

# Study on M/M/1 Queueing Network with Preparatory Work and Feedback using Three Nodes when Catastrophe Occurs

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## Abstract

In this paper we study M/M/1 open queueing network with instantaneous Bernoulli feedback and preparatory work with three nodes when catastrophe occurs. We derive n number of customers in the system, queue length (all three nodes), system length and system time. The numerical examples are given to test the feasibility of the model.

**Keywords:** Queueing Network, Feedback, Preparatory work, Catastrophes

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## 1. Introduction

The appearance cycle, the assistance interaction, the size of servers, how much framework spaces, and the quantity of clients — who may be individuals as a rule, information bundles, vehicles, or whatever else — are totally inspected by lining hypothesis. This present reality utilizations of hypothesis of lines range various areas.

Agner KrarupErlang [3], a Danish mathematician and engineer, pioneered queueing theory with his first paper published in 1909, which was the basis for applied queueing theory. Telecommunications, transportation, logistics, finance, and other fields use queueing theory.

James R. Jackson pioneered the use of queueing networks in 1957. Jackson [5] discovered an earlier product-form solution for tandem queues. The queueing network's most crucial element is Jackson's network. There are three types of queueing networks: open, closed, and mixed. Buyers access an open network from the outside, use the systems to offer them with services, and then leave the network. A closed network prevents both new users from joining and current ones from leaving. In a mixed network, some classes of customers may have access to the network while others do not.

When the functioning of a system of queue is time-varying, the system is said to be in transient. A queueing network is thought of to be in steady state when its functional characteristics are independent of time.Parthasarathy [7] and Parthasarathy, Sharafali [8] have described an efficient solution for an M/M/1 queue using an easy strategy and a multi-server Poisson queue.

In actual service systems, where jobs may necessitate multiple services, the feedback queue is essential. Queueing systems with feedback are those that allows a customer to return in the event that they have not been satisfied or want more assistance. Feedback may be observed in a variety of real-world settings, including supermarkets, communication networks, and changes made to industrial systems. Takacs [12]

pioneered the use of queues with feedback mechanisms in 1963. Thangaraj and Vanitha [13] reviewed the continued fraction method for the M/M/1 queue with comments.

Unpredictable instances that take place at a service system are called catastrophes. Gelenbe [4] suggested the concept of disasters, which has drawn a lot of research passion currently due to its extensive deployment in industrial, computing, and service systems. Science and technology have benefited from the notion of calamity. In the majority of situations in life, it occurs at random, causing all units for extinguishing and the servicing facility to activate until a fresh arrival. A system failure results in the instantaneous destruction of all clients and the deactivation of the server. When someone fresh shows up, the server is prepared to serve.

Chandrasekaran, Saravananarajan [1] discussed the M/M/1 feedback queue's transient and reliability study in the event of disasters, server troubles, and servicing. [2]. KrishnaKumar and Arivudainambi [6] investigated the transient solution of an M/M/1 queue with disasters. Shanmugasundaram and Vanitha [11] proposed analysis of M/M/1 Retrial Queueing Network in Steady-state with catastrophes. Thangaraj and Vanitha [14] have evaluated Using continued fractions to analyse .To avoid a too-long queue, every server request and customer that needs setup time (preparatory work time) before getting service should be accommodated to prevent an excessively lengthy line. Examined was the preparatory work done for arriving clients who only had one server feedback queue by Santhakumaran and Shanmugasundaram [9]. When a disaster occurs, Shanmugasundaram and Chitra [10] discussed the Time dependent solution of a single server feedback queue customer having a service with and without preparatory work

## 2. Description of the Model

In this paper we see M/M/1 queueing network with instantaneous Bernoulli feedback and preparatory work for three node open queue. The arrival rate of the customer entering the queue is a Poisson process with rate  $\lambda$ . Before getting service the customers either directly goes with preparatory work or without preparatory work. Here the customers with preparatory work does not get feedback, but the customers without preparatory work are allowed for feedback. After getting service, the customers with preparatory work join the node one with probability  $\bar{p}$ , after getting service from node one, they move to node two with probability  $q$  or to the node three with probability  $1-\bar{q}$ , from node two the customers leaves the system after completing the service with the probability  $r$ , or moves to the node three with probability  $1-r$ , after getting service in node three the customers leaves the system. The customer without preparatory work decides whether to go for feedback or not. If the customer makes the decision for feedback, then he joins the feedback with probability  $1-\bar{p}$ , if not the customer joins the preparatory work and get the service. Here the service times are independent but not identically distributed. Service rates are exponentially distributed with rate  $\mu_1, \mu_2$ , where  $\mu_1$  is the service rate for the customer with preparatory work and  $\mu_2$  is the service rate for the customer without preparatory work. Service rate for node one, node two and node three are  $\mu_3, \mu_4$  and  $\mu_5$ . The capacity of the queue is infinite and the discipline is FIFO. Catastrophes occurs as a result from arrival and service processes, here service process follows poisson process with rate  $\gamma$ . The system is shown in Fig.1.

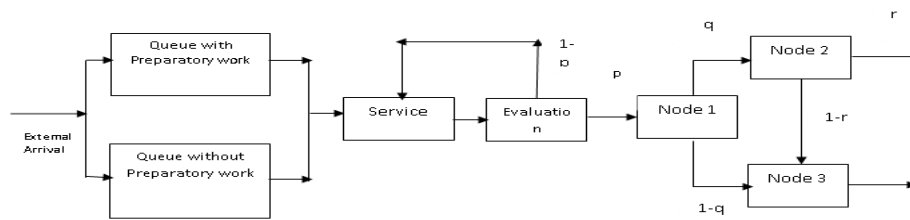


Fig . 1 Queueing Network with preparatory work and feedback

Let  $Q_n(t) = Q\{X(t) = n\}$ ,  $n = 0, 1, 2, \dots$  denote the transient state probability that there are  $n$  number of customers in the system at time  $t$ .

Let  $Q(x, t) = \sum_{n=0}^{\infty} Q_n(t)x^n$  be the probability generating function.

If time  $t = 0$ , generally it is assumed that there are no customers in the system. i.e.  $Q_0(0) = 1$

From the above assumption, the probability  $Q_n(t)$  following the system of differential – difference equations :

In steady state,  $\lim_{n \rightarrow \infty} Q_n(t) = Q_n$  and  $Q'_n(t) = 0$  as  $t \rightarrow \infty$

$$\begin{aligned}
 0 &= -\lambda Q_0 + \{[\bar{p}\mu_1 + \bar{q}\mu_3 + r\mu_4] + [\bar{p}\mu_1 + \bar{q}\mu_3 + (1-r)\mu_4 + \mu_5] + [\bar{p}\mu_1 + (1-\bar{q})\mu_3 + \mu_5]\} \\
 &\quad + [(1-\bar{p})\mu_2 + \bar{q}\mu_3 + r\mu_4] + [(1-\bar{p})\mu_2 + \bar{q}\mu_3 + (1-r)\mu_4 + \mu_5] \\
 &\quad + [(1-\bar{p})\mu_2 + (1-\bar{q})\mu_3 + \mu_5] \} Q_1 + \gamma(1 - Q_0) \\
 &= -\lambda Q_0 + [3\bar{p}\mu_1 + 3(1-\bar{p})\mu_2 + 2(1+\bar{q})\mu_3 + 2\mu_4 + 4\mu_5] Q_1 + \gamma(1 - Q_0) \\
 &= -\lambda Q_0 + \alpha Q_1 + \gamma(1 - Q_0) \tag{1}
 \end{aligned}$$

Where  $\alpha = 3\bar{p}\mu_1 + 3(1-\bar{p})\mu_2 + 2(1+\bar{q})\mu_3 + 2\mu_4 + 4\mu_5$

$$\begin{aligned}
 0 &= -\lambda Q_{n-1} + \{\lambda + [3\bar{p}\mu_1 + 3(1-\bar{p})\mu_2 + 2(1+\bar{q})\mu_3 + 2\mu_4 + 4\mu_5] + \gamma\} Q_n + \\
 &\quad [3\bar{p}\mu_1 + 3(1-\bar{p})\mu_2 + 2(1+\bar{q})\mu_3 + 2\mu_4 + 4\mu_5] Q_{n+1} \\
 0 &= -\lambda Q_{n-1} + [\lambda + \alpha + \gamma] Q_n + \alpha Q_{n+1} \tag{2}
 \end{aligned}$$

Equation (1) gives

$$\begin{aligned}
 0 &= -\lambda Q_0 + \alpha Q_1 + \gamma(1 - Q_0) \\
 (\lambda + \gamma) Q_0 &= \alpha Q_1 + \gamma \\
 Q_0 &= \frac{\gamma}{(\lambda + \gamma) - \alpha \frac{Q_1}{Q_0}} \tag{3}
 \end{aligned}$$

Equation (2) gives

$$\begin{aligned}
 0 &= -\lambda Q_{n-1} + [\lambda + \alpha + \gamma] Q_n + \alpha Q_{n+1} \\
 [\lambda + \alpha + \gamma] Q_n &= \lambda Q_{n-1} + \alpha Q_{n+1} \\
 \frac{Q_n}{Q_{n-1}} &= \frac{\lambda}{[\lambda + \alpha + \gamma] - \alpha \frac{Q_{n+1}}{Q_n}} \tag{4}
 \end{aligned}$$

Put  $n=1$  in equation (4)

$$\frac{Q_1}{Q_0} = \frac{\lambda}{[\lambda + \alpha + \gamma] - \alpha \frac{Q_2}{Q_1}}$$

Sub the above value in equation (3)

$$Q_0 = \frac{\gamma}{(\lambda + \gamma) - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \alpha \frac{Q_2}{Q_1}}}$$

$$Q_0 = \frac{\gamma}{(\lambda + \gamma) - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \dots}}}}$$

$$Q_0 = \frac{\gamma}{(\lambda + \gamma) - \beta} \tag{5}$$

Where  $\beta = \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \dots}}$

Equation (5) satisfies the quadratic equation

$$\beta^2 - (\lambda + \alpha + \gamma)\beta + \lambda\alpha = 0$$

The roots of the above equation are  $\frac{u \pm \sqrt{u^2 - 4\lambda\alpha}}{2}$ , where  $u = \lambda + \alpha + \gamma$

Let the roots be  $\beta_1, \beta_2$ , we take the unique real root lies within  $[0,1)$

Substituting  $\beta_2$  in equation (5), we get

$$Q_0 = \frac{\gamma}{(\lambda + \gamma) - \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2} \right]}$$

After some algebraic calculation, we get

$$Q_0 = \frac{\gamma \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\lambda\alpha} \right]}{1 - \alpha \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\lambda\alpha} \right]} \tag{6}$$

Expanding binomially, we get

$$Q_0 = \sum_{n=0}^{\infty} \alpha^n \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\lambda\alpha} \right]^{n+1} + \gamma \sum_{n=0}^{\infty} \alpha^n \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\lambda\alpha} \right]^{n+1}$$

The remained steady state probabilities can be calculated in equations of  $Q_0$ , from equation (4)

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{[\lambda + \alpha + \gamma] - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \frac{\alpha\lambda}{[\lambda + \alpha + \gamma] - \dots}}}}$$

By similar argument as before, the above equation reduces to

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{\frac{u + \sqrt{u^2 - 4\lambda\alpha}}{2}}$$

After some calculations, we get

$$Q_n = \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\alpha} \right]^n Q_0, \quad n = 1, 2, 3, \dots \tag{7}$$

Where  $Q_0 = \frac{\gamma \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\lambda\alpha} \right]}{1 - \alpha \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2\lambda\alpha} \right]}$  and  $\alpha = 3p\mu_1 + 3(1 - p)\mu_2 + 2(1 + q)\mu_3 + 2\mu_4 + 4\mu_5$

### 3. Asymptotic behavior of the average queue length

**Theorem:**

If  $\gamma > 0$ , the asymptotic behavior of the average queue length  $L_q$  at steady state is

$$L_q = \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2[\lambda + \gamma] - \left[ (\lambda + \gamma + \alpha) - \sqrt{(\lambda + \gamma + \alpha)^2 - 4\lambda\alpha} \right]}$$

Where  $\alpha = 3\bar{p}\mu_1 + 3(1 - \bar{p})\mu_2 + 2(1 + \bar{q})\mu_3 + 2\mu_4 + 4\mu_5$

**Proof :**

Consider equation (1) and (2) with the initial condition  $Q_0(0) = 1$

$$\frac{\partial Q(x,t)}{\partial t} = \left[ \lambda x + \frac{\alpha}{x} - (\lambda + \gamma + \alpha) \right] Q(x,t) + \alpha \left( 1 - \frac{1}{x} \right) Q_0(x) + \gamma \tag{8}$$

The mean size is  $h(t) = \sum_{n=1}^{\infty} nQ_n(t) = \frac{\partial Q(x,t)}{\partial t}$  at  $x=1$

Differentiating the equation (8) with respect to  $x$  at  $x=1$ , we get

$$\frac{dh(t)}{dx} + \gamma h(t) = \lambda - \alpha(1 - Q_0)$$

Solving the above differential equation for  $h(t)$  with  $h(0) = \sum_{n=1}^{\infty} nQ_n(t) = 0$

$$h(t) = \frac{\lambda}{\gamma} (1 - e^{-\gamma t}) - \frac{\alpha}{\gamma} (1 - e^{-\gamma t}) + \alpha \int_0^t Q_0(u) e^{-\gamma(t-u)} du \tag{9}$$

Taking Laplace Transform for Equation (1) and (2), we get

$$Q_0^*(x) = \frac{1 + \frac{\gamma}{x}}{(x + \lambda + \gamma) - \left[ \frac{u - \sqrt{u^2 - 4\lambda\alpha}}{2} \right]} \tag{10}$$

Taking Laplace Transforms for equation (9), Let  $h^*(x)$  be the Laplace transform of  $h(t)$

$$h^*(x) = \frac{\lambda - \alpha}{x(x + \gamma)} + \frac{\alpha}{(x + \gamma)} Q_0^*(x) \tag{11}$$

$$\lim_{t \rightarrow \infty} h(t) = \lim_{x \rightarrow 0} x h^*(x)$$

Using equation (10) and the above concept for the equation (11), we get

$$L_q = \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]}$$

Where  $\alpha = 3p\mu_1 + 3(1 - p)\mu_2 + 2(1 + q)\mu_3 + 2\mu_4 + 4\mu_5$

#### 4. Balance Equations

By using Little's formula, we get the following equations

**Average queue length ( $L_q$ )**

$$L_q = \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]}$$

Where  $\alpha = 3\bar{p}\mu_1 + 3(1 - \bar{p})\mu_2 + 2(1 + \bar{q})\mu_3 + 2\mu_4 + 4\mu_5$

**Waiting time of a customer ( $W_q$ )**

$$W_q = \frac{L_q}{\lambda} = \left( \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]} \right) \frac{1}{\lambda}$$

**Average number of customer ( $L_s$ )**

$$L_s = L_q + \frac{\lambda}{\mu} = \left( \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]} \right) + \frac{\lambda}{\mu}$$

**Waiting time of a customer in the system ( $W_s$ )**

$$W_s = \frac{L_s}{\lambda} = \left[ \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]} + \frac{\lambda}{\mu} \right] \frac{1}{\lambda}$$

**Mean Queue Length for node one, node two, node three are as follows**

$$L_{q1} = \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]}$$

Where  $\alpha = \bar{p}\mu_1 + (1 - \bar{p})\mu_2 + \bar{q}\mu_3$

$$L_{q2} = \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]}$$

Where  $\alpha = \bar{p}\mu_1 + (1 - \bar{p})\mu_2 + \bar{q}\mu_3 + r\mu_4$

$$L_{q3} = \frac{\lambda - \alpha}{\gamma} + \frac{2\alpha}{2(\lambda + \gamma) - \left[ (\lambda + \alpha + \gamma) - \sqrt{(\lambda + \alpha + \gamma)^2 - 4\lambda\alpha} \right]}$$

Where  $\alpha = \bar{p}\mu_1 + (1 - \bar{p})\mu_2 + (1 - \bar{q})\mu_3 + (1 - r)\mu_4 + \mu_5$

Using little's formula, we can calculate the remaining parameters

$$W_{q_1}, L_{s_1}, W_{s_1}, W_{q_2}, L_{s_2}, W_{s_2}, W_{q_3}, L_{s_3}, W_{s_3}$$

### 5. Particular Case

When  $\mu_1 = 3\mu_1$ ,  $\mu_2 = 3\mu_2$  and  $\mu_3 = \mu_4 = \mu_5 = 0$  and  $\bar{p} = \bar{p}$ ,  $\bar{q} = 1 - \bar{p}$ , there is a customer with and without preparatory work, then the system's average queue length's asymptotic nature  $L_q$  for the feedback when  $\gamma > 0$  is

$$L_q = \frac{\lambda - (\bar{p}\mu_1 + \bar{q}\mu_2)}{\gamma} + \frac{2(\bar{p}\mu_1 + \bar{q}\mu_2)}{2(\lambda + \gamma) - [(\lambda + (\bar{p}\mu_1 + \bar{q}\mu_2) + \gamma) - \sqrt{(\lambda + (\bar{p}\mu_1 + \bar{q}\mu_2) + \gamma)^2 - 4\lambda(\bar{p}\mu_1 + \bar{q}\mu_2)}]}$$

Using little's formula, we can calculate the remaining parameters

$$W_{q_1}, L_{s_1}, W_{s_1}, W_{q_2}, L_{s_2}, W_{s_2}, W_{q_3}, L_{s_3}, W_{s_3}.$$

### 6. Numerical Examples

#### Number of customers in all the three queues for the system

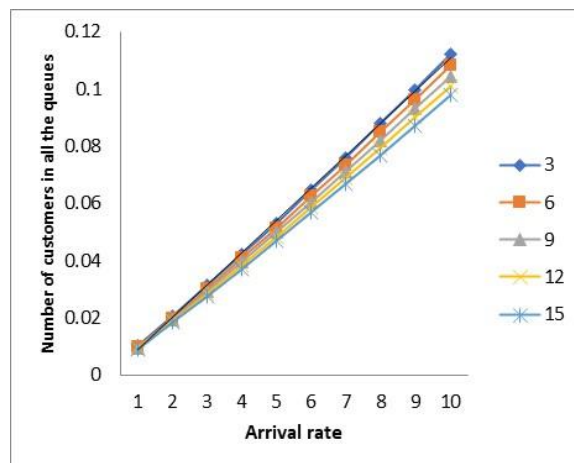
The consumers in all the three queues for the system are calculated in table 1 for  $\bar{p} = 0.3$ ,  $\bar{q} = 0.5$ ,  $r = 0.4$ ,  $\mu_1 = 6$ ,  $\mu_2 = 4$ ,  $\mu_3 = 8$ ,  $\mu_4 = 9$ ,  $\mu_5 = 10$ ,  $\lambda = 1, 2, 3 \dots 10$ ,  $\gamma = 3, 6, 9, 12, 15$  (catastrophe effect).

In fig 2, As the value of  $\lambda$  increases, the number of customers in each of the three queues increases, and as the value of  $\gamma$  increases, the number of customers in each of the queues decreases.

Table : 1

	3	6	9	12	15
1	0.0102	0.0099	0.0096	0.0094	0.0091
2	0.0206	0.02	0.0194	0.0189	0.0183
3	0.0313	0.0303	0.0294	0.0285	0.0277
4	0.0421	0.0408	0.0395	0.0384	0.0373
5	0.0532	0.0515	0.0499	0.0484	0.0469
6	0.0645	0.0624	0.0604	0.0585	0.0568
7	0.0761	0.0735	0.0711	0.0689	0.0668
8	0.0879	0.0848	0.082	0.0794	0.077
9	0.0999	0.0964	0.0931	0.0901	0.0873
10	0.1122	0.1082	0.1044	0.101	0.0978

Fig : 2 Number of customers in all the queues



Waiting time of a customer in all the three queues

The waiting time of the customers in all the three queues are calculated in table 2 for  $\bar{p} = 0.3$ ,  $\bar{q} = 0.5$ ,

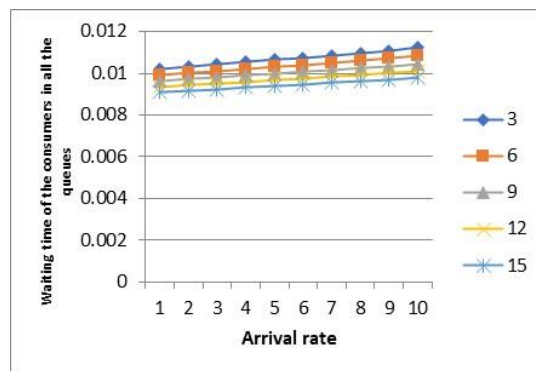
$r = 0.4$ ,  $\mu_1 = 6$ ,  $\mu_2 = 4$ ,  $\mu_3 = 8$ ,  $\mu_4 = 9$ ,  $\mu_5 = 10$ ,  $\lambda = 1, 2, 3 \dots 10$ ,  $\gamma = 3, 6, 9, 12, 15$  (catastrophe effect).

In fig 2, As the value of  $\lambda$  increases, Customers' waiting times in each of the three lines are getting higher, and as the value of  $\gamma$  increases, the waiting time of customers in each of the queues decreases.

Table : 2

	3	6	9	12	15
1	0.0102	0.0099	0.0096	0.0094	0.0091
2	0.0103	0.01	0.0097	0.0094	0.0092
3	0.0104	0.0101	0.0098	0.0095	0.0092
4	0.0105	0.0102	0.0099	0.0096	0.0093
5	0.0106	0.0103	0.01	0.0097	0.0094
6	0.0108	0.0104	0.0101	0.0098	0.0095
7	0.0109	0.0105	0.0102	0.0098	0.0095
8	0.011	0.0106	0.0103	0.0099	0.0096
9	0.0111	0.0107	0.0103	0.01	0.0097
10	0.0112	0.0108	0.0104	0.0101	0.0098

Fig: 3 Waiting time of consumers in all the three queues



Number of consumers in the system

The number of the vendors in the system are calculated in table 3 for  $\bar{p} = 0.3, \bar{q} = 0.5, r = 0.4, \mu_1 = 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \mu_2 = 4, \mu_3 = 8, \mu_4 = 9, \mu_5 = 10, \lambda = 5, \gamma = 3, 6, 9, 12, 15$

(catastrophe effect).

In fig 4, as the service cost rises and for distinct values of  $\gamma$ , the number of consumers in the system decreases.

Table: 3

	3	6	9	12	15
5	1.0537	1.052	1.0503	1.0488	1.0473
6	0.8865	0.8848	0.8832	0.8817	0.8803
7	0.767	0.7653	0.7637	0.7622	0.7608
8	0.6772	0.6756	0.674	0.6725	0.6712
9	0.6073	0.6057	0.6041	0.6027	0.6013
10	0.5513	0.5497	0.5482	0.5467	0.5454
11	0.5053	0.5038	0.5023	0.5009	0.4996
12	0.467	0.4655	0.464	0.4626	0.4614
13	0.4345	0.433	0.4316	0.4302	0.429
14	0.4066	0.4051	0.4037	0.4024	0.4011

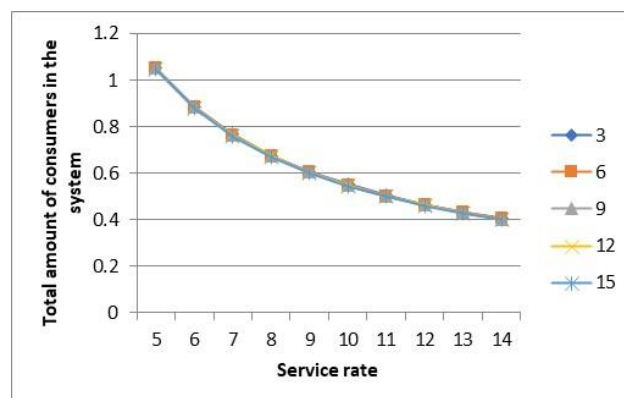


Fig: 4 Total amount of consumers in the system

**Waiting time of the consumers in the system**

The total amount of consumers in the system are calculated in table 4 for  $\bar{p} = 0.3, \bar{q} = 0.5, r = 0.4, \mu_1 = 5, 6, 7, 8, 9, 10, 11, 12, 13, 14$  ,  $\mu_2 = 4$  ,  $\mu_3 = 8, \mu_4 = 9, \mu_5 = 10$  ,  $\lambda = 5$  ,  $\gamma = 3, 6, 9, 12, 15$

(Catastrophe effect).

In fig 5, When the service cost rises and for different values of  $\gamma$ , the total amount of vendors in the system decreases.

Table : 4

	3	6	9	12	15
5	0.2107	0.2104	0.2101	0.2098	0.2095
6	0.1773	0.177	0.1766	0.1763	0.1761
7	0.1534	0.1531	0.1527	0.1524	0.1522
8	0.1354	0.1351	0.1348	0.1345	0.1342
9	0.1215	0.1211	0.1208	0.1205	0.1203
10	0.1103	0.1099	0.1096	0.1093	0.1091
11	0.1011	0.1008	0.1005	0.1002	0.0999
12	0.0934	0.0931	0.0928	0.0925	0.0923
13	0.0869	0.0866	0.0863	0.086	0.0858
14	0.0813	0.081	0.0807	0.0805	0.0802

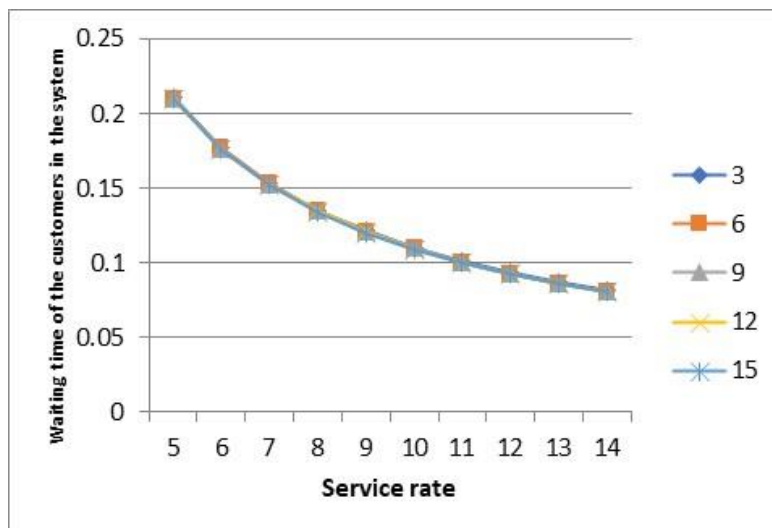


Fig : 5 Waiting time of the customer in the system

**Node one -Number of customers**

The customers in node one are calculated in table 5 for  $\bar{p} = 0.3, \bar{q} = 0.5, r = 0.4, \mu_1 = 6, \mu_2 = 4, \mu_3 = 8, \mu_4 = 9, \mu_5 = 10, \lambda = 1, 2, 3 \dots 10, \gamma = 3, 6, 9, 12, 15$  (catastrophe effect).

In fig 6, As the value of  $\lambda$  increases, the number of customers in node one increases, and as the value of  $\gamma$  increases, the number of customers in node one decreases.

Table : 5

	3	6	9	12	15
1	0.0919	0.0713	0.0584	0.0495	0.043
2	0.1963	0.1483	0.1199	0.101	0.0873
3	0.3144	0.231	0.1845	0.1542	0.1328
4	0.4473	0.3196	0.2521	0.2093	0.1794
5	0.5961	0.4138	0.3225	0.2661	0.2272
6	0.7611	0.5136	0.3957	0.3244	0.276
7	0.9425	0.6188	0.4716	0.3844	0.3258
8	1.1397	0.729	0.5499	0.4457	0.3765
9	1.3521	0.844	0.6305	0.5084	0.4281
10	1.5784	0.9634	0.7133	0.5724	0.4806

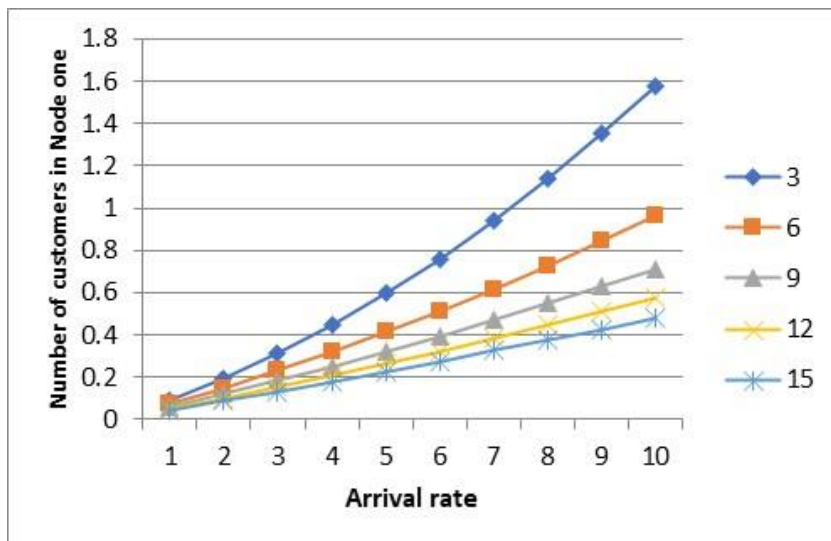


Fig : 6 Number of customers in Node one

**The total amount of consumers in the Node two**

The customers in node two are calculated in table 6 for  $\bar{p} = 0.3, \bar{q} = 0.5, r = 0.4, \mu_1 = 6, \mu_2 = 4, \mu_3 = 8, \mu_4 = 9, \mu_5 = 10, \lambda = 1, 2, 3 \dots 10, \gamma = 3, 6, 9, 12, 15$  (catastrophe effect).

In fig 7, As the value of  $\lambda$  increases, the number of vendors in node two increases, and as the value of  $\gamma$  increases, the number of customers in node two decreases.

Table : 6

	3	6	9	12	15
1	0.0694	0.057	0.0485	0.0422	0.0374
2	0.1466	0.1183	0.0995	0.0861	0.0759
3	0.2326	0.184	0.1532	0.1317	0.1157
4	0.3283	0.2544	0.2096	0.179	0.1566
5	0.4346	0.3295	0.2686	0.2279	0.1986
6	0.5526	0.4094	0.3302	0.2785	0.2417
7	0.683	0.4942	0.3944	0.3307	0.2859
8	0.8265	0.5838	0.4611	0.3844	0.331
9	0.9835	0.6782	0.5303	0.4396	0.3772
10	1.1543	0.7774	0.6018	0.4962	0.4244

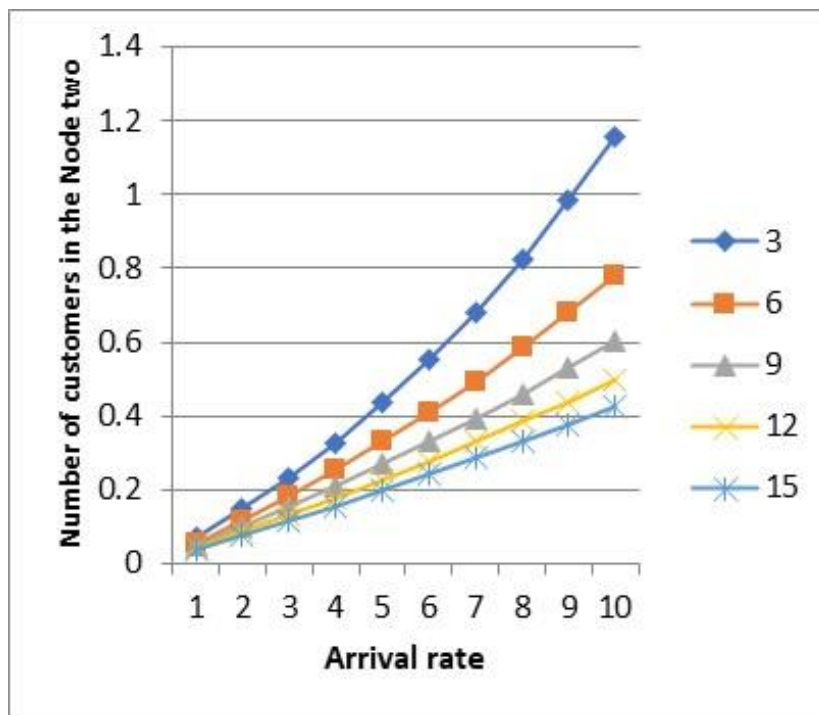


Fig : 7 The total amount of consumers in the Node two

### Number of consumers in Node Three

The customers in node three are calculated in table 7 for  $\bar{p} = 0.3$ ,  $\bar{q} = 0.5$ ,  $r = 0.4$ ,  $\mu_1 = 6$ ,  $\mu_2 = 4$ ,  $\mu_3 = 8$ ,  $\mu_4 = 9$ ,  $\mu_5 = 10$ ,  $\lambda = 1, 2, 3 \dots 10$ ,  $\gamma = 3, 6, 9, 12, 15$  (catastrophe effect).

In fig 8, As the value of  $\lambda$  increases, the Quantity of consumers in node three increases, and as the value of  $\gamma$  increases, the Quantity of customers in node three decreases.

Table : 7

	3	6	9	12	15
1	0.0383	0.0342	0.031	0.0283	0.026
2	0.0792	0.0704	0.0634	0.0577	0.0529
3	0.1231	0.1085	0.0972	0.0881	0.0806
4	0.1701	0.1487	0.1325	0.1196	0.1092
5	0.2206	0.1912	0.1694	0.1523	0.1386
6	0.2749	0.2361	0.2078	0.1861	0.1689
7	0.3333	0.2834	0.2479	0.2211	0.2
8	0.3963	0.3333	0.2898	0.2573	0.232
9	0.4641	0.386	0.3333	0.2947	0.2649
10	0.5373	0.4415	0.3787	0.3333	0.2987

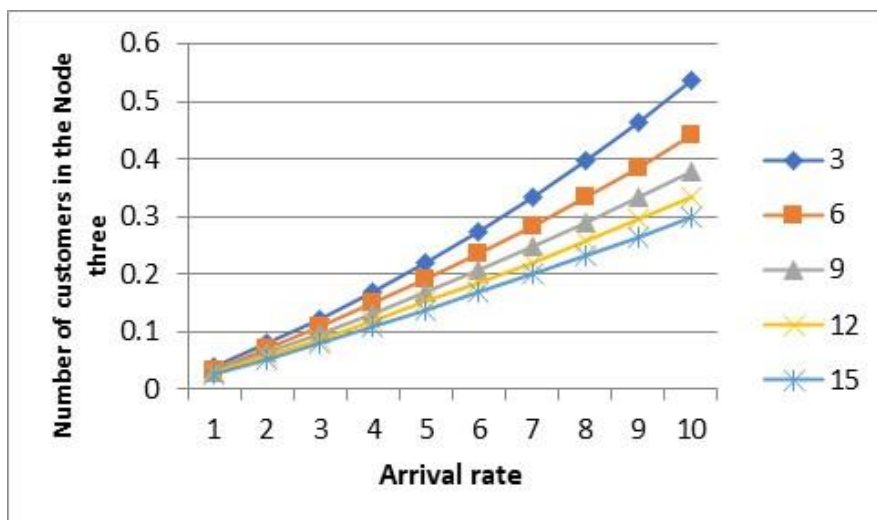


Fig : 8 Count of vendors in Node three

## 7. Conclusion

In this study, we derive probability of n count of vendors in the system with queue length, queue time and system time of markovian queue with single server network along with preparatory work and feedback for three nodes when catastrophes occur. The numerical examples shows that when the number of customers increases with arrival (fig.2) waiting time increase with arrival (fig.3) , The count of consumers in the nodes one,two,three increases with arrival rate ( fig. 6 ,7, 8). It shows the coincides of the result.

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