check{\mathcal{P}}(m)(q)$

Hence  $\check{\mathcal{P}}(m)(0) \geq \check{\mathcal{P}}(m)(q)$  for all  $q \in \mathcal{X}$  and any parameter  $m$  in  $I$ .

**Theorem 3.5:** Let  $(\check{\mathcal{P}}, I)$  be a fs-W-algebras over  $\mathcal{X}$ . If  $J$  is a subset of  $I$ , then  $(\check{\mathcal{P}}/J, J)$  is a fs-W-algebras over  $\mathcal{X}$ .

The example that follows demonstrates that a fs-set  $(\check{\mathcal{P}}, I)$  exists over a W-algebra  $\mathcal{X}$  in such a way that,

- (i)  $(\check{\mathcal{P}}, I)$  is not a fs-W-algebra over  $\mathcal{X}$ .
- (ii) there exists a subset  $\mathcal{B}$  of  $I$  such that  $(\check{\mathcal{P}}/J, J)$  is a fs-W-algebra over  $\mathcal{X}$ .

**Example 3.6:** Let  $(\mathcal{M}, \odot, pink)$  is a W-algebra as in example-3.2 define parameters  $I = \{glorious, elegant, modest, clever, gentle\}$ .

If  $(\check{\mathcal{P}}, I)$  be a fs-set over  $\mathcal{M}$ , then  $\check{\mathcal{P}}[glorious]$ ,  $\check{\mathcal{P}}[elegant]$ ,  $\check{\mathcal{P}}[modest]$ ,  $\check{\mathcal{P}}[clever]$  &  $\check{\mathcal{P}}[gentle]$  defined as

$\check{\mathcal{P}}$	pink	gray	red	olive	tan
glorious	0.7	0.7	0.7	0.4	0.4
elegant	0.8	0.7	0.4	0.6	0.4
modest	0.6	0.2	0.6	0.2	0.2
clever	0.2	0.3	0.4	0.5	0.5
gentle	0.3	0.2	0.6	0.6	0.2

Then  $(\check{\mathcal{P}}, I)$  is not fs-W-algebra over  $\mathcal{M}$ . Since  $\check{\mathcal{P}}[\text{clever}]$  &  $\check{\mathcal{P}}[\text{gentle}]$  are not fuzzy W-algebra in  $\mathcal{M}$ . But  $J = \{\text{glorious, elegant, modest}\}$  then  $(\check{\mathcal{P}}/J, J)$  is a fs-W-algebra over  $\mathcal{M}$ .

**Definition 3.7:** [1] The soft set  $(\check{H}, \mathcal{D})$  where  $\mathcal{D} = I \cup J$  and for every  $t \in \mathcal{D}$  is the extended intersection (Ei) of two soft sets  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$  over  $\mathcal{M}$ .

$$\check{H}[t] = \left\{ \begin{array}{l} \check{\mathcal{P}}[t] : t \in I \setminus J \\ \check{\mathcal{Q}}[t] : t \in J \setminus I \\ \check{\mathcal{P}}[t] \cap \check{\mathcal{Q}}[t] : t \in I \cap J \end{array} \right\} \text{ we write } (\check{\mathcal{P}}, I) \check{\cap}_t (\check{\mathcal{Q}}, J) = (\check{H}, \mathcal{D}).$$

**Definition 3.8:** Let  $(\check{\mathcal{P}}, I)$  &  $(\check{\mathcal{Q}}, J)$  be two soft sets over  $\mathcal{M}$  such that  $I \cap J \neq \emptyset$  the Ei of  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$  is denoted by  $(\check{\mathcal{P}}, I) \check{\cap}_s ((\check{\mathcal{Q}}, J)$  and is defined as  $(\check{\mathcal{P}}, I) \check{\cap}_s (\check{\mathcal{Q}}, J) = (\check{H}, \mathcal{D})$  where  $\mathcal{D} = I \cap J$  and for all  $u \in \mathcal{D}$ ,  $\check{H}[u] = \check{\mathcal{P}}[u] \cap \check{\mathcal{Q}}[u]$ .

**Theorem 3.9:** The Ei of  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$  is a fs-W-algebra over  $\mathcal{X}$ , If  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$  are fs-W-algebras over  $\mathcal{X}$ .

**Proof:** The Ei of  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$  is  $(\check{\mathcal{P}}, I) \check{\cap}_t (\check{\mathcal{Q}}, J) = (\check{H}, \mathcal{D})$  then  $\mathcal{D} = I \cup J$  for any  $u \in \mathcal{D}$ .  $\check{H}(u) = \check{\mathcal{P}}(u)$  (resp  $\check{H}(u) = \check{\mathcal{Q}}(u)$ ) is a fuzzy W-algebra if  $u \in I \setminus J$  (resp  $u \in J \setminus I$ ).  $\check{H}[u] = \check{\mathcal{P}}[u] \cap \check{\mathcal{Q}}[u]$  is a fuzzy W-algebra for all  $u \in I \cap J$  if  $I \cap J \neq \emptyset$ . Given that two fuzzy W-algebra intersect to form a fuzzy W-algebra. Therefore, over W-algebra  $\mathcal{X}$ ,  $(\check{H}, \mathcal{D})$  is a fs-W-algebra.

**Corollary 3.10:** If  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$  are two fs-W-algebras over  $\mathcal{X}$ , then  $(\check{\mathcal{P}}, I) \check{\cap} (\check{\mathcal{Q}}, J)$  is a fs-W-algebra over  $\mathcal{X}$  through its Ei.

**Corollary 3.11:** A fs-W-algebra is the limited Ei of two fs-W-algebras.

**Theorem 3.12:** A W-algebra  $\mathcal{X}$  has two fs-W-algebras:  $(\check{\mathcal{P}}, I)$  and  $(\check{\mathcal{Q}}, J)$ . The union  $(\check{\mathcal{P}}, I) \check{\cup} (\check{\mathcal{Q}}, J)$  is a fuzzy W-algebra over  $\mathcal{X}$ , If  $I$  &  $J$  are disjoint.

Proof: We can write  $(\check{\mathcal{P}}, I) \check{\cup} (\check{\mathcal{Q}}, J) = (\check{H}, \mathcal{D})$  where  $\mathcal{D} = I \cup J$  and for all  $t \in \mathcal{D}$

$$\check{H}[t] = \left\{ \begin{array}{l} \check{\mathcal{P}}[t] : t \in I \setminus J \\ \check{\mathcal{Q}}[t] : t \in J \setminus I \\ \check{\mathcal{P}}[t] \cap \check{\mathcal{Q}}[t] : t \in I \cap J \end{array} \right\}$$

Since  $I \cap J = \emptyset$ , for every  $u \in \mathcal{D}$ , either  $u \in I \setminus J$  (or)  $u \in J \setminus I$ . Because  $(\check{\mathcal{P}}, I)$  is a fuzzy W-algebra over  $\mathcal{X}$ . If  $u \in I \setminus J$ , then  $\check{H}(u) = \check{\mathcal{P}}(u)$  is a fuzzy W-algebra in a  $\mathcal{X}$ .  $\check{H}(u) = \check{\mathcal{Q}}(u)$  is a fuzzy W-algebra in a  $\mathcal{X}$ , if  $u \in J \setminus I$ , for  $(\check{\mathcal{Q}}, J)$ .

Hence  $(\check{H}, \mathcal{D}) = (\check{\mathcal{P}}, I) \check{\cup} (\check{\mathcal{Q}}, J)$  is a fuzzy W-algebra over  $\mathcal{X}$ .

If  $I$  &  $J$  are not disjoint, theorem 3.12 is not valid as the following example displays.

**Example 3.13:** Let  $\mathcal{M} = \{\text{pink, gray, red, olive, tan}\}$  be a universe, and determine of a soft engine  $\odot$  that generates the items listed below.

$$\text{pink} \star \varrho = \left\{ \begin{array}{l} \text{pink} : \varrho \in \{\text{pink, gray}\} \\ \text{red} : \varrho = \text{red} \\ \text{olive} : \varrho = \text{olive} \\ \text{tan} : \varrho = \text{tan} \end{array} \right\}$$

$$\text{gray} \star \varsigma = \left\{ \begin{array}{l} \text{gray} : \varsigma = \text{pink} \\ \text{pink} : \varsigma = \text{gray} \\ \text{red} : \varsigma = \text{red} \\ \text{olive} : \varsigma = \text{olive} \\ \text{tan} : \varsigma = \text{tan} \end{array} \right\}$$

$$\text{red} \star \text{z} = \left\{ \begin{array}{l} \text{pink} : \text{z} = \text{red} \\ \text{red} : \text{z} \in \{\text{pink, gray}\} \\ \text{tan} : \text{z} = \text{olive} \\ \text{olive} : \text{z} = \text{tan} \end{array} \right\}$$

$$\text{olive} \star \text{u} = \left\{ \begin{array}{l} \text{pink} : \text{u} = \text{olive} \\ \text{olive} : \text{u} \in \{\text{pink, tan}\} \\ \text{tan} : \text{u} = \text{red} \\ \text{red} : \text{u} = \text{tan} \end{array} \right\}$$

$$\text{tan} \star \text{v} = \left\{ \begin{array}{l} \text{pink} : \text{v} = \text{tan} \\ \text{red} : \text{v} = \text{olive} \\ \text{olive} : \text{v} = \text{red} \\ \text{tan} : \text{v} \in \{\text{pink, tan}\} \end{array} \right\}$$

Then  $(\mathcal{M}, \star, \text{pink})$  is a W-algebra, define  $I$  &  $J$  as,

$$I = \{\text{glorious, elegant, modest, clever}\}$$

$$J = \{\text{modest, clever, gentle}\}$$

$I$  and  $J$  are therefore not disjoint. If  $(\check{\mathcal{P}}, I)$  be a fs-set over  $\mathcal{M}$ , then fuzzy sets in  $\mathcal{M}$  include  $\check{\mathcal{P}}[\text{glorious}]$ ,  $\check{\mathcal{P}}[\text{elegant}]$ ,  $\check{\mathcal{P}}[\text{modest}]$  &  $\check{\mathcal{P}}[\text{clever}]$ . Then, define as:

$\check{\mathcal{P}}$	<i>pink</i>	<i>gray</i>	<i>red</i>	<i>olive</i>	<i>tan</i>
<i>glorious</i>	0.8	0.7	0.4	0.4	0.4
<i>elegant</i>	0.7	0.6	0.5	0.3	0.3
<i>modest</i>	0.9	0.6	0.2	0.4	0.2
<i>clever</i>	0.6	0.6	0.3	0.3	0.5

Then a fs-W-algebra over  $\mathcal{M}$  is  $(\check{\mathcal{P}}, I)$ . If  $(\check{\mathcal{Q}}, J)$  be a fs-W-algebra over  $\mathcal{M}$ , then the fuzzy sets in  $\mathcal{M}$   $\check{\mathcal{Q}}[\text{modest}]$ ,  $\check{\mathcal{Q}}[\text{clever}]$  and  $\check{\mathcal{Q}}[\text{gentle}]$ . Here is how we define them:

$\check{\mathcal{Q}}$	<i>pink</i>	<i>gray</i>	<i>red</i>	<i>olive</i>	<i>tan</i>
<i>modest</i>	0.7	0.6	0.4	0.2	0.2
<i>clever</i>	0.6	0.6	0.4	0.4	0.6
<i>gentle</i>	0.8	0.5	0.3	0.5	0.3

Hence, a fs-W-algebra over  $\mathcal{M}$  is  $(\check{Q}, \mathcal{J})$ . However, the union  $(\check{\mathcal{P}}, I) \cup (\check{Q}, \mathcal{J})$  is not a fs-W-algebra over  $\mathcal{M}$ .

$$\begin{aligned}
 & (\check{\mathcal{P}}[\text{modest}] \cup \check{Q}[\text{modest}])(\text{olive} \star \text{red}) = (\check{\mathcal{P}}[\text{modest}] \cup \check{Q}[\text{modest}])(\text{tan}) \\
 & = \max\{\check{\mathcal{P}}[\text{modest}](\text{tan}), \check{Q}[\text{modest}](\text{tan})\} = 0.2 \text{ and} \\
 & \min\{(\check{\mathcal{P}}[\text{modest}] \cup \check{Q}[\text{modest}])(\text{olive}), \check{\mathcal{P}}[\text{modest}] \cup \check{Q}[\text{modest}](\text{red})\} \\
 & = \min\{\max\{\check{\mathcal{P}}[\text{modest}](\text{olive}), \check{Q}[\text{modest}](\text{olive})\}, \max\{\check{\mathcal{P}}[\text{modest}](\text{red}), \\
 & \quad \check{Q}[\text{modest}](\text{red})\}\} \\
 & = \min\{\max\{0.4, 0.2\}, \max\{0.2, 0.4\}\} = 0.4.
 \end{aligned}$$

**Definition 3.14.** A fs-set  $(\check{\mathcal{P}}, I)$  over a W-algebra  $\mathcal{M}$ .  $(\check{\mathcal{P}}, I)$  is a fs-ideal of  $\mathcal{M}$  based on a parameter  $u$ , if there exists a parameter  $u \in I$  such that  $\check{\mathcal{P}}[u]$  is a fuzzy ideal of  $\mathcal{M}$ . We say that  $(\check{\mathcal{P}}, I)$  is a fs-ideal of  $\mathcal{M}$  if and only if it is based on all parameters.

**Example 3.15.** Let  $\mathcal{M} = \{\text{guava, mango, beetroot, plum, turnip}\}$  be a universe, and look at a soft machine that produces the items listed below.

guava  $\pitchfork$  q = guava for all  $q \in \mathcal{M}$ ,

$$\text{mango} \pitchfork \varsigma = \left\{ \begin{array}{l} \text{guava} : \varsigma \in \{\text{mango, plum, turnip}\} \\ \text{banana} : \varsigma \in \{\text{guava, beetroot}\} \end{array} \right\}$$

$$\text{carrot} \pitchfork z = \left\{ \begin{array}{l} \text{beetroot} : z \in \{\text{guava, mango, turnip}\} \\ \text{guava} : z \in \{\text{beetroot, plum}\} \end{array} \right\}$$

$$\text{plum} \pitchfork u = \left\{ \begin{array}{l} \text{plum} : u = \text{guava,} \\ \text{mango} : u = \text{beetroot,} \\ \text{guava} : u = \text{plum,} \\ \text{beetroot} : u \in \{\text{mango, turnip}\} \end{array} \right\}$$

$$\text{turnip} \pitchfork v = \left\{ \begin{array}{l} \text{guava} : v = \text{turnip,} \\ \text{turnip} : v \in \{\text{guava, beetroot}\} \\ \text{mango} : v \in \{\text{mango, plum}\} \end{array} \right\}$$

then  $(\mathcal{M}, \pitchfork, \text{guava})$  is a W-algebra. Define  $\mathcal{K} = \{\text{puss, goat, yak, hen, sheep, rabbit}\}$ .

Let  $(\check{\mathcal{P}}, \mathcal{K})$  be a fs-set over  $\mathcal{M}$ . Then  $\check{\mathcal{P}}[\text{puss}], \check{\mathcal{P}}[\text{goat}], \check{\mathcal{P}}[\text{yak}], \check{\mathcal{P}}[\text{hen}], \check{\mathcal{P}}[\text{sheep}]$  &  $\check{\mathcal{P}}[\text{rabbit}]$  are fuzzy sets in  $\mathcal{M}$ .

$\check{\mathcal{P}}$	guava	mango	beetroot	plum	turnip
puss	0.6	0.4	0.4	0.4	0.4
goat	0.7	0.5	0.7	0.5	0.5
yak	0.8	0.8	0.3	0.3	0.8
hen	0.4	0.2	0.4	0.2	0.2
sheep	0.5	0.5	0.9	0.7	0.4
rabbit	0.6	0.5	0.2	0.2	0.5

A fs-ideal that is based on the parameters “puss”, “goat”, “yak”, “hen” and “rabbit”, then  $(\check{\mathcal{P}}, \mathcal{K})$ . However, based on the parameters “sheep”,  $(\check{\mathcal{P}}, \mathcal{K})$  is not a fs-ideal of  $\mathcal{M}$ , as  $\check{\mathcal{P}}[\text{sheep}](\text{turnip}) = 0.4 < 0.5 = \min \{ \check{\mathcal{P}}[\text{sheep}](\text{turnip} \cap \text{plum}), \check{\mathcal{P}}[\text{sheep}](\text{plum}) \}$ .

We know for the most part, that sheeps prefer beetroot. Based on the parameter “sheep”, we may determine that  $(\check{\mathcal{P}}, \mathcal{K})$  in the case above is not a fs-ideal of  $\mathcal{M}$ . This implies that  $(\check{\mathcal{P}}, \mathcal{K})$  cannot be a fuzzy ideal of  $\mathcal{M}$ , just as sheep prefers a beetroot over another.

#### 4. CONCLUSION

In this paper, we combine the notion of fs-sets with the theory of W-algebras. Fs-ideals and fs-W-algebras were introduced, and then some of their associated features were investigated. We shall investigate the applicability of fs-sets to different W-algebraic ideals on the basis of these results.

#### REFERENCES

- [1] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M., *On some new operations in soft set theory*, Comput. Math. Appl. 57, 1547-1553, 2009.
- [2] Ceterchi Rodica, *The Lattice Structure of Pseudo-Wajsberg Algebras*, Journal of Universal Computer Science, 6, 22, 2000.
- [3] Ceterchi Rodica, *Pseudo-Wajsberg Algebras*, Multi-Valued Logic, 6, 67, 2001.
- [4] Chen, D., Tsang, E.C.C., Yeung, D.S., Wang, X., *The parametrization reduction of soft sets and its applications*, Comput. Math. Appl. 49, 757-763, 2005.
- [5] Font, J. M., Rodriguez, A. J., and Torrens, A., *Wajsberg Algebras*, Stochastica, 8, 5, 1984.
- [6] Ibrahim, A., and Indhumathi, M., *PWI-Ideals of Lattice Pseudo-Wajsberg algebras*, Advances in Theoretical and Applied Mathematics, 13, 1, 2018.
- [7] Ibrahim, A., and Indhumathi, M., *Classes of p-ideals of Lattice pseudo-Wajsberg Algebras*, International Journal of Research in Advent Technology, 7, 172, 2019.
- [8] Ibrahim, A., and Indhumathi, M., *Various Types of PWI-Ideals of a Lattice pseudo-Wajsberg Algebras*, Advances in Mathematics, 3, 285, 2019.
- [9] Ibrahim, A., and Indhumathi, M., *On fuzzy Extended PWI-Ideals of Lattice pseudo-Wajsberg Algebras*, International Journal of Advanced Science and Technology, 29, 1163, 2020.
- [10] Ibrahim, A., and Indhumathi, M., *On anti-fuzzy EPWI-ideals of lattice pseudo W-algebras*, Malaya Journal of Matematik, 8, 1587, 2020.
- [11] Maji, P.K., Roy, A.R., Biswas, R., *Fuzzy soft sets*, J. Fuzzy Math. 9, 589-602, 2001.
- [12] Maji, P.K., Roy, A.R., Biswas, R., *An application of soft sets in a decision making problem*, Comput. Maths. Appl. 44, 1077-1083, 2002.
- [13] Maji, P.K., Roy, A.R., Biswas, R., *Soft set theory*, Comput. Math. Appl. 45, 555-562, 2003.
- [14] Molodtsov, D., *Soft set theory-First results*, Comput. Math. Appl. 37, 19-31, 1999.
- [15] Wajsberg, M., *Beitrage zum Metaaussagenkalkul I*, Monatshefte fur Mathematik, 42, 221, 1935.
- [16] Young Bae Jun, Kyoung Ja Lee, Chul Hwan Park., *Fuzzy softset theory applied to BCK/BCI-algebras*, Computers and Mathematics with Applications, 59, 3180-3192, 2010.
- [17] Yousif, Y.Y., Hussain, M. A., Hussain, L.A., *Fibrewise pairwise soft separation axioms*, Journal of Physics: Conference Series, 2019.