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Vague Strong Implicative Filters of Lattice Wajsberg Algebras

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Abstract:

In this paper, we introduce the notation of a vague strong implicative filter of lattice wajsberg algebra. Also, we investigate some of its properties with illustrations. Further, we obtain the relation between vague implicative filter and anti vague strong implicative filter In lattice wajssberg algebra. Finally, we establish the equivalent condition of a vague strong implicative filter.

Keywords: wajsberg algebra; Lattice wajsberg algebra; Implicative filter; strong Implicative filter, vague Implicative strong filter; vague strong implicative filter; vague implicative filter, vague strong implicative filter.

1. Introduction:

The concept of Lattice was first defined by Dedekind in 1897 and then developed by Birkhoft.G, imposed an operation an open problem "Is there a common abstraction which includes Boolean algebra, Boolean rings and lattice ordered group or L-group is an algebraic structure connecting lattice and group. To answer this problem many common abstractions, namely dually residuated lattice ordered semigroups, commutative lattice ordered groups. lattice ordered rings,latice ordered near rings and lattice ordered semirings are presented.Amoung them the algebraic structure lattice ordered semirings or L-semiring was introduced by RangaRao.P.,[9].Also the concept proposed by Zadeh.L.A.[13] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function $\mu_A(x)$ defined from X in to [0 1] has revolutionized the theory of Mathematical modeling. Decision making etc.,in handling the imprecise real life situations mathematically. Now several branches of fuzzy mathematics like fuzzy algebra,fuzzy topology ,fuzzy control theory ,fuzzy measure theory etc.,haveemerged.But in the decision making, the fuzzy theory takes care of membership of an element x only, that is the evidence against x belonging to A .

Gau and Buehrer.D.J and some other areas of Mathematical modeling.Since then the theory of fuzzy sets developed extensively and embraced almost all subjects like engineering science and technology. But the membership function $\mu_{A(x)}$ gives only a approximation belong to A .To avid this and obtain a better estimation and analysis of data decision making. Gau.W.L and Bueher D.J. [3] have initiated the study of vague sets with the hope that they form a better tool to understand, interpret and solve real life problems which are in general vague, than the theory of vague sets do. Ranjit Biswas[6] initiated the study of vague groups by Ramakrishna.N [4],[5],[7] are grate extended the study of vague algebra. The objective of this paper is to contribute further to the study of vague algebra by In this section, we introduce vague strong implicative filter of lattice of wajsberg algebra and vague strong implicative filter of lattice of wajsberg algebra A with illustrations and investigate some properties with suitable examples.

2. Preliminaries

In this section, we recall some basic definitions and properties which are useful to develop the main results.

Definition 2.1 [2] Let (A, →, *,1) be an algebra with a binary operation "→"and a quasi complement " * " is called a wajsberg algebra if and only if it satisfies the following axioms for all x,y,z ∈ A,

1. $1 \to x = x$
2. $(x \to y) \to ((y \to z) \to (x \to z)) = 1$
3. $(x \to y) \to y = (y \to x) \to x$
4. $(x^* \to y^*) \to (y \to x) = 1$.

Definition 2.2[2] The wajsberg algebra (A, → **, *,** 1) satisfies the following properties for all x,y,z ∈ A,

(i). $x \to x = x$

(ii). If $(x \to y) = y \to x = 1$ then $x = y$

(iii). $x \to 1 = 1$

(iv). $x \to (y \to x) = 1$

(v). If $x \to y = y \to z = 1$ then $x \to z = 1$

(vi). If $(x \to y) \to ((z \to x) \to (z \to y)) = 1$

(vii). $x \to (y \to z) = y \to (x \to z)$

(viii). $x \to 0 = x \to 1^* = x^*$

(ix). $(x^*)^* = x$

(x). $x^* \to y^* = y \to x$.

Definition 2.3 [2] The wajsberg algebra (A, →,*,1) is called a lattice wajsberg algebra if it satisfies the following properties for all x,y ∈ A,

(1) a partial ordering "≤" on a lattice wajsberg algebra A, such that x≤y if and only if $x \to y = 1$
(2) $(x \vee y) = (x \to y) \to y$

(3) $(x\wedge y)=((x^{*}\rightarrow y^{*})\rightarrow y^{*})^{*}$ Thus ,we have (A,⋁,⋀,*0,1) is a lattice wajsberg algebra with lower bound 0 and upper bound 1.

Theorem 2.4 [2] The wajsberg algebra (A, →,*,1) satisfies the following properties for all x,y, $z \in$ A,

1. If x≤y then x →z ≥ y→ z

2. If x≤y then z →x ≤ z→ y

3. If x≤y→z if and only if y ≤ x→ z

4.$(x\vee y)^{*}=(x^{*}\vee y^{*})$

5.$(x\wedge y)^{*}=(x^{*}\wedge y^{*})$

6. (x⋁y)→z=(x→ z)⋀(y→ z

7.x→(y⋀z)=(x→ y)⋀(x→ z)

8.(x→ y)⋁(y→x)=1

9.x→ (y ⋁z)=(x→ y)⋁(x→ z)

10.(x⋀y) → z =(x→ y)⋁(x→ z)

11.(x⋀y) ⋁z =(x⋁z)⋀(y⋁z)

12.(x⋀y) →z =(x→ y) →(x→ z) for all x,y,z in A.

Definition 2.5[2] A lattice wajsberg algebra (A, → **, *,** 1) is called a lattice H-Wajsberg algebra,if it satisfies x⋁y⋁((x⋀y)→z)=1 for all x,y.z ∈ A. In a lattice H-wajsberg algebra A ,the following hold.

1.x→(x→ y)=(x→ y)

2.x→ $(y \rightarrow z)$ =(x→ y) →(x→ z) foa all x,y,z in A.

Definition 2.6[2] Let (A_1, → **, *,** 1) and (A_2, → **, *,** 1) be lattice wajsberg algebras, a maping

f: $A_1 \rightarrow A_2$ is called implication homomorphism if f(x → **y) =**f(x) →f(y) holds,

Definition 2.7[2] Let (A_1, → **, *,** 1) and (A_2, → **, *,** 1) be lattice wajsberg algebras, f: $A_1 \rightarrow A_2$ is implication homomorphism from A_1 to A_2 satisfies the following properties.

1. f(x ⋀ **y) =**f(x)⋀f(y) 2. f(x ⋁ **y) =**f(x)⋁f(y) 3.$f(x^{*})=[f(x)]^{*}$

Definition 2.8[2] Let (A_1, → **, *,** 1) be a lattice wajsberg algebra. A subset F of A is called an implicative filter of A if it satisfies the following properties for all x,y ∈ A.

1.1∈ F

2. x∈F and x→**y** ∈ F implies x,y ∈ F.

Definition 2.9[1] Let X be a set. A function μ: X→[0,1] is called a fuzzy subset on X, for all x ∈ X the value of μ(x) describes a degree of membership of x in μ.

Definition 2.10[11] Let μ be a fuzzy subset in set A. Then for t ∈ [0 ,1],the st μ^t ={ x ∈A : μ(x) ≥ t } is called a level subset of μ.

Definition 2.11 [11]Let (A_1, → **, *,** 1) be a lattice wajsberg algebra. A fuzzy subset μ of A is called a fuzzy implicative filter of A if it satisfies the following properties for all x,y in A.

1. μ(1)≥ μ(x)
2. μ(z)≥min{ μ(y), μ((y→z) }.

Definition 2.12 [12]Let μ be a fuzzy implicative filter of a lattice wajsberg algebra A, then A is called x≤y implies μ(x)≤ μ(y) for all x,y in A

Definition 2.13 [11]Let (A_1, → **, *,** 1) be a lattice wajsberg algebra. A fuzzy subset μ of A is called a strong implicative filter if it satisfies

1. $1 \in F$
2. x→ $(y \to$ z) $\in F$ and x $\to y \in$ F implies x→ $z \in$ F.

Definition 2.14[2] Let μ be a fuzzy implicative filter of a lattice wajsberg algebra.A fuzzy sub set μ of A is called fuzzy strong implicative filter of A if it satisfies the properties.

1.μ(1)≥ μ(x)

2.μ($x \to z$)≥min{ μ($x \to y$), μ((x→y→z) }.

Definition 2.15 [3]: A vague set A in the universe of discourse X is a pair (t_A , f_A) where t_A :X →[0 ,1] , f_A: X → [0,1] with $t_A(x)+f_A(x) \le 1$ for all x in X. Here t_A is called the membership function and f_A is called non-membership function and also called true membership function, false membership function respectively.

3. Conclusions:

In this paper, we have introduced the definitions of vague WI-ideal lattice ideal of lattice Wajsberg algebra. We have discussed some of their properties with illustrations. Also, we have shown that every vague WI-ideal of lattice Wajsberg algebra is an Finally, we have shown that collection of WI-ideals of lattice Wajsberg algebras is an lattice ideal of lattice Wajsberg algebra. But, the converse part is true only in the lattice H-Wajsberg algebras. Finally, we have shown that collection of WI-ideals of lattice Wajsberg algebras is an

Results : 3. Vague strong Implicative Filters

In this section, we introduce vague implicative filter of lattice of wajsberg algebra and vague strong implicative filter of lattice of wajsberg algebra A with illustrations and investigate some properties.

Definition 3.1 Let w =(A, → **, *,** 1) be a lattice wajsberg algebra. An vague set A= (t_A , f_A) of w is called vague implicative filter of A if it satisfies the properties.

1. $m_A{}^{(1) \geq}$ $m_A{}^{(x)}$ and $n_A{}^{(1) \leq}$ $n_A{}^{(x)}$

2. $m_A{}^{(y)} \geq \min\{ m^{(x)}, m^{(x\to y)} \}$

$n^{(y)} \leq \max \{n_A{}^{(x)}, n_A{}^{(x\to y)} \}$ for all x, y ,z ∈ A.

Definition 3.2 Let w= (A, → **, *,** 1)be a lattice wajsberg algebra. An vague set A= (m_A , n_A) of w is called vague strong implicative filter of A if it satisfies the properties.

1. $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$

$m_A{}^{(x\to z)} \geq \min\{ m_A{}^{(x\to y)}, m_A{}^{(x\to(y\to z))} \}$

3. $n_A{}^{(x\to z)} \leq \max\{ n_A{}^{(x\to y)}, n_A{}^{(x\to(y\to z))} \}$.

Example 3.2 Let a set A={0,a,b,c,d,1} with the following figures 3.2.1,3.2.2 and 3.2.3 as a partial ordering .Define a quasi complement " * " and a binary operation → on A as in the tables 3.2.1 and 3.2.2.

Table 3.2.1 (Implication)

→	0	A	b	C	D	1
0	1	1	1	1	1	1
A	D	1	a	C	C	1
b	c	1	1	c	c	1
C	B	A	b	1	A	1
D	A	1	a	1	1	1
1	0	A	b	C	D	1

Table:3.2 .2 (complement)

X	x^*
0	1
A	C
B	D
C	A
D	B
1	0

Define '∨' and '∧' operations on A as follows:

a) (x∨y)=(x→y)→ y

b) (x∧y)=((x^* →y^*)→y^*))* for all x,y in A, then A is a lattice wajsberg algebra.

Let A={(0,0.4,0.1)(a,0.7,0.2)(b,0.4,0.2)(c,0.6,0.3)(d 0.5,0.1),(1,0.7,0.3} be a vague strong implicative filter of A but not an vague strong implicative filter of lattice wajsberg algebra w.

Sol:- (i) $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$

(ii) $m_A{}^{(y)} \geq \min\{ m_A{}^{(y)}, m_A{}^{(x\to y)} \}$ and (iii). $n_A{}^{(y)} \leq \max \{n_A{}^{(x)}, n_A{}^{(x\to y)} \}$

for all x, y ,z ∈ A.

Clearly (1) $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$ for all x in A

(ii) $m_A{}^{(y)} = m_A{}^{(b)}$=0.4 $m_A{}^{(a\to b)}$=$m_A{}^{(a)}$=0.7 and min{$m_A{}^{(b)}$, $m_A{}^{(a)}$}=min{0.4,0.7}=0.4

(iii)$n_A{}^{(a\to b)}$=$n_A{}^{(a)}$=0.2 max{$n_A{}^{(x\to y)}$, $n_A{}^{(x\to(y\to z)}$ }

=max $\{n_A^{(a)}, n_A^{(a\to c)}\}$=max$\{n_A^{(a)}, n_A^{(c)}\}$=0.7

Therefore $n_A^{(x\to z)} \le \max\{ n_A^{(x\to y)}, n_A^{(x\to(y\to z)} \}$

Hence A={(0,0.4,0.1)(a,0.7,0.2)(b,0.4,0.2)(c,0.6,0.4)(d 0.5,0.1),(1,0.7,0.3} be a vague implicative filter of A but not an vague strong implicative filter of lattice wajsberg algebra W.

Theorem 3.3 Let W be a lattice wajsberg algebra, and let A= (m_A, n_A) be a vague strong implicative filter of W, then A= (m_A, n_A) is an vague implicative filter of W.

Proof:- Let w **=**(A, → **, *,** 1) be a lattice wajsberg algebra ,by the definition of w

1. $m_A^{(1)} \geq m_A^{(x)}$ and $n_A^{(1)} \leq n_A^{(x)}$

2. $m_A^{(x\to z)} \ge \min\{ m_A^{(x\to y)}, m_A^{(x\to(y\to z))} \}$

3. $n_A^{(x\to z)} \le \max\{ n_A^{(x\to y)}, n_A^{(x\to(y\to z))} \}$

Put x=1 in above (2) and (3) then we get 2. $m_A^{(1\to z)} \ge \min\{ m_A^{(1\to y)}, m^{(1\to(y\to z))} \}$

Implies $m_A^{(z)} \ge \min\{ m_A^{(y)}, m_A^{(y\to z)} \}$ and

$n_A^{(1\to z)} \le \max\{ n_A^{(1\to y)}, n_A^{(1\to(y\to z))} \}$ implies that $n_A^{(z)} \le \max\{ n_A^{(y)}, n_A^{(y\to z)} \}$

Hence A= (m_A, n_A) is an vague implicative filter of W.

Theorem 3.4 Let w be a lattice wajsberg algebra. Then w is a H-wajsberj algebra if and only if each a vague implicative filter of w is an vague strong implicative filter.

Proof:- Let w **=**(A, → **, *,** 1) be a lattice wajsberg algebra and let A= (m_A, n_A) be a vague strong implicative filter of w, we have 1. $m_A^{(1)} \geq m_A^{(x)}$ and $n_A^{(1)} \leq n_A^{(x)}$

2. $m_A^{(y)} \ge \min\{ m_A^{(y)}, m_A^{(x\to y)} \}$

3. $n_A^{(x\to z)} \le \max\{ n_A^{(x\to y)}, n_A^{(x\to(y\to z)} \}$ also we have by the definition of H-wajsberj algebra x→ $(y \to z)$ =(x→ y) →(x→ z) foa all x,y,z in W.

Implies $\min\{ m_A^{(x\to y)}, m_A^{(x\to(y\to z))} \} = \min\{m_A^{(x\to y)}, m_A^{(x\to y)\to(x\to z)}\} \le m_A^{(x\to z)}$.

Therefore $m_A^{(x\to z)} . \ge \min\{m_A^{(x\to y)}, m_A^{(x\to y)\to(x\to z)}\}$…….(3.4.1)

And $\max\{ n_A^{(x\to y)}, n_A^{(x\to(y\to z))} \} = \max\{n_A^{(x\to y)}, n_A^{(x\to y)\to(x\to z)}\} \ge n_A^{(x\to z)}$.

Therefore $n_A^{(x\to z)} \le \max\{n_A^{(x\to y)}, n_A^{(x\to y)\to(x\to z)}\}$ for all x,y,z in W…….(3.4.2)

From (3.4.1) and (3.4.2)" w" is an vague strong implicative filter.

Conversely, we consider an vague strong implicative filter A= {(0,0,0.7)(a,0,0.7)(b,0,0.7),(c,0,0.7)(d,0,0.7)(1,0.8,0)} of w is vague strong implicative filter. Then A is implicative filter, and then implies that

A is lattice H- wajsberg algebra.

Theorem 3.5 Let w be a lattice wajsberg algebra and A= (m_A ,n_A)be a vague set of A,if A=(m_A, n_A) vague strong implicative filter, Then the following are satisfied and equivalent.

(i) If A= (m_A ,n_A) is an vague implicative filter and for all x,y $\in A$,

$m_A{}^{(x\to y)} \geq \min\{ m_A{}^{(x\to(x\to y)}$ and $n_A{}^{(x\to y)} \leq \min\{ n_A{}^{(x\to(x\to y)} \}$

(ii)If A= (m_A ,n_A) is an vague implicative filter and for all x,y,z $\in A$,

$m_A{}^{(x\to y)\to(x\to z)} \geq m_A{}^{(x\to(y\to z))}$ and $n_A{}^{(x\to y)\to(x\to z)} \leq n_A{}^{(x\to(y\to z))}$.

(iii) $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$ for all x,y,z in A.

(iv) $m_A{}^{(x\to y)} \geq \min\{ m_A{}^{(z\to(x\to(x\to y)))}, m_A{}^{(z)} \}$ and $n_A{}^{(x\to y)} \leq \max\{ n_A{}^{(z\to(x\to(x\to y)))}, n_A{}^{(z)} \}$, for all x,y,z in A.

Proof:- (i) implies (ii):

Let A= (m_A ,n_A) is an vague strong implicative filter and for all x,y $\in A$,

We have $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$

$m_A{}^{(x\to z)} \geq \min\{ m_A{}^{(x\to y)} , m_A{}^{(x\to(y\to z)} \}$and $n_A{}^{(x\to z)} \leq \max\{ n_A{}^{(x\to y)} , n_A{}^{(x\to(y\to z)} \}$

we have 1. $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$

$m_A{}^{(x\to z)} \geq \min\{ m_A{}^{(x\to y)}, m_A{}^{(x\to(y\to z))} \}$ and $n_A{}^{(x\to z)} \leq \max\{ n_A{}^{(x\to y)}, n_A{}^{(x\to(y\to z))} \}$

Put x= 1 then $m_A{}^{(z)} \geq \min\{ m_A{}^{(y)}, m_A{}^{(y\to z))} \}$ and

$n_A{}^{(z)} \leq \max\{ n_A{}^{(y)}, n_A{}^{(y\to z))} \}$ for all x,y z $\in A$.

Therefore A= (m_A ,n_A) is an vague implicative filter and for all x,y,z $\in A$.

From $m_A{}^{(x\to z)} \geq \min\{ m_A{}^{(x\to y)}, m_A{}^{(x\to(y\to z))} \}$ and $n_A{}^{(x\to z)} \leq \max\{ n_A{}^{(x\to y)}, n_A{}^{(x\to(y\to z))} \}$

Put z=y in the above condition then

$m_A{}^{(x\to y)} \geq m_A{}^{(x\to(x\to y))}$ and $n_A{}^{(x\to y)} \leq n_A{}^{(x\to(y\to z))}$

If for any x, y, z $\in A$ $m_A{}^{(x\to y)} \geq m_A{}^{(x\to(x\to y)}$, and $n_A{}^{(x\to y)} \leq n_A{}^{(x\to y)}, n_A{}^{(x\to(x\to y))} \}$.

Implies $m_A{}^{(x\to y)\to(x\to z)} = m_A{}^{(x\to((x\to y)\to z))} \geq m_A{}^{((x\to x\to((x\to y)\to z))).}$

Therefore A= (m_A ,n_A) is an vague implicative filter and for all x,y,z $\in A$,

$m_A{}^{(x\to y)\to(x\to z)} \geq m_A{}^{(x\to(y\to z))}$ and $n_A{}^{(x\to y)\to(x\to z)} \leq n_A{}^{(x\to(y\to z))}$.

(ii) implies (iii):- Let (ii) be hold .Then, it is clear $m_A{}^{(1)} \geq m_A{}^{(x)}$ and $n_A{}^{(1)} \leq n_A{}^{(x)}$. If any x,y $\in$ A, We have $m_A{}^{(x\to y)\to(x\to z)} \geq m_A{}^{(x\to(y\to z)}$ and $n_A{}^{(x\to y)\to(x\to z)} \leq n_A{}^{(x\to(y\to z))}$.Put y=x, then, we get $m_A{}^{(x\to x)\to(x\to z)} \geq m_A{}^{(x\to(x\to z)}$ implies $m_A{}^{(x\to z)} \geq m_A{}^{(x\to(x\to z)}$ and we have for any x,y in A $m_A{}^{(x\to y)} \geq m_A{}^{(x\to(x\to y)}$ since $m_A{}^{(x)}$ is an implicative filter and t hen ,we have $m_A{}^{(x\to(x\to y)} \geq \min\{ m_A{}^{(z\to(x\to(x\to y))}, m_A{}^{(z)} \}$.

Hence $m_A{}^{(x\to z)} \geq \min\{ m_A{}^{(z\to(x\to(x\to y))}, m_A{}^{(z)} \}$.

Similarly $n_A{}^{(x\to z)} \leq \max\{ n_A{}^{(z\to(x\to(x\to y))}, n_A{}^{(z)} \}$.

(iii) Implies (i): Let (iii) be hold.

Put x=1then , we get $m_A^{(y)} \geq \min\{ m_A^{(z\to y)}, m_A^{(z)} \}$ and $n_A^{(x\to y)} \leq n_A^{(x\to(x\to y)}$

Implies $m_A^{(1)} \geq m_A^{(x)}$ and $n^{(1)} \leq n_A^{(x)}$ for all x, y, z in A.

Theorem 3.5 A= (m_A ,n_A) be a vague strong implicative filter of lattice wajsberg algebra A if and only if and the fuzzy sets m_A ,$n_A{}^c$ strong implicative filter A where $n_A{}^c(x)$=1- $n_A^{(x)}$ for all x ∈ A.

Proof:- Let A= (m_A ,n_A) be a vague strong implicative filter of lattice wajsberg algebra A. From the definition we have the fuzzy subset m_A (Membership function) is strong implicative filter of A. Now $n_A{}^{C(1)}$ =1-$n_A{}^{(1)}$≥1-$n_A{}^{(x)}$ =$n_A{}^{C(x)}$ and

$n_A{}^C(x\to z)$ =1-$n_A(x\to z)$≥1-$n_A(x\to z)$ =$n_A{}^C(x\to z)$≥1-max{ $n_A(x\to y)$, $n_A(x\to (y\to z))$ }

=min{ $1 - n_A(x\to y)$, $1 - n_A(x\to (y\to z))$ }=min{ $n_A{}^C(x\to z)$, $n_A{}^C(x\to (y\to z))$ }

$n_A{}^C(x\to z)$≥ min{ $n_A{}^C(x\to z)$, $n_A{}^C(x\to (y\to z))$ }

Therefore $n_A{}^c$ strong implicative filter A.

Conversely, if m_A ,$n_A{}^c$ fuzzy strong implicative filter A. Then we have $m_A^{(1)} \geq m_A^{(x)}$ and

1-$n_A^{(1)}$= $n_A{}^c(1)$≥$n_A{}^c(x)$=1- $n_A^{(x)}$ implies that $n_A^{(x)} \geq n_A^{(1)}$ also we have

$m_A^{(x\to z)} \geq \min\{ m_A^{(x\to y)}, m_A^{(x\to(y\to z))} \}$ and

$n_A{}^C(x\to z)$≥ min{ $n_A{}^C(x\to z)$, $n_A{}^C(x\to (y\to z))$ }

Implies that $1 - n_A(x\to z)$≥min{$1 - n_A(x\to y)$, $1 - n_A(x\to (y\to z))$ }

=1- max{$n_A(x\to y)$, $n_A(x\to (y\to z))$ }.

Therefore $n_A(x\to z)$≤ max{$n_A(x\to y)$, $n_A(x\to (y\to z))$ }.

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