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Some Bench Mark Results on Total Domination Subdivision Stable Graph

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Abstract:

For a graph G, the total dominating set defined as a set of vertices in S such that all the vertices in V(G) has at least one neighbor in S, the least cardinality is noted as □t(G). The total domination number of each and every graph while subdividing any edge xy of G is equal to the total domination number of G, which results in the total domination subdivision stable graph abbreviated as TDSS and the symbolic expression is Gtsd(xy). The research paper, we introduce TDSS and proposed conditions under which a graph is TDSS and not TDSS.

Keywords: Total domination, Total domination subdivision, Total domination subdivision stable (TDSS).

1. Introduction

All graphs considered here simple, connected and undirected graph with V and E which follows vertex set and edge set. For basic terminology and notations for graphs and domination parameters which is not defined here refer [1] and [2] respectively. The boundary of D defined [2] as $B(D) = N(D) - D$. Let $x \in G$, the vertex x is called good [3] such that if all possible γ_t – sets contained the vertex x otherwise it is called bad vertex. If a vertex x is needed only to dominate itself in the minimum dominating set then x is called selfish. Any vertex we call t-dominated in $V - D$ at least t vertices needed to dominate that vertex. If, after removing a vertex x from G, we attain the graph G –

x, which is less than the total domination number of *G*, that is, $\gamma_t(G - x) < \gamma_t(G)$, then that vertex *x* is considered down. The total domination subdivision number was first investigated in [4].

The total domination subdivision number increased by subdividing least number of edges of G which is discussed in [5]. It has been studied by several authors in [6,7,8,9 & 10]. The authors [11] introduced the graph named as domination subdivision stable graph is the domination number of each and every graph while subdividing any edge of *G* is equal to the domination number of *G*, which is abbreviated as DSS. Let $e = xy$ be an edge with end points $\{x, y\}$ of *G*. While subdividing it, we access a new one say *z* and having new edges say $\{x, z\}$ and $\{z, y\}$ of the resulting graph, it is expressed as $G_{sd}\, xy$.

Based on this concept, we extend this to total domination and introduce a new graph named as total domination subdivision stable graph.

2. Main Results

The focus of this section, we defined Total domination subdivision stable graph and discussed basic properties for obtaining *TDSS* from a graph *G*.

Definition 2.1

For a given graph *G*, the total domination number of all graphs derived by subdividing any edge *xy* of *G* is same for the total domination number of *G* which named as total domination subdivision stable. Using this graph operation (subdivision) of any edge *xy*, we obtain a new vertex *z*, the expression of this denoted $G_{tsd}\, xy = z$.

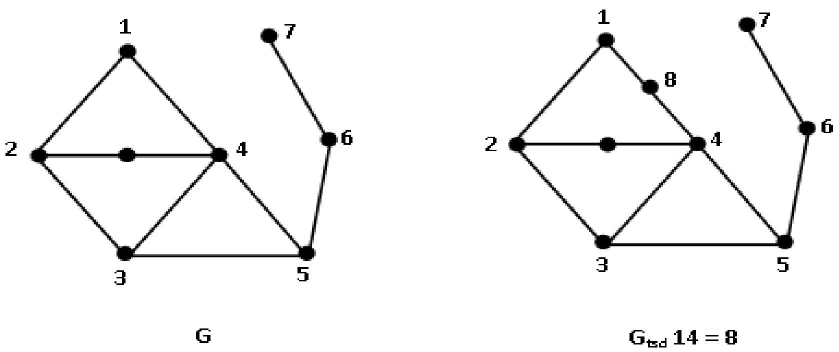

Fig. 1 Graph G and G_{tsd} 14

In Fig 1, $\gamma_t(G) = \gamma_t(G_{tsd}\, 14) = 4$. According to the Fig 1, the total domination number is the same for every pair. Therefore $\gamma_t (G_{tsd}\, xy) = \gamma_t (G)$.

Theorem 2.2

For every graph *G*, $\gamma_t (G_{tsd}\, xy) \geq \gamma_t (G)\ \forall\ xy \in E (G)$.

Proof

Let us assume the graph to be *G* and γ_t – set of *G* to be *D*. Consider $G_{tsd}\, xy$ where $e = xy \in E (G)$ and assume D_1 to be a γ_t – set for $G_{tsd}\, xy$.

If possible let $| D_1 | < | D |$.

Case 1 $z \in D_1$

In this case the possible conditions are either,

1. $x, y \in D_1$ or
2. x *or* $y \in D_1$.

Let $x, y \in D_1$ ie., $x, y, z \in D_1$, then we obtain D_2 where $D_2 = D_1 - \{z\}$ is a γ_t – set for G so $| D_2 | < | D |$, we get a contradiction.

If either x or $y \in D_1$ say $x \in D_1$, then we obtain D_3 where $D_3 = D_1 - \{z\} \cup \{y\}$ as a γ_t – set for G such that $| D_3| < | D |$, and we get a contradiction.

Case 2 $z \notin D_1$

In this case the possible conditions are either,

1. $x, y \in D_1$ or
2. x *or* $y \in D_1$.

In both cases, we get a contradiction for a γ_t – set D_1. Since for G, D_1 itself is a γ_t – set we get $| D_1 | < | D |$. Thus $\gamma_t (G_{tsd}\, xy) \geq \gamma_t (G)\ \forall e = xy$ in $E (G)$. □

Theorem 2.3

For a given graph G such that $\gamma_t (G) = \gamma_{2t} (G)$, then G is TDSS graph

Proof

Assume that a total 2 – dominated graph is G and the γ_t – set for G is D.

Let $e = xy \in E (G)$.

Case 1 x, y *in* D

Let $G_{tsd}\, xy = z$. Then we get a γ_t – set for $G_{tsd}\, xy$ is $D - \{x\} \cup \{z\}$

ie., $\gamma_t (G_{tsd}\, xy) = \gamma_t (G)$.

Case 2 x not in D, y in D

Claim

If y is a 2 – dominated vertex such that y is adjacent to x, z where $x, z \in \gamma_t (G)$, then we get $\gamma_t (G_{tsd}\, xy) = \gamma_t (G)$ and also $\gamma_t (G_{tsd}\, zy) = \gamma_t (G)$.

Proof

Let G be any graph and the 2- dominated vertex is x. Let $G_{tsd}\, xy = s$. In $G_{tsd}\, xy$, vertex x dominates s and z dominates y. Hence $\gamma_t (G_{tsd}\, xy) = \gamma_t (G)$. Similarly $\gamma_t (G_{tsd}\, zy) = \gamma_t (G)$.

By the above claim, $\gamma_t (G_{tsd}\, xy) = \gamma_t (G)$.

Case 3 x in D, y not in D

Similarly if x in D and y not in D, we get $\gamma_t (G_{tsd}\, xy) = \gamma_t (G)$. This is true $\forall$ $e = xy \in E(G)$. Therefore, G is TDSS. □

Converse need not be true.

Example

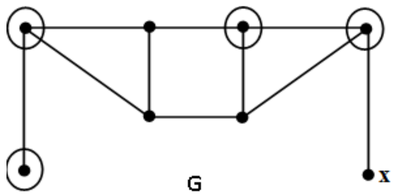

Fig. 2 *TDSS* graph

In Fig.2, G is *TDSS* but x is not a 2 – dominated vertex.

Corollary

If G is a graph with $x, y \in V(G)$, $\exists$ a γ_t – set D where x is adjacent to y, $x \in D$ and y is 2 – dominated then G is *TDSS*.

Proof

Let $x, y \in V(G)$ and γ_t – set for G be D such that $x \in D$ and y is 2 – dominated. Let $G_{tsd}\, xy = z$. By Theorem 2.3, we have D is a γ_t – set for $G_{tsd}\, xy$. It is true for remaining

$x, y \in V(G)$. Therefore, G is *TDSS*. □

Example

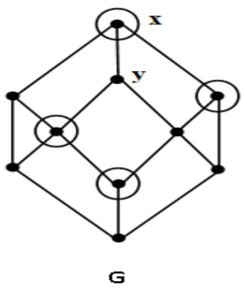

Fig. 3 Graph G

In Fig. 3, the vertices x and y are such that $x \in \gamma_t (G)$ and y is 2 – dominated for the γ_t – set of G. Therefore G is *TDSS*.

Remark

If every vertex is 2 – dominated in $V – D$, then it follows the above corollary.

Theorem 2.4

For every vertex is 2 – dominated in $V – D$ $\forall\, \gamma_t$ – set of G then,

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(1) G does not have any pendant vertex.

(2) If for $x, z \in \gamma_t(G)$, x is a down vertex and x adjacent to z. Then $\exists$ one s which is not adjacent to x but s is adjacent to z.

Proof

Let G be a given graph in which all vertices in $V - D$ is 2 – dominated for all the γ_t – set of G.

(1) Assume that y is a pendant vertex of G. Since all the vertices in $V - D$ is 2 – dominated, this implies that every pendant vertex must be included in every γ_t – set of G. Consider x be a support vertex of y where x is neighbor of y. Therefore, $\exists$ atleast one z such that $z \neq y$, $z \notin D$, $z \in N(x)$. Then, we have a γ_t – set $D_1 = D - \{y\} \cup \{z\}$ such that y is single dominated, we get a contradiction. Hence, D does not have any pendant vertex, ie., $\forall x \in V(G)$, $N(x) \geq 2$.

(2) For each and every $x \in \gamma_t(G)$, $Pn[x, D] = \phi$. Let us consider x, z such that x is adjacent to z that belongs to $\gamma_t(G)$. Consider the graph $G - x$, we have the γ_t – set for $G - x$ is $\gamma_t(G) - \{z\} - \{x\} \cup \{s\}$ where $s \in N(z)$. Therefore, we get $\gamma_t(G - x) = \gamma_t(G) - 1$. Thus x is a down vertex if $x \in \gamma_t(G)$. □

Remark

1. By theorem 2.3 and 2.4 we see that if G is a TDSS graph such that every γ_t – set of G is 2 – dominating, then every γ_t – set of G includes all pendant and support vertices.
2. If x is a t – dominated vertex say y adjacent to $y_1, y_2, \ldots, y_t$ where $y_1, y_2, \ldots, y_t \in \gamma_t(G)$, $\gamma_t(G_{tsd}\, xy_1) = \gamma_t(G_{tsd}\, xy_2) = \ldots = \gamma_t(G_{tsd}\, xy_k) = \gamma_t(G)$ ie., a t – dominated graph is TDSS.

Theorem 2.5

If either x or y is selfish and $\gamma_t(G) = 2$ such that $x, y \in \gamma_t(G)$ then G is *TDSS*.

Proof

Let $x, y \in \gamma_t(G)$, the vertex y is selfish. Let $G_{tsd}\, xs = t$ for some $s \in V(G)$. Then we have a γ_t – set for $G_{tsd}\, xs$ as $\gamma_t(G) - \{y\} \cup \{t\}$.Therefore, G is *TDSS*. □

Converse need not be true.

Example

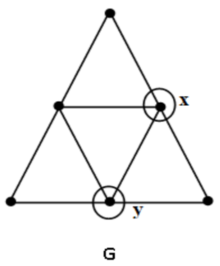

Fig. 4 *TDSS* Graph with neither x nor y is selfish

In Fig.4, neither x nor y is selfish but G is a *TDSS* graph.

Theorem 2.6

Let G be a graph with $x \in V(G)$. Let γ_t – set for G is D such that $x, y \in D$, x adjacent to y where y is selfish, $B(\{x, y\}) \cap D = \phi$. If this is possible for each and every vertices of G, then G is a *TDSS*.

Proof

Let G be a graph with $x \in V(G)$ and γ_t – set for G is D. By given condition $\exists$ one y such that $x, y \in D$, where y is selfish, x adjacent to y and $B(\{x, y\}) \cap D = \phi$.

Let $G_{tsd}\, xs = z$ where $s \in N(x)$. Then we have γ_t – set for $G_{tsd}\, xs$ is $D_1 = D - \{y\} \cup \{z\}$. That is true for each and every $x \in V(G)$ of G. Therefore G is *TDSS*. □

Corollary

For a graph G such that $\forall x, y \in V(G)$, there exists a γ_t – set D such that $B(\{x, y\}) \cap D = \phi$ and either x or y is selfish. Then, G is TDSS.

Proof

Let G be a graph with $x, y \in V(G)$ such that x adjacent to y. Given that $\exists$ one γ_t – set D where $x, y \in D$, such that $B(\{x, y\}) \cap D = \phi$ and either x or y is selfish.

Let us assume that x is selfish and $G_{tsd}\, xy = z$. Then we have γ_t – set for $G_{tsd}\, xy$ is $D - \{x\} \cup \{z\}$. By theorem 2.5, this is true for all $x, y \in V(G)$. Therefore G is TDSS. □

Converse need not be true.

Example

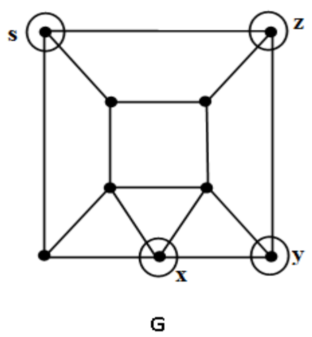

Fig. 5 *TDSS* graph with $B(\{x, y\}) \cap D \neq \phi$

In Fig.5, we have $B(\{x, y\}) \cap D \neq \phi$. Then G is TDSS with $x, y \in D$, x adjacent to y such that y is selfish.

Theorem 2.7

If G is *TDSS*, then every pendant vertex of G is included in some γ_t – set.

Proof

Let G be TDSS with $x, y \in V(G)$, here the pendant vertex is y and support vertex is x. Let $G_{tsd}\, xy = z$. In $G_{tsd}\, xy$, either $y \in \gamma_t\,(G_{tsd}\, xy)$ or $y \notin \gamma_t\,(G_{tsd}\, xy)$.

If $y \in \gamma_t\,(G_{tsd}\, xy)$, then $z \in \gamma_t\,(G_{tsd}\, xy)$, $x \notin \gamma_t\,(G_{tsd}\, xy)$. Here, the γ_t – set for G containing y is $\gamma_t\,(G_{tsd}\, xy) - \{z\} \cup \{x\}$.

If $y \notin \gamma_t\,(G_{tsd}\, xy)$, then $x, z \in \gamma_t\,(G_{tsd}\, xy)$. Here also the γ_t – set for G containing y is $\gamma_t\,(G_{tsd}\, xy) - \{z\} \cup \{y\}$. In all cases, there is a γ_t – set containing y. □

Corollary

If G has at least one pendant vertex that is *TDSS*. Then G has at least one selfish vertex.

Proof

By the above Theorem 2.7 mentioned if $\exists$ a γ_t – set D containing y which is pendant and $x, y \in D$ where x is support vertex. For this γ_t – set D, y is selfish. □

3. Conclusion

In the research paper, we introduce new graph class by using subdivision of an edge which is total domination subdivision stable graph and studied the basic results for a graph to be *TDSS*. Further, we proposed the condition by obtaining *TDSS* graph from a given graph G and proved few results of *TDSS* graph.

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