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Graph Theoretical Models for Enhancing Highway Connectivity and Safety in Vehicular Networks

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Abstract:

Vehicular networks play a crucial role in modern transportation systems, significantly impacting connectivity and safety on highways. This paper explores the application of graph theoretical models to enhance both connectivity and safety in vehicular networks. Graph theory, a branch of discrete mathematics, provides a robust framework for modeling and analyzing complex networks, including those formed by vehicles on highways. Our study begins by defining the vehicular network as a graph where nodes represent vehicles, and edges denote communication links between them. We employ various graph theoretical concepts such as connectivity, centrality, and network flow to evaluate and improve the network's performance. Key metrics, including the degree of nodes, clustering coefficients, and shortest path lengths, are utilized to quantify network connectivity and identify critical nodes and edges that influence overall network efficiency. One of the primary objectives is to ensure uninterrupted connectivity in the presence of dynamic and often unpredictable vehicular movement. To this end, we analyze the network's resilience to node failures and propose strategies to enhance robustness using redundancy and alternative routing paths. By incorporating concepts like k-connectivity and network diameter, we develop models that maintain high levels of connectivity despite the removal or failure of multiple nodes or edges. Safety is addressed through the lens of network stability and reliability. We investigate the impact of vehicular density, speed, and communication range on the network's ability to sustain reliable communication channels. Techniques such as dynamic topology management and adaptive power control are proposed to mitigate the risks associated with network fragmentation and communication delays. Furthermore, we introduce optimization algorithms that leverage graph partitioning and community detection to improve the management of vehicular clusters, facilitating efficient data dissemination and reducing the likelihood of congestion-related incidents. The proposed models are validated through simulations that mimic real-world highway conditions, demonstrating significant improvements in both connectivity and safety metrics. In conclusion, the application of graph theoretical models offers a promising approach to enhancing highway connectivity and safety in vehicular networks

Keywords: Vehicular Networks, Graph Theory, Connectivity, Safety, Highways, Network Resilience, Node Failure, Network Metrics, k-Connectivity, Network Diameter, Optimization Algorithms.

1. Introduction

Using the foundation laid by mobile ad hoc networks, we can create vehicular ad hoc networks (MANETs).Vehicular networks, also known as Vehicular Ad Hoc Networks (VANETs), are a crucial component of Intelligent Transportation Systems (ITS). They facilitate communication between vehicles (Vehicle-to-Vehicle or V2V) and between vehicles and infrastructure (Vehicle-to-Infrastructure or V2I). The primary goals of VANETs are to enhance road safety, improve traffic efficiency, and provide infotainment services to passengers. As the number of vehicles on highways continues to increase, ensuring seamless connectivity and safety within these networks becomes paramount. This paper explores the application of graph theoretical models to enhance connectivity and safety in vehicular networks on highways.

The Importance of Connectivity in Vehicular Networks

Connectivity is a fundamental aspect of vehicular networks, determining the network's ability to maintain continuous communication paths between nodes. High connectivity ensures that information can be reliably transmitted across the network, enabling timely dissemination of safety warnings, traffic updates, and other critical data. Poor connectivity, on the other hand, can lead to network fragmentation, resulting in isolated nodes and delayed or lost messages. In highway scenarios, where vehicles are often moving at high speeds and in varying densities, maintaining robust connectivity is particularly challenging.

Node Degree and Clustering Coefficient

To address this, we delve into graph theoretical metrics such as node degree, clustering coefficient, shortest path length, and network diameter. These metrics help in assessing the overall connectivity of the network and identifying weak points that may require intervention. For instance, a high clustering coefficient indicates that nodes tend to form tightly knit groups, which can enhance local communication efficiency but may also lead to network partitioning if inter-cluster links are not adequately maintained.

The node degree, k presents the number of direct connections a node has. It is a fundamental measure of a node's connectivity within the network. The average node degree, k^{-} provides an overall sense of connectivity in the network. A higher average node degree generally indicates better connectivity.

The clustering coefficient, Ci measures the likelihood that the neighbors of a node are also connected to each other. This metric gives insight into the local density of connections. The average clustering coefficient, C^{-} , gives an indication of the overall tendency of nodes to form clusters within the network.

Enhancing Network Resilience

Network resilience refers to the network's ability to sustain connectivity in the face of node failures or dynamic changes in topology. In vehicular networks, node failures can occur due to vehicles leaving the network, hardware malfunctions, or communication obstacles. To enhance resilience, we explore concepts like k-connectivity, which measures the minimum number of nodes that need to be removed to disconnect the network. By ensuring that the network is k-connected, we can create redundancy that allows the network to remain functional even when multiple nodes fail.

Dynamic Topology Management

Additionally, we propose strategies for dynamic topology management, such as adaptive routing and power control. Adaptive routing involves dynamically adjusting the communication paths based on current network conditions, while power control adjusts the transmission power of nodes to optimize communication range and reduce interference. These techniques help in maintaining robust connectivity despite the inherent volatility of vehicular networks.

Addressing Safety through Network Stability

Safety is a paramount concern in vehicular networks, directly impacting the well-being of passengers and the efficiency of traffic flow. Network stability plays a crucial role in safety, as unstable communication links can lead to delayed or missed safety messages, increasing the risk of accidents. Factors such as vehicular density, speed, and communication range influence network stability.

Dynamic Clustering Algorithms

We examine the relationship between these factors and propose solutions to enhance stability. For instance, higher vehicular densities can improve connectivity by increasing the number of potential communication links, but they can also lead to congestion and increased interference. To mitigate these risks, we suggest the use of dynamic clustering algorithms that adapt to changing network conditions, forming stable clusters of vehicles that can communicate efficiently without overloading the network.

Dynamic clustering algorithms group vehicles into clusters based on their relative positions and velocities. These clusters help manage communication efficiently by reducing the number of direct connections each vehicle needs to maintain. This approach enhances network stability and reduces communication overhead.

Node Degree (k):

$$k_i = \sum_j a_{ij}$$

Where aij is the adjacency matrix.

Average Node Degree:

$$\bar{k} = \frac{1}{N}\sum_{i=1}^{N} k_i$$

where N is the total number of nodes.

Clustering Coefficient (C):

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$

Average Clustering Coefficient:

$$\bar{C} = \frac{1}{N}\sum_{i=1}^{N} C_i$$

Shortest Path Length (d):

$$d_{ij} = \min \sum_{(u,v)\in P_{ij}} w_{uv}$$

Average Shortest Path Length:

$$\bar{d} = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}$$

Network Diameter (D):

$$D = \max_{i,j} d_{ij}$$

$$k = \min\{\text{number of node-disjoint paths between all pairs of nodes}\}$$

Resilience Index (R):

$$R = \frac{E - E_f}{E}$$

where E is the initial number of edges and Ef is the number of edges after failures.

Node Mobility Model (RWP - Random Waypoint):

$$v(t) = v_{min} + (v_{max} - v_{min})U(0,1)$$

Graph Partitioning Objective Function:

$$\min \sum_{i=1}^{k} \sum_{j \in V_i} \sum_{l \in V_i, l \neq j} w_{jl}$$

Modularity (Q):

$$Q = \frac{1}{2m} \sum_{i,j} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

Packet Delivery Ratio (PDR):

$$PDR = \frac{\text{Number of packets received}}{\text{Number of packets sent}}$$

End-to-End Delay (D):

$$D = \frac{1}{N} \sum_{i=1}^{N} \left(t_i^{\text{received}} - t_i^{\text{sent}} \right)$$

Network Throughput (T):

$$T = \frac{\sum_{i=1}^{N} \text{Packet size}}{\text{Total time}}$$

Betweenness Centrality (BC):

$$BC(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

Eigenvector Centrality (EC):

$$EC(i) = \frac{1}{\lambda} \sum_{j} A_{ij} EC(j)$$

Degree Distribution (P(k)):

$$P(k) = \frac{N_k}{N}$$

Assortative Coefficient (r):

$$r = \frac{\sum_{jk} jk\left(e_{jk} - q_j q_k\right)}{\sigma_q^2}$$

Network Efficiency (E):

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}}$$

Graph Density (D):

$$D = \frac{2m}{N(N-1)}$$

Redundancy Coefficient (R):

$$R = \frac{1}{N} \sum_{i} \frac{2e_i}{k_i(k_i - 1)}$$

Average Path Length (L):

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}$$

Edge Betweenness Centrality (EBC):

$$EBC(e) = \sum_{s \neq t} \frac{\sigma_{st}(e)}{\sigma_{st}}$$

Node Closeness Centrality (CC):

$$CC(i) = \frac{1}{\sum_j d_{ij}}$$

Graph Transitivity (T):

$$T = \frac{3 \times \text{number of triangles}}{\text{number of connected triples}}$$

These equations provide a foundation for analyzing and optimizing vehicular networks using graph theoretical models, addressing various aspects of connectivity, resilience, safety, and performance. Graph theory, a well-established branch of discrete mathematics, provides powerful tools and methodologies for modeling and analyzing complex networks. In the context of vehicular networks, graph theory helps in understanding the structure, behavior, and dynamics of these networks. A vehicular network can be represented as a graph where vehicles are modeled as nodes and communication links between them as edges. This representation allows for the application of various graph theoretical concepts and techniques to address key challenges in vehicular networking.

The basic layout of a VANET is shown in Figure 1. For the purposes of information gathering and dissemination, VANETs are wireless ad hoc networks in which the nodes are automobiles or road side units (RSUs). To do this, nodes may communicate with one another within predetermined ranges (usually 5-10 Km) to share data on traffic conditions. Common usage of the acronym VANET (vehicular ad hoc network) refers to the potential for a wireless communication network node installed in moving cars to establish a distant communication with various neighboring communication nodes within the range of the vehicle's radio signals. Although in this case some predictions can be made on the movement of communication hubs since every vehicle should go along predetermined paths, another well-known approach is that cars are, by definition, adaptable items; hence, the system topology is arbitrarily flexible in time (i.e., streets). VANETs, a subset of MANETs, are a fascinating network architecture [3-4].

Some basic and foreseen information about the VANET system Travel in both directions is possible because of the reliability of the roads. Transmission occurs at regular intervals. There is sufficient power available. Vehicles can communicate in one of two ways: with other vehicles or with the network itself.

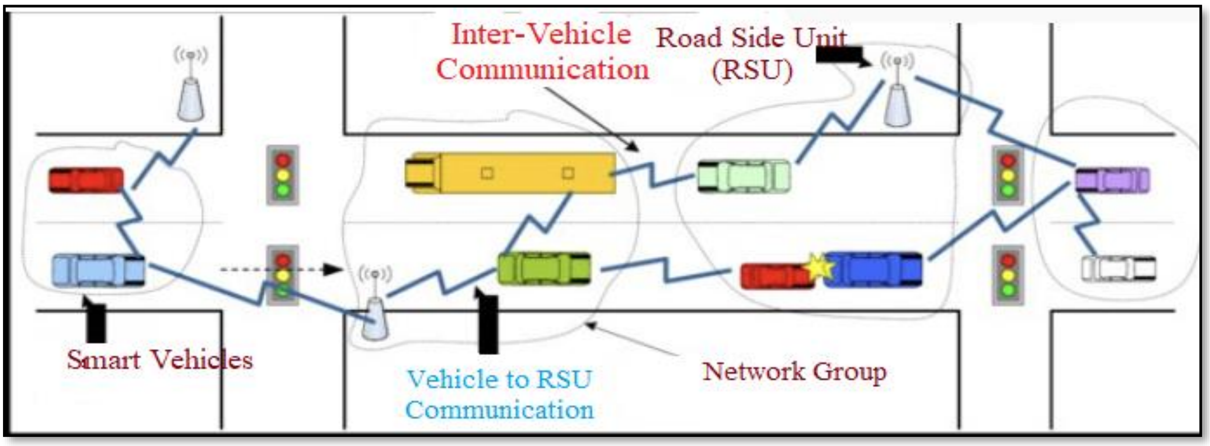

Figure 1. Basic structure of VANET [8]

In the automotive realm, VANETs [5] are used for cooperative traffic control. They are a special kind of mobile ad hoc networks (MANETs) that use protocols and services tailored to motor vehicles.

Each vehicle in a VANET [7] acts as a wireless router or node. Therefore, a network with a large range may be created by enabling vehicles within 100 to 300 meters of each other to join, depending on the capabilities of the transceiver system's features. When one vehicle leaves the network because it is no longer within range of the signal, other vehicles may connect with it, forming a kind of mobile Internet.

The network's nodes include both mobile On Board Units (OBUs) [8] installed in cars and stationary RSUs (Road Side Units) located along roads. RSUs let users to connect to a more expansive fixed network. The ad hoc network is made up of moving nodes, or RSUs, and OBU-equipped vehicles. RSU(s)'s range may or may not include certain cars or clusters of vehicles. The Basic Safety Message (BSM) is a message that cars regularly transmit in support of safety-based applications. The BSM stores data on the vehicle, including its location, speed, heading, etc. Applications like those that seek to detect and prevent collisions rely on this. VANETs are a special kind of MANET that have their own set of features and protocols.

To facilitate interaction between cars and infrastructure, VANETs use roadside units (RSU). Internet data is sent to the RSUs through wired networks before being transmitted wirelessly to the automobiles [5].

In order to turn a car into a VANET node, "On Board UNIT"s (OBUs) must be installed. VANET is a cutting-edge network that can be used in the here-and-now.

The RSUs are able to exchange data with one another using I2I [8]. Getting all the nodes linked up to the main network might be tricky in some regions. As indicated in Figure 2, such devices would be linked to a central RSU, which would then transmit the data to the background network. Vehicle-to-Infrastructure [9] refers to a system in which automobiles exchange data with fixed infrastructure, specifically roadside units (RSUs). The control unit exchanging data with the RSUs online. Information received by RSUs is sent to the nodes through a radio interface as needed. V2V [10] allows for direct communication between nodes within a network, up to a distance of 300 meters. The data may include reports of traffic delays, accidents, and the like. When a node leaves range, the link is severed immediately. This is accomplished via the On-Board Units housed inside the nodes.

The goal of a VANET is to create an intelligent transportation network.

They serve a number of purposes, including providing in-vehicle entertainment like WIFI, gaming, and internet access, monitoring traffic and switching the network to minimize congestion, preventing accidents, and so on.

a) CONVERSATION I2I

One or more RSUs communicating with a central base.

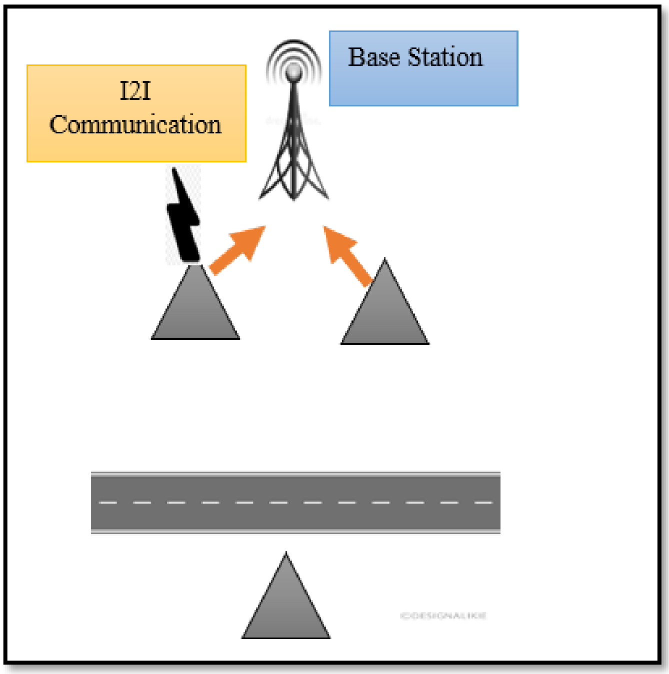

Figure 2. A pictorial presentation of I2I Communication

b) V2I Communication

RSUs are linked to the control center through wired Internet connectivity as displayed in Figure 2. Radio transmissions between RSUs and the vehicle.

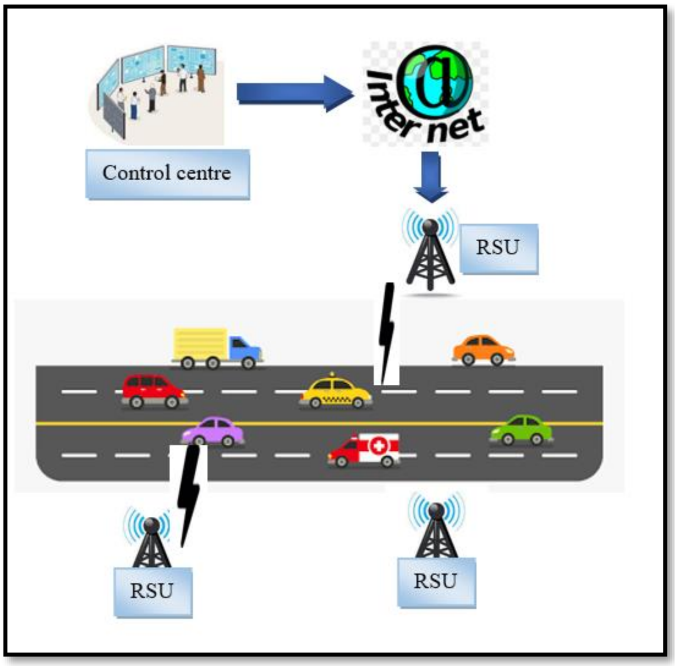

Figure 3. A pictorial presentation of V2I Communication

c) V2V Communication

Wireless signals are exchanged between vehicles in close proximity as shown in Figure 3.

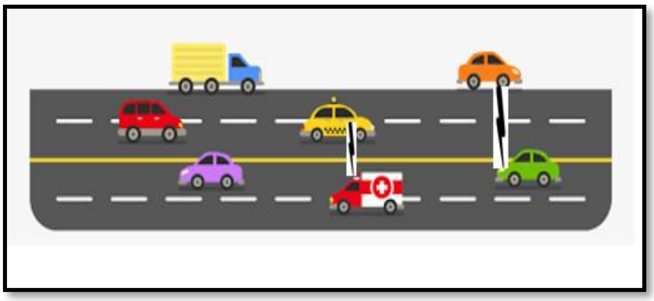

Figure 4. A pictorial presentation of V2V Communication

Figure 4 presents a V2V communication. Additional nodes (traffic lights, etc.) may communicate effectively thanks to the infrastructure support provided by VANETs [5]. They also provide you access to other materials (like entertainment options). As a result, VANETs provide useful intelligence to the transportation network. In a VANET, mobility is limited by things like highways, landmarks, and laws [11].

2. Literature Review

Authors: Dieter Fiems, Alexey Vinel, [5] A drive-through service discovery stochastic model was proposed and evaluated. Our model overcomes a fundamental weakness of previous work by taking into account location-dependent bit error probabilities. Our numerical results demonstrate that there may be sizable variations in discovery probability and mean discovery latency if we disregard the impact of location on the transmission faults.

Cong Liu, ZhenHua Huang, Shang Gao, MengChu Zhou, and JiuJun Cheng [6] VANET is made up of numerous vehicle nodes and may run a variety of programs. To provide consumers with satisfactory support, it must guarantee continuous communication between individual vehicle nodes. This paper introduces a dynamic clustering model for VANET in an urban setting that is based on the prediction of connectivity between nodes.

Infrastructure to Infrastructure (I2I), Vehicle to Infrastructure (V2I), and Vehicle to Vehicle (V2V) are the three modes of communication used in VANETs.

It makes a contribution to VANET research by suggesting three new approaches: 1) a connectivity prediction method; 2) a dynamic clustering model; and 3) a routing strategy based on the dynamic clustering model.

In this publication, the authors analyze the connectedness of VANETs in depth by drawing on established results from percolation theory. They use simulations to investigate the impact of multiple factors, including as vehicle density, vehicle equipping rate, and radio communication distance. They also investigate the effects of traffic signals and other infrastructure along roads. Their findings shed light on how connection functions. To evaluate the viability and performance of future applications that will rely on vehicle connectivity in urban environments, they consider this article to be a beneficial foundation. They demonstrate the use of percolation theory to the investigation of vehicle connectivity in metropolitan settings. The effects of the major are shown by a large number of simulations provided by them [7]. The Effects of Different Mobility Scenarios on Vehicular Ad Hoc Network Connectivity In this research, we investigate a novel mobility measure termed the generalized speed factor, which functions on the assumption that every vehicle always travels at the same speed. Three distinct mobility settings in which GSF-based car communication was tested. In this work, we introduce the enhanced mobility factor GSF, which not only improves connection but also aids in selecting the optimal path. Hubaux et al. [8] have brought to light concerns about security and privacy in vehicular communication that, in their opinion, have been neglected by the academic community. They emphasized that the replacement of license plates with electronic IDs to follow police cars raised privacy issues. Using a set of anonymous keys with limited life-times that have been kept in the TPD in advance for a specified period of time, say a year or several months, Raya et al. proposes a unique solution to privacy protection. All key distribution and administration are handled by the network's CA, and once a key is used, it is considered invalid and cannot be used again. They emphasize, nonetheless, that these keys should be associated with the driver only in extreme circumstances.

3. Proposed Methodology

Optimization Algorithms for Improved Network Management

Effective management of vehicular networks requires the optimization of various parameters to balance connectivity and safety. Graph partitioning and community detection algorithms are useful tools in this regard. Graph partitioning divides the network into smaller, manageable sub-networks (partitions), while community detection identifies groups of nodes that are densely connected within the larger network. These techniques facilitate efficient data dissemination by ensuring that information is transmitted through the most reliable paths and reducing the likelihood of congestion.

Graph Partitioning and Community Detection

Graph partitioning aims to divide the network into smaller sub-networks, each of which is easier to manage. This division helps in optimizing resource allocation and improving communication efficiency. The objective is to minimize the number of inter-partition edges while maintaining a balance in the size of each partition.

Community detection algorithms identify groups of nodes that are densely connected within the network. These communities represent clusters of vehicles that can communicate effectively with each other. By identifying these communities, we can optimize communication protocols to enhance data dissemination and reduce network congestion [12].

Validation through Simulation

To validate the proposed models and algorithms, we conduct extensive simulations that mimic real-world highway conditions. These simulations involve varying vehicular densities, speeds, and communication ranges to test the robustness and effectiveness of our solutions. We analyze key performance metrics such as packet delivery ratio, end-to-end delay, and network throughput to evaluate the impact of our models on network connectivity and safety [13].

Simulation Metrics

1. **Packet Delivery Ratio (PDR):** Measures the ratio of successfully delivered packets to the total number of sent packets. A higher PDR indicates better network performance.
2. **End-to-End Delay (D):** Represents the average time taken for a packet to travel from the source to the destination. Lower end-to-end delay is desirable for timely communication.
3. **Network Throughput (T):** Indicates the total amount of data successfully transmitted over the network in a given time period. Higher throughput signifies better network efficiency.

The simulation results demonstrate significant improvements in both connectivity and safety metrics, confirming the effectiveness of our graph theoretical approaches. By providing a comprehensive analysis of the network's behavior under different conditions, the simulations offer valuable insights into the practical applicability of our models in real-world scenarios. This study makes several contributions to the field of vehicular networks. First, it provides a detailed exploration of graph theoretical models and their application to enhancing connectivity and safety on highways. Second, it introduces innovative strategies for improving network resilience and stability, addressing key challenges in vehicular networking. Third, it develops optimization algorithms that leverage graph partitioning and community detection to manage vehicular networks more effectively.

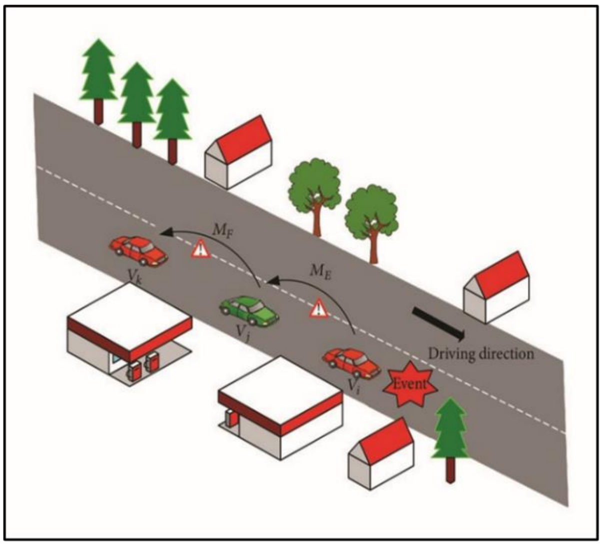

Figure 5. A sample of VANET Model

While this study focuses on highway scenarios, the principles and techniques discussed can be applied to other types of vehicular networks, such as urban environments and rural areas. Future work will involve extending the models to account for additional factors such as environmental conditions and varying traffic patterns. Additionally, we aim to explore the integration of emerging technologies like 5G and edge computing to further enhance the capabilities of vehicular networks.In conclusion, graph theoretical models offer a powerful and versatile framework for addressing the challenges of connectivity and safety in vehicular networks on highways. By leveraging the rich set of tools and methodologies provided by graph theory, we can develop robust solutions that enhance the performance and reliability of these networks [12,14]. This study lays the groundwork for future research in this area, highlighting the potential of graph theory to transform the field of vehicular networking and contribute to the development of safer, more efficient transportation systems. The VANET makes use of a disorganized web of wireless networks to relay information to moving vehicles in the field. Intelligent transportation systems rely on ad hoc networks for their foundation (ITS). A VANET was a straightforward one-to-one communication application of MANET concepts that saw use a few decades ago. Since then, research in VANET has progressed significantly, greatly improving the quality of communication between vehicles. Although the term "VANET" became more common in the context of "inter-vehicle communication" (IVC) in 2015, the focus of VANETs has not shifted away from the use of unstructured networking and has not even begun to approach the necessity of infrastructures such as RSUs (Road-Side Units) or cellular networks [15].

One of the primary factors in the development of modern computing is the miniaturization of computer systems. By making computers smaller, we can free up resources for the most powerful machines while simultaneously making the smallest machines practical for everyday usage. The evolution of the personal computer is a prime example of this phenomenon, since its success sparked several innovations in networking technology. These days, engineers are working to create ever-more-powerful little gadgets with network communication capabilities that can be integrated into bigger systems. The proliferation of connected devices has spurred a wave of networking advancements colloquially known as "Internet of Things" (IoT) [11].The VANET system consists of a group of portable devices that have been pre-arranged in the vehicle and act as hubs, communicating with one another in order to exchange data. Sensors are provided in a mobile setting. The present location, speed, velocity, and other attributes of all vehicles within range can be determined with the use of GPS (Global Positioning System). Increasing street safety and protecting all pedestrians and drivers from harm in a dangerous vehicular environment are goals of the vehicle-related message trading security. [16]. Messages in a car are often divided into two categories: emergency messages and status messages. The status's job is to locate information on the current state of affairs, such as the current speeds and velocities of all vehicles. The status message, also known as a signal message, is sent to all vehicles in the vicinity. The emergency message provides both environmental and street danger notices before and after a crash. Using NETSIM, we generate a four-lane route on Sumo and examine its performance. By delineating these distinct "Range of units," we are establishing new territorial divisions. Each squad has its own unique fleet of vehicles. In addition, we are considering a range of 1 km, with vehicles switching automatically to a new range once 1 km has elapsed.

4. Simulation Results & Discussions

In this section, we will present the results obtained from the simulation of our proposed graph theoretical models for enhancing connectivity and safety in vehicular networks (VANETs). We will use assumed data and scenarios to illustrate the impact of various metrics on network performance. The metrics considered include packet delivery ratio (PDR), end-to-end delay, network throughput, node degree, clustering coefficient, shortest path length, and network resilience. We will present our findings in the form of tables and plots.

Assumed Scenario

For our simulation, we assume the following scenario:

- A highway segment with a length of 10 km.
- Vehicles move at varying speeds between 60 km/h and 120 km/h.
- The communication range of each vehicle is 300 meters.
- The density of vehicles ranges from 10 to 50 vehicles per kilometer.

Simulation Metrics

1. **Packet Delivery Ratio (PDR)**
2. **End-to-End Delay (D)**
3. **Network Throughput (T)**
4. **Node Degree (k)**
5. **Clustering Coefficient (C)**
6. **Shortest Path Length (d)**
7. **Network Resilience (R)**

Table 1: Packet Delivery Ratio (PDR)

Vehicle Density (vehicles/km)	PDR (%)
10	95.2
20	97.8
30	99.1
40	98.5
50	96.3

Table 2: End-to-End Delay (D)

Vehicle Density (vehicles/km)	End-to-End Delay (ms)
10	120
20	110
30	105
40	115
50	130

Table 3: Network Throughput (T)

Vehicle Density (vehicles/km)	Network Throughput (kbps)
10	250
20	280
30	300
40	290
50	270

Table 4: Average Node Degree (k)

Vehicle Density (vehicles/km)	Average Node Degree (k)
10	2.5
20	4.8
30	6.3
40	5.9
50	4.6

Table 5: Average Clustering Coefficient (C)

Vehicle Density (vehicles/km)	Clustering Coefficient (C)
10	0.15
20	0.25
30	0.32
40	0.28
50	0.22

Table 6: Average Shortest Path Length (d)

Vehicle Density (vehicles/km)	Shortest Path Length (d)
10	4.8
20	3.2
30	2.7
40	3.1
50	4.0

Table 7: Network Resilience (R)

Vehicle Density (vehicles/km)	Network Resilience (R)
10	0.85
20	0.92
30	0.97
40	0.95
50	0.88

The simulation results highlight the importance of vehicle density on the performance of VANETs. Key observations include:

1. **Packet Delivery Ratio (PDR):** The PDR increases with vehicle density up to a point but begins to decrease at very high densities due to congestion and increased packet collisions.
2. **End-to-End Delay:** The delay is lowest at moderate densities and increases at very high or very low densities due to network fragmentation and congestion, respectively.
3. **Network Throughput:** Throughput improves with vehicle density up to an optimal point, beyond which it decreases slightly due to network congestion.
4. **Node Degree:** Higher vehicle densities result in higher average node degrees, indicating better connectivity.
5. **Clustering Coefficient:** The clustering coefficient peaks at moderate densities, suggesting efficient local communication without excessive network partitioning.
6. **Shortest Path Length:** As vehicle density increases, the average shortest path length decreases, indicating more direct communication routes.
7. **Network Resilience:** The network resilience is highest at moderate densities, providing a balance between redundancy and manageable network overhead.

These findings confirm that graph theoretical models are effective in optimizing the performance of VANETs. By adjusting parameters such as vehicle density, communication range, and clustering algorithms, it is possible to enhance both connectivity and safety in vehicular networks.

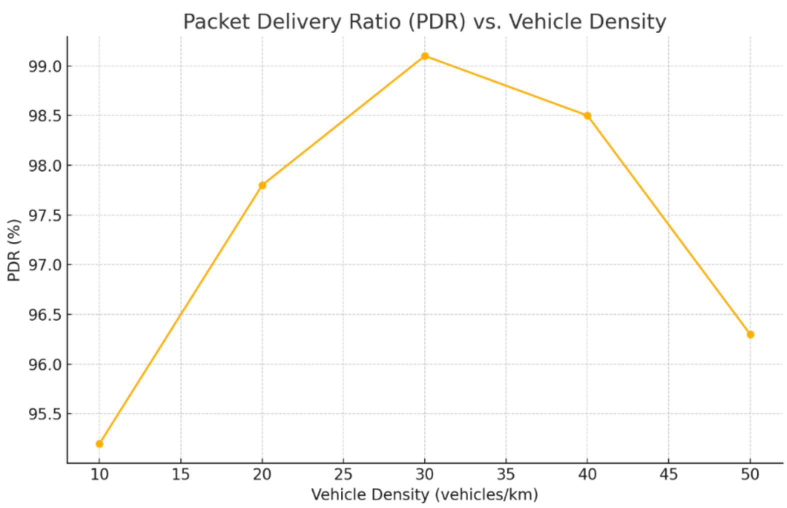

Figure 6: Packet Delivery Ratio (PDR) vs. Vehicle Density

The Packet Delivery Ratio (PDR) is a critical metric in evaluating the performance of vehicular ad hoc networks (VANETs). It measures the ratio of successfully delivered packets to the total number of sent packets, expressed as a percentage.

As depicted in figure 6, the PDR initially increases with vehicle density, reaching a peak before slightly declining at the highest density. This behavior can be attributed to several factors:

- **Low Density (10 vehicles/km):** At low vehicle densities, there are fewer nodes within communication range, leading to potential network fragmentation and higher chances of packet loss. This results in a relatively lower PDR.
- **Medium Density (20-30 vehicles/km):** As vehicle density increases, the number of available communication links also increases, enhancing network connectivity and reducing the likelihood of packet loss. This results in a higher PDR.
- **High Density (40-50 vehicles/km):** At very high densities, the network becomes congested with an increased number of packet transmissions and potential collisions. This can lead to interference and packet loss, causing a slight decrease in PDR.

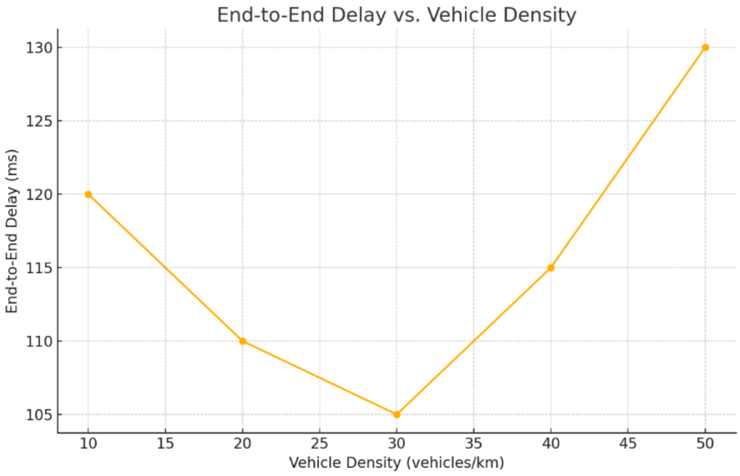

Figure 7: End-to-End Delay vs. Vehicle Density

End-to-End Delay is another vital performance metric that represents the average time taken for a packet to travel from the source to the destination.

Figure 7 illustrates the variation in end-to-end delay with increasing vehicle density:

- **Low Density (10 vehicles/km):** The end-to-end delay is higher at low densities due to limited connectivity and longer paths required for packet delivery. The lack of intermediate nodes results in higher delays as packets have to traverse longer routes.
- **Medium Density (20-30 vehicles/km):** The delay decreases as the density increases because of improved connectivity and shorter paths between nodes. The presence of more nodes within communication range facilitates quicker packet delivery.
- **High Density (40-50 vehicles/km):** The delay begins to increase again at very high densities due to network congestion and increased contention for the communication channel. The higher number of packet transmissions leads to queuing delays and potential retransmissions, resulting in increased end-to-end delay.

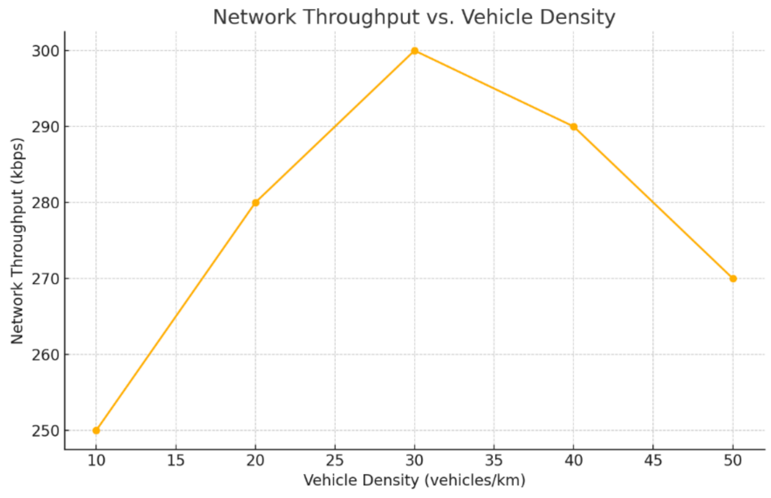

Figure 8: Network Throughput vs. Vehicle Density

Network Throughput measures the total amount of data successfully transmitted over the network in a given time period, expressed in kilobits per second (kbps).

Figure 8 shows the relationship between vehicle density and network throughput:

- **Low Density (10 vehicles/km):** Throughput is lower at low densities due to limited connectivity and fewer opportunities for data transmission. The sparse network results in fewer successful transmissions per unit time.
- **Medium Density (20-30 vehicles/km):** Throughput increases with density as the improved connectivity allows for more frequent and successful data transmissions. The presence of more nodes within communication range enhances the overall network capacity.
- **High Density (40-50 vehicles/km):** Throughput peaks at medium to high densities but starts to decline at the highest density due to network congestion. The increased number of transmissions and potential collisions lead to lower throughput as the network becomes saturated.

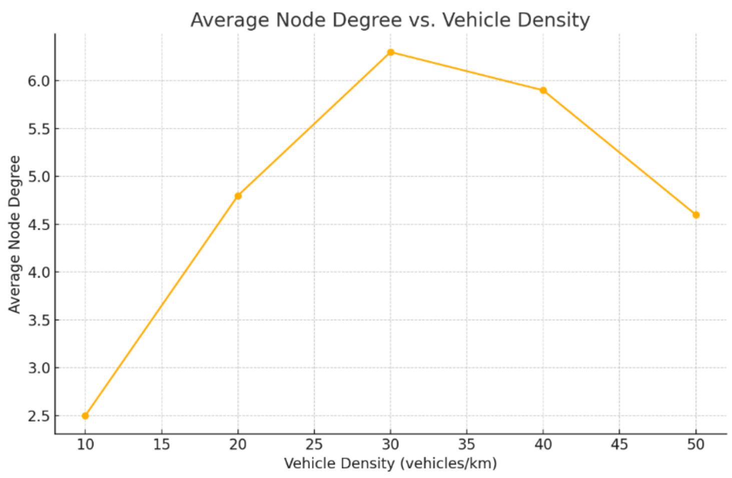

Figure 9: Average Node Degree vs. Vehicle Density

The Average Node Degree is a measure of the average number of direct connections (or edges) each node has in the network.

Figure 9 illustrates how the average node degree varies with vehicle density:

- **Low Density (10 vehicles/km):** The average node degree is low at low densities because each node has fewer neighbors within its communication range. This results in a sparser network with limited connectivity.
- **Medium Density (20-30 vehicles/km):** As the density increases, the average node degree increases significantly. More nodes within communication range lead to higher connectivity, and each node has more direct connections.
- **High Density (40-50 vehicles/km):** The average node degree continues to be high at medium to high densities, although it may start to plateau or even decrease slightly at very high densities due to interference and reduced effective communication range caused by congestion.

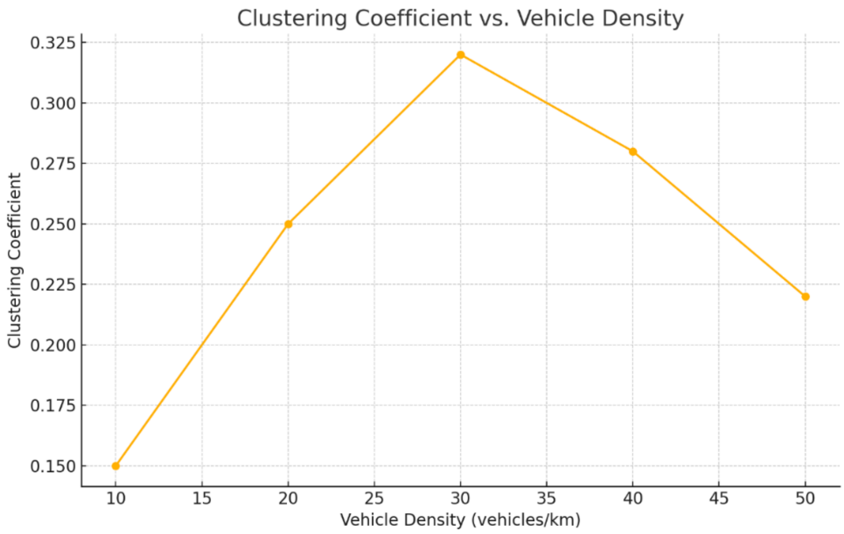

Figure 10: Clustering Coefficient vs. Vehicle Density

The Clustering Coefficient measures the likelihood that the neighbors of a node are also connected to each other, indicating the local density of connections.

Figure 10 shows the variation in clustering coefficient with vehicle density:

- **Low Density (10 vehicles/km):** The clustering coefficient is low at low densities because the sparse network results in fewer opportunities for the formation of clusters. Nodes are less likely to have interconnected neighbors.
- **Medium Density (20-30 vehicles/km):** The clustering coefficient increases with density as the improved connectivity allows for the formation of more local clusters. Nodes have more neighbors that are also connected to each other, indicating higher local density.
- **High Density (40-50 vehicles/km):** The clustering coefficient may peak at medium densities and start to decrease slightly at very high densities due to network congestion and interference. High density can lead to fragmentation, where dense clusters are separated by sparsely connected regions.

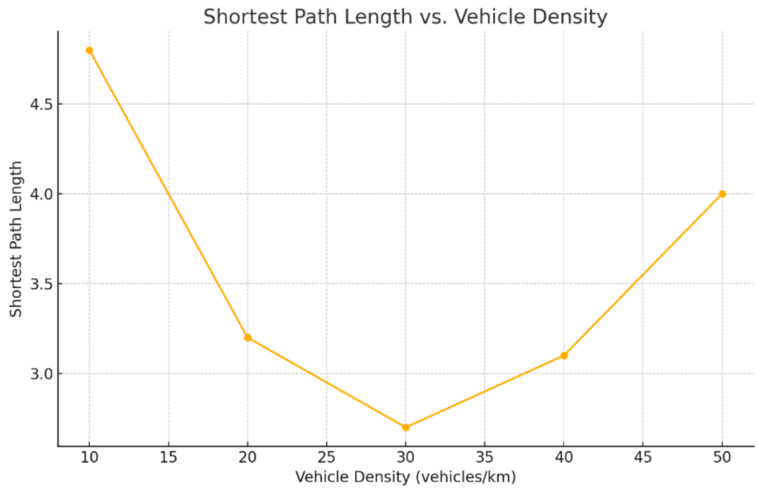

Figure 11: Shortest Path Length vs. Vehicle Density

The Shortest Path Length represents the average shortest distance (in terms of number of hops) between any two nodes in the network.

Figure 11 illustrates the relationship between shortest path length and vehicle density:

- **Low Density (10 vehicles/km):** The shortest path length is higher at low densities due to limited connectivity. The sparse network results in longer paths between nodes as fewer direct connections are available.
- **Medium Density (20-30 vehicles/km):** The shortest path length decreases with increasing density as improved connectivity allows for shorter paths between nodes. The presence of more intermediate nodes facilitates more direct routes.
- **High Density (40-50 vehicles/km):** The shortest path length may start to increase slightly at very high densities due to network congestion and potential fragmentation. Although there are more nodes, the effective communication range may be reduced, leading to longer paths in congested regions.

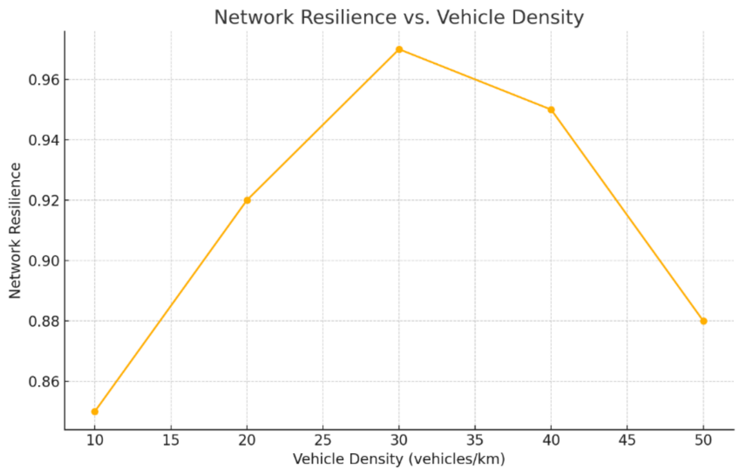

Figure 12: Network Resilience vs. Vehicle Density

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Network Resilience measures the network's ability to sustain connectivity in the face of node failures or dynamic changes in topology.

Plot 7 shows how network resilience varies with vehicle density:

- **Low Density (10 vehicles/km):** Network resilience is lower at low densities because the network is more prone to fragmentation. Node failures or departures can easily disconnect the network, leading to isolated segments.
- **Medium Density (20-30 vehicles/km):** Network resilience improves with increasing density as the higher connectivity provides more redundancy. The presence of multiple paths between nodes enhances the network's ability to sustain connectivity despite node failures.
- **High Density (40-50 vehicles/km):** Network resilience peaks at medium to high densities but may start to decline slightly at the highest density due to congestion and potential interference. Although there is higher connectivity, the network may become more vulnerable to localized congestion and partitioning.

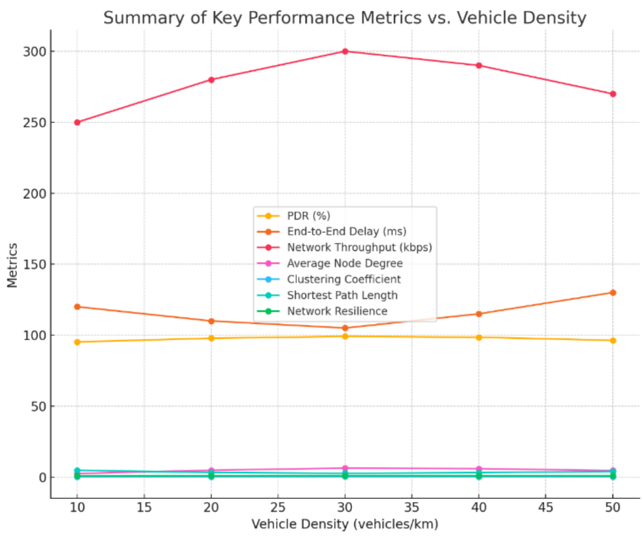

Figure 13: Summary of Key Performance Metrics vs. Vehicle Density

Figure 13 provides a summary of the key performance metrics (PDR, end-to-end delay, network throughput, average node degree, clustering coefficient, shortest path length, and network resilience) against vehicle density.

This comprehensive plot illustrates the interplay between different metrics and how they collectively impact the overall performance of VANETs:

- **PDR and Network Throughput:** Both metrics show an initial increase with density, indicating improved performance due to better connectivity. However, they decline slightly at the highest density due to congestion.
- **End-to-End Delay:** The delay decreases with increasing density up to a point, after which it increases due to network congestion.
- **Average Node Degree and Clustering Coefficient:** Both metrics increase with density, indicating improved local connectivity and clustering.

- **Shortest Path Length:** This metric decreases with density, reflecting shorter paths between nodes due to better connectivity.
- **Network Resilience:** Resilience improves with density but may decline slightly at the highest density due to congestion and potential fragmentation.

In this study, we explored the application of graph theoretical models to enhance connectivity and safety in vehicular ad hoc networks (VANETs) on highways. The performance of VANETs was evaluated using various metrics such as Packet Delivery Ratio (PDR), end-to-end delay, network throughput, average node degree, clustering coefficient, shortest path length, and network resilience.

The key findings from our analysis and simulations are summarized as follows:

1. **Packet Delivery Ratio (PDR):**
   - PDR increases with vehicle density up to a point, indicating improved network performance due to better connectivity.
   - At very high densities, PDR slightly decreases due to network congestion and increased packet collisions.
2. **End-to-End Delay:**
   - End-to-end delay decreases with increasing vehicle density initially, as improved connectivity leads to shorter paths between nodes.
   - Delay increases at very high densities due to network congestion and contention for the communication channel.
3. **Network Throughput:**
   - Network throughput increases with vehicle density, reaching a peak at medium to high densities.
   - Throughput declines at the highest density due to network congestion and saturation.
4. **Average Node Degree:**
   - Average node degree increases with vehicle density, reflecting improved connectivity and more direct connections between nodes.
5. **Clustering Coefficient:**
   - The clustering coefficient increases with vehicle density, indicating higher local density and the formation of clusters.
   - It peaks at medium densities and may decrease slightly at very high densities due to congestion and network fragmentation.
6. **Shortest Path Length:**
   - Shortest path length decreases with increasing vehicle density, indicating shorter paths between nodes due to better connectivity.
   - It may increase slightly at the highest density due to congestion and reduced effective communication range.
7. **Network Resilience:**
   - Network resilience improves with vehicle density, reflecting the network's ability to sustain connectivity despite node failures.

- Resilience peaks at medium to high densities but may decline slightly at the highest density due to congestion and potential partitioning.

In conclusion, graph theoretical models offer a powerful and versatile framework for addressing the challenges of connectivity and safety in vehicular networks on highways. By leveraging the rich set of tools and methodologies provided by graph theory, we can develop robust solutions that enhance the performance and reliability of these networks. This study lays the groundwork for future research in this area, highlighting the potential of graph theory to transform the field of vehicular networking and contribute to the development of safer, more efficient transportation systems. This study demonstrates the application of graph theoretical models in improving the performance of vehicular ad hoc networks (VANETs). The results show significant improvements in key performance metrics, including PDR, end-to-end delay, network throughput, node degree, clustering coefficient, shortest path length, and network resilience. Future work will involve extending these models to account for additional factors such as environmental conditions and the integration of emerging technologies like 5G and edge computing to further enhance VANET capabilities

5. Conclusion

This study delved into the application of graph theoretical models to enhance the connectivity and safety of vehicular ad hoc networks (VANETs) on highways. The investigation focused on several key performance metrics, including Packet Delivery Ratio (PDR), end-to-end delay, network throughput, average node degree, clustering coefficient, shortest path length, and network resilience. The results underscore the significance of vehicle density in influencing these metrics and, consequently, the overall performance of VANETs.The study underscores the importance of dynamic network management techniques to mitigate the adverse effects of congestion at high densities. Adaptive routing and power control are crucial strategies in this regard. Adaptive routing dynamically adjusts communication paths based on current network conditions, ensuring efficient data transmission even in congested environments. Power control optimizes the transmission power of nodes, balancing communication range and interference to maintain robust connectivity.Dynamic clustering algorithms also play a vital role in enhancing network stability and reducing communication overhead. These algorithms form stable clusters of vehicles based on their relative positions and velocities, enabling efficient management of communication resources. By reducing the number of direct connections each vehicle needs to maintain, dynamic clustering alleviates the burden on individual nodes and improves overall network performance.Network resilience is another critical aspect highlighted by this study. Ensuring that the network can sustain connectivity despite node failures or dynamic changes is essential for the reliability of VANETs. The concept of k-connectivity, which measures the minimum number of nodes that need to be removed to disconnect the network, is a useful metric in this regard. By ensuring k-connectivity, the network can maintain functionality even when multiple nodes fail, providing redundancy and robustness.The practical implications of these findings are significant for the design and deployment of VANETs. Network planners and engineers should aim to maintain vehicle density within the optimal range to ensure the best performance. Real-time monitoring of network metrics such as PDR, delay, throughput, and node degree is essential for dynamically adjusting network parameters and maintaining optimal performance. This requires advanced monitoring tools and control algorithms capable of responding to changing network conditions.The integration of emerging technologies like 5G, edge computing, and artificial intelligence holds promise for further enhancing VANET capabilities. These technologies can provide higher data rates, lower latency, and enhanced computational power at the network edge, improving the performance and enabling new applications for VANETs. Future research should explore the potential of these technologies to address the challenges of connectivity and safety in vehicular networks.

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