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Determination of Continuous Deterioration Level due to Changing Demand Rates in Inventory

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Abstract:

A significant portion of the retail sector deals with products whose freshness deteriorates with time and lowers demand at the same price. Later, when it is normal to offer a discount to stimulate sales, the item may start to deteriorate. If chosen wisely, inventory management could have numerous advantages for the retailer.

The demand rates are changing with and without shortages of the products. This served as an inspiration for creating and analysing an inventory model in which the demand for a degrading product is first driven solely by its selling price and then by its level of freshness. For an inventory model with lost sales shortage, researchers consider general deterioration distribution and general demand function. It is demonstrated that net profit is a conditionally concave function of discount and a concave function of the period with positive inventory. Significant management insights gained from sensitivity analysis indicate that certain policies run opposed to those that merchants often implement, while others align with popular practices.

In this study, a brief introduction will be provided where the guidelines will be applied to manage inventory and keep up the quality of the products so that the demand rate is satisfied. The research will focus on the problems of the deterioration of Inventory with and without shortages and Different Demand rates. The chosen methodology will be a mixed approach with primary qualitative and secondary qualitative data analysis. The research finds out that meeting customer demand and maintaining product stock is possible by applying a quick functioning inventory control system. The research has also given recommendations on maximising the profit function and control inventories while keeping them at optimum level.

Keywords: Inventory Model, Dependent Demand, Deteriorating Conditions, Shortage levels.

1. Introduction

Inventory theory seeks to provide guidelines that managers can apply to reduce the expenses related to keeping inventory levels high and satisfying consumer demand. The "Inventory management system" was created to replace the manual system that was in use at the time and solve its issues. This software is intended to remove, or at least lessen, the difficulties this current system presents. Additionally, this system is tailored to the specific requirements of the business to ensure efficient and successful operation.

In addition to reducing errors as much as possible during data entry, the application also displays error messages when incorrect data is entered. To operate this system, a user is not required to possess any formal knowledge. This alone demonstrates that it is user-friendly. As previously mentioned, an inventory management system can result in a secure, dependable, quick, and error-free management system. Instead of focusing on the recordkeeping, it can help the user focus on other things. As a result, it will aid the Organisation in making better use of its resources.

All organisations, regardless of size, face obstacles in handling inventory, stock, sales, product, category, and customer data. By creating unique personnel management systems that are tailored to managerial demands because every inventory management system has various stock needs. This is intended to help with strategic planning and will guarantee that the company has the appropriate amount of data and specifics in place to support long-term objectives. Additionally, the solutions have remote access capabilities that lets a company manage staff from anywhere at any time, perfect for busy executives who are constantly on the road. In the end, these technologies will enable a person to handle resources more effectively [6].

Men, machinery, materials, money, and other resources that are usable inventory. The inventory is frequently referred to as "stock" when material resources are involved. If resources are controlled or if there is at least one expense that goes down as inventory rises, then there is an inventory problem. The goal is to reduce the overall cost, whether it be anticipated or real. But when inventory influences demand, the goal might also be to maximise profit.

Using the model helps the supplier at every stage by lowering the order amount and reducing production loss from deterioration. In addition, by selecting the following variables, the practitioner must determine not only the conventional order quantity decision but also the quality control deterioration rate.

1) The commodities' diminishing value

2) Cutting expenses while preserving

3) Demand sensitivity to quality.

2. Notations for Mathematical Model

The following are the notions involved in the study:

$$I(t) = Inventoryleavlattimet$$

$$D(t) = Demantrateattimet$$

$$R(t) = Replenishmentrateattimet$$

$$h(t) = Deteriorationrateattimet$$

$$C(t) = Deteriorationlevelattimet$$

$$S(t) = Shortagelevelattimet$$

$$H = Holdingcostperunitofinventorypertimeperiod$$

$$C = Shortagecostperunitofunsatisfieddemand$$

$$h = Deteriorationcostperunitofinventorypertimeperiod$$

$$I_0 = InitialInventerylevel$$

$$C_0 = InitialDeteriorationlevel$$

$$S_0 = InitialShortagelevel$$

Assumptions

1. **Continuous inventory level**

$$\frac{dI}{dt} = R(t) - D(t) - h(t)I(t)(2.1)$$

In the equation (2.1) $R(t)$ is the replenishment rate, $D(t)$ is the demand rate, $I(t)$ is the inventory level. For dairy products $h(t)$ is rate of spoilage or deterioration. The above equation represents continuous change in inventory level over the time.

2. **Continuous Deterioration level**

$$\frac{dC}{dt} = \left(1 - P(t)\right).h(t).I(t)(2.2)$$

In equation (2.2) $h(t)$ is the deterioration time, $I(t)$ is the inventory level, $P(t)$ is the preservative factor. This equation accounts for the effects of a preservative factor $P(t)$ on reducing the deterioration rate $h(t)$ of dairy products.

3. **Shortage level**

$$S(t) = max\left(0, D(t) - I(t)\right)(2.3)$$

In equation (2.3) $I(t)$ is the inventory level and $D(t)$ is the deterioration time. This equation shows the shortage level at time *t*.

The above equations (1), (2) and (3) helps firms understand the gap between demand and available supply, guiding decisions on inventory replenishment and management. These equations provide a mathematical framework for analysing the effects of deterioration on dairy product inventory, exploring preservation strategies, and ensuring the availability of products to meet demand. By incorporating these equations into inventory management models, firms can optimize their strategies to minimize losses due to deterioration, maximize product quality, and meet customer demand effectively.

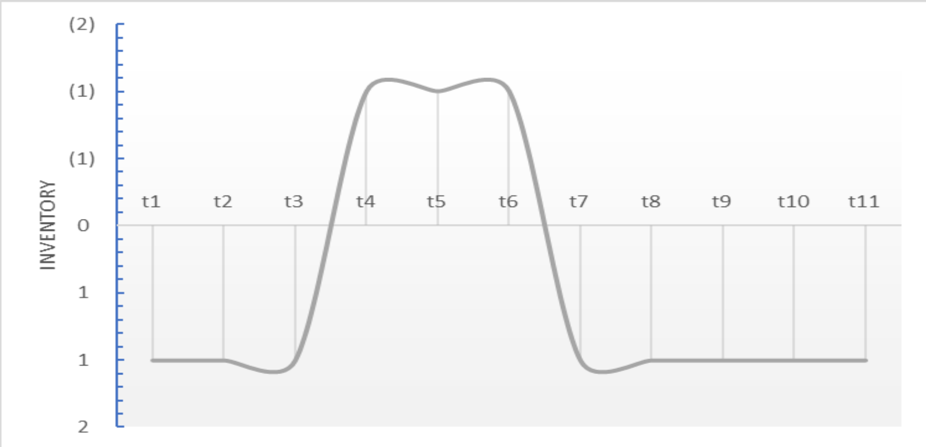

Figure 1

3. Model Formulation and Solution

Mathematical modelling provides a powerful tool to analyse and optimize inventory management strategies. The model formulation involves expressing the relationships between inventory levels, replenishment rates, demand rates, and deterioration rates through differential equations. These equations capture the dynamic behavior of the inventory system over time.

The provided set of equations represents the change in inventory level I_t over time interval $[0, T]$ for different intervals, each characterized by different rates of replenishment and demand. Let's break down these equations and their intervals

$$\frac{dI_t}{dt} = \{\theta I_t - p_1 - a, 0 \leq t \leq t_1 \ \theta I_t - p_2 - a, t_1 \leq t \leq t_2 \ \theta I_t - a, t_2 \leq t \leq t_3 \ -a, t_3 \leq t \leq t_4 \ p_2 - a, t_4 \leq t \leq T \qquad (3.1)$$

The boundary conditions here is $I(0) = I_0$Continuity of Inventory Level at $t_1, t_2, t_3 \wedge t_4 is\ I$

These conditions ensure that the inventory level remains continuous at the boundaries of each time interval.

The solutions for the equation (3.1) are as follows:

$$I_t = \frac{\theta}{\theta - p_1}\left(I_0 - (p_1 + a)e^{-\theta t}\right) + (p_1 + a)e^{-\theta t}, 0 \leq t \leq t_1 \qquad (3.3)$$

$$Ce^{\theta(t - t_1)} + \frac{p_2 + a}{\theta}, t_1 \leq t \leq t_2 \qquad (3.4)$$

$$De^{\theta(t - t_2)} + \frac{a}{\theta}, t_2 \leq t \leq t_3 \qquad (3.5)$$

$$-at + E, t_3 \leq t \leq t_4 \qquad (3.6)$$

$$\frac{p_2 - a}{\theta}\left(1 - e^{-\theta(t - t_4)}\right) + F \qquad (3.7)$$

In the above equations (3.4), (3.5), (3.6), (3.7) C, D, E, F are the constants determined by the continuity conditions.

The solutions for the inventory level equations (3.3) to (3.7) offer insights into the effects of deterioration on inventory dynamics. By introducing preservation factors $P(t)$ into the deterioration term of the differential equation, we can modify the deterioration rate and observe how it affects the inventory over time. This allows us to analyse how different preservation methods can help mitigate the impact of deterioration.

$$\frac{dI_t}{dt} = P(t).(\theta I_t - p_1 - a) \tag{3.8}$$

The boundary condition here is $I(0) = I_0$ Here $I_0 = C_1$

Separating the variable

$$\frac{dI}{\theta I_t - p_{1-a}} = P(t)dt \tag{3.9}$$

$$\int_{⬚}^{⬚} ⬚ \frac{dI}{\theta I_t - p_1 - a} = \int_{⬚}^{⬚} ⬚\, P(t)dt \tag{3.10}$$

Here $u = \theta I_t - p_{1-a}$ then $du = \theta dI$ then,

$$\int_{⬚}^{⬚} ⬚ \frac{1}{u} du = \theta \int_{⬚}^{⬚} ⬚\, dt \tag{3.11}$$

$$ln|u| = \theta t + C_1 \tag{3.12}$$

C_1 is the Constant of Integration

Providing exponentials on both the side on equation (3.12)

$$|u| = e^{\theta t + C_1} \tag{3.13}$$

Removing absolute value from the equation (3.13)

$$u = \pm e^{\theta t + C_1} \tag{3.14}$$

Denoting $C_2 = \pm e^{C_1}$ so the equation (3.14) becomes,

$$\theta I - p_1 - a = C_2 e^{\theta t} \tag{3.15}$$

Rearranging equation (3.12) in order to solve I

$$I = \frac{C_2 e^{\theta t} + p_1 + a}{\theta} \tag{3.16}$$

This the general equation (3.16) for I(t). To find specific solution, we consider initial condition $I(0) = I_0$

Applying initial conditions with $t = 0$ and $I = I_0$ in the general equation (3.16)

$$I_0 = \frac{C_2 e^{\theta t} + p_1 + a}{\theta} \tag{3.17}$$

Solving equation (3.17) for C_2

$$C_2 = \theta I_0 - p_1 - a \tag{3.18}$$

Substituting the value of C_2 in general equation (3.16)

$$I(t) = \frac{(\theta I_0 - p_1 - a)e^{\theta t} + p_1 + a}{\theta} \tag{3.19}$$

Now solving this equation (3.19)

$P(t) = P_0$ for simplicity, assuming its constant over each time interval. θ, p_1, a, I_0 are the given constants.

Substituting the values in equation (3.19) for different time intervals.

For $0 \le t \le t_1$

$$I(T) = \frac{(\theta I_0 - p_1 - a)e^{\theta T} + p_1 + a}{\theta} \tag{3.20}$$

For $t_1 \le t \le t_2$

$$I(T) = \frac{(\theta I(t_1) - p_1 - a)e^{\theta(T-t_1)} + p_1 + a}{\theta} \tag{3.21}$$

For $t_2 \le t \le t_3$

$$I(T) = \frac{(\theta I(t_2) - p_1 - a)e^{\theta(T-t_2)} + p_1 + a}{\theta} \tag{3.22}$$

For $t_3 \le t \le t_4$

$$I(T) = \frac{(\theta I(t_3) - p_1 - a)e^{\theta(T-t_3)} + p_1 + a}{\theta} \tag{3.23}$$

For $t_4 \le t \le T$

$$I(T) = \frac{(\theta I(t_4) - p_1 - a)e^{\theta(T-t_4)} + p_1 + a}{\theta} \tag{3.24}$$

The equation (3.21), (3.22), (3.23) and (3.24) provides the value of I(T) for different time intervals. This equation solved based on initial conditions and given parameters.

Shortage level equation

$$S(t) = max\big(0, D(t) - I(t)\big)$$

Here, considering the cases where demand $D(t)$ exceeds inventory level $I(t)$ and where inventory level is sufficient to meet demand.

$$D(t) \le I(t)$$

There is no shortage, so $S(t) = 0$ (3.25)

$$D(t) > I(t)$$

In this case, the shortage is equal to the difference between demand and inventory level so,

$$S(t) = D(t) - I(t) \quad (3.26)$$

Hence, from equation (3.25) and (3.26) the solution for the shortage level equation is written as:

$$S(t) = \{0 if D(t) \leq I(t) \; D(t) - I(t) if D(t) > I(t) \quad (3.27)$$

This equation (3.27) represents the shortage level at time t based on the comparison between demand and inventory level. If demand exceeds inventory, the shortage is the difference between demand and inventory. Otherwise, there is no shortage.

4. Numerical Examples

Example 4.1: Let's use some arbitrary values for the parameters and functions involved replenishment rate R(t) is 100 units per day, Deman rate D(t) is 80 + 10t units per day, Deterioration rate h(t) is 0.1 units per day, Initial inventory level I (0) is 500 units.

Solutions

$$\frac{dI}{dt} = R(t) - D(t) - h(t)I(t)$$

$$\frac{dI}{dt} = 100 - (80+10.1) - (0.1)(500)$$

$$\frac{dI}{dt} = 100 - 90 - 50$$

$$\frac{dI}{dt} = -40$$

Now, integrating both the side with respect to *t*:

$$\int_{\square}^{\square} \square \, dI = \int_{\square}^{\square} \square \, (-40)dt$$

$$I(t) = -40t + C$$

Applying the initial condition $I(0) = 500$

$$500 = -40 * 0 + C$$

$$C = 500$$

Hence, the solution I(t) is:

$$I(t) = -40t + 500$$

Example 4.2: Let's assume Preservative factor P(t) is 0.3 and Initial deterioration level C(0) is 50 units

Solutions

$$\frac{dC}{dt} = \left(1 - P(t)\right).h(t).I(t)$$

$$\frac{dC}{dt} = (1 - 0.3) * 0.1 * (-40t + 500)$$

$$\frac{dC}{dt} = 0.07 * (-40t + 500)$$

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$$\frac{dC}{dt} = -2.8t + 35$$

Now, integrating both sides with respect to *t*:

$$\int dC = \int (-2.8t + 35)dt$$

$$C(t) = -1.4t^2 + 3.5t + D$$

Applying the initial solution C(0) is 50

$$50 = -1.40^2 + 35 * 0 + D$$

D = 50

The solution for C(t) is

$$C(t) = -1.4t^2 + 3.5t + 50$$

Example 4.3 Solutions for Shortage Level Equation

$$S(t) = max\big(0, D(t) - I(t)\big)$$

Substituting the values of D(t) and I(t)

$$S(t) = max\ \big(0, (80 + 10t) - (-40t + 500)\big)$$

S(t) = max (0, 80 + 10t + 40t -500)

S(t) = max (0,50t – 420)

Since S(t) cannot negative, the expression inside the max function must be greater than or equal to zero:

Solving for t

$$50t \geq 420$$

$$t \geq \frac{420}{50}$$

$$t \geq 8.4$$

The shortage level S(t) will be zero for $t < 8.4$days and it will be positive for $t \geq 8.4$ days.

Sensitivity analysis of example 4.1

Changing parameters	**Change(t)**	**Change in percentage**		
		Δ% in R(t)	**Δ% in D(t)**	**Δ% in C(t)**
I(t)	t= 0	500	550	10%
	t=1	510	560	9.80%
	t=2	520	570	9.60%
	t=3	530	580	9.40%

R(t)	t=0	100	120	20%
	t=1	100	120	20%
	t=2	100	120	20%
	t=3	100	120	20%
D(t)	t=0	80	90	13%
	t=1	90	100	11.10%
	t=2	100	110	10%
	t=3	110	120	9%
H(t)	t=0	0.1	0.12	20%
	t=1	0.1	0.12	20%
	t=2	0.1	0.12	20%
	t=3	0.1	0.12	20%

Sensitivity analysis of example 4.2

Changing parameters	**Change(t)**	**Change in percentage**	
		Equation C(t)	**Δ% in C(t)**
p(t)	0.2	40+40t	0
	0.3	50+35t	25%
	0.4	60+30t	50%
	0.5	70+25t	75%

Sensitivity analysis of example 4.3

Changing parameters	**Change**	**Change in percentage**		
		D(t)	**I(t)**	**Δ% in S(t)**
D(t)	t=0	80	90	12.50%
	t=1	90	100	11%
	t=2	100	110	10%
	t=3	110	120	9%
I(t)	t=0	500	550	10%
	t=1	460	510	10.87%
	t=2	420	470	11.90%
	t=3	380	430	13.16%

5. Conclusion

The research findings emphasise the significance of efficient inventory management in retail operations, specifically in the management of deteriorating inventory items. The study's key findings are as follows: Effective inventory management systems are essential for meeting customer demand, ensuring product availability, and optimising profitability. The study highlights the necessity of efficient inventory control systems to ensure seamless operations and reduce disruptions. Preservation technology is crucial in reducing the impact of deterioration on inventory items, particularly perishable goods such as dairy products. Allocating resources towards preservation technology can effectively mitigate financial losses, elevate the standard of service, and bolster competitiveness within the retail industry. This study investigates the intricacies of inventory systems, encompassing diverse categories of inventory items and the challenges associated with their management. Challenges arising from supply shortages, improper management, and deterioration necessitate the implementation of effective inventory control strategies. Ethical considerations are fundamental to research on inventory management, encompassing the reduction of environmental impact, the promotion of social responsibility, the guarantee of data privacy and security, and the preservation of transparency and consumer information. In summary, the study highlights the crucial significance of efficient inventory management strategies, preservation technology, and ethical considerations in retail operations. Through the utilisation of mathematical modelling and analytical techniques, businesses can enhance inventory management practices, reduce expenses, and effectively meet market demands while maintaining the quality of perishable goods. The study's mathematical modelling provides valuable insights into strategies for managing inventory, especially when dealing with deteriorating inventory items. The study showcases the dynamic nature of inventory systems and the influence of factors like demand rates, replenishment rates, and deterioration rates on inventory levels through the formulation and analysis of differential equations and their solutions [14]. The mathematical modelling yielded several significant findings: The study formulates differential equations to model the temporal evolution of inventory levels, considering variables such as replenishment rates, demand rates, and deterioration rates. These equations offer a mathematical framework for examining inventory dynamics and optimising inventory management strategies. By integrating and solving differential equations, this study obtains solutions for inventory levels, deterioration levels, and shortage levels at various time intervals. These solutions provide valuable understanding of inventory system dynamics and the impact of deterioration on inventory levels over a period. The study performs a sensitivity analysis to investigate the influence of altering parameters, such as demand rates, preservation factors, and initial inventory levels, on the dynamics of inventory. This analysis facilitates the identification of crucial factors that influence decisions regarding inventory management and enables the optimisation of strategies for controlling inventory. Numerical examples exemplify the utilisation of mathematical models in practical situations, showcasing how these models can be employed to scrutinise inventory systems, forecast inventory levels, and enhance inventory management practices. In summary, the mathematical modelling presented in the study offers a rigorous analytical framework for comprehending and enhancing inventory management strategies when dealing with deteriorating inventory items.

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