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Analyze the Stochastic Solid Fuzzy Transportation Problem with Mixed Constraints through Weibull Distribution

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Abstract:

This paper proposed a general formulation of the stochastic solid transportation problem (SSTP) with mixed constraints such as supply, demand and conveyance capacity taken as uncertain under stochastic environment, following the Weibull distribution (WD). The aim of this study is to minimize the transportation cost includes probabilistic constraints have inequalities of stochastic solid transportation problem (SSTP). SSTP with probabilistic constraints is represented as a chance constrained programming problem. Obtain alpha cut representation from cost coefficient of the fuzzy objective function. We have developed four models for stochastic solid transportation problem. The suggested models are demonstrated by taken as numerical example. A sensitivity analysis is performed to understand parameter's sensitivity in the proposed model.

Introduction: In system of transportation, goods are moved from various sources to destinations using different vehicles and organizational systems, involving both technology and human efforts. Efficient resource allocation in transportation system is crucial for industries and imprecision from factors like fluctuating demand, unreliable supply chains and unpredictable traffic. To address these complexities, advanced mathematical models are needed to manage stochasticity, fuzziness and mixed constraints. The study explores the stochastic solid fuzzy transportation problem with mixed constraint by utilizing the Weibull distribution to model uncertainties inherent in transportation systems. This research addresses the complexity introduced by stochastic variables and fuzzy parameters, particularly in situations where demand, supply and cost of transportation are not deterministic.

Objectives: The aim of this study is to minimize the cost of transportation includes probabilistic constraints have inequalities of stochastic solid transportation problem (SSTP).

Methods: Obtain alpha cut representation form the cost coefficient of the fuzzy objective function and four models are developed for stochastic solid transportation problem. These models are demonstrating by using a numerical example and a sensitivity analysis is conducted to understand the sensitivity of the parameters in the propose model.

Results: Obtained optimal solutions for developed four models of SFSTPMC and sensitivity analysis shows that cost of transportation and flow of unit are sensitive to change in probabilities of demand. Improve transportation system by understanding sensitivity patterns that help decision maker choose appropriate supply availability probabilities.

Conclusions: This study presented an approach for solving the SFSTPMC using the Weibull distribution for probabilistic constraints and fuzzy objective functions for transportation cost. Developed and optimized four models, focusing on stochastic parameters. Sensitivity analysis demonstrated the impact of these parameters on transportation cost and unit flow. The results validate the model's effectiveness in practical resource allocation and decision making under uncertainty.

Keywords: SFSTPMC (Stochastic Fuzzy Solid Transportation Problem with Mixed Constraints), Weibull distribution, mixed constraints.

1. Introduction

In transportation, goods are carrying from various sources to various destinations by using types of vehicles within various organizational systems. It encompasses not just various technologies such as vehicles, energy, and infrastructure but also involves the time and efforts of individuals. In essence, the transportation problem is a specific category within linear programming, linked to everyday activities in our lives, primarily focusing on logistic. Issues arise during distribution or transportation of goods between different locations, this help in optimizing these issues. To fulfil specific requirements, items are transported from various sources to various destinations. The objective is to fulfil destination's demand by minimizing cost of transportation while adhering to the limitations of the exits supply.

[1] introduced the transportation problem. System of transportation includes mixed transportation modes for delivery of items, it might be possible to minimize cost of transportation through various transportation ways. In such type of cases solid transportation is appropriate which includes supply, demand and conveyance capacity. Develop a solution methodology to solve solid transportation problem and comparison between solid transportation problem and classical transportation problem done by [2], [3]. H. Isermann proposed an approach to solve mixed constraint solid transportation problem. [4] firstly stated fuzzy set theory. [5] suggested a solution methodology to solve solid transportation problem including mixed constrain in different environment. [6] formulated several models and solution methodologies of fixed charge fuzzy stochastic solid transportation problem. [7] proposed solution procedure of multi-objective solid transportation problem with multiple items. [8] developed a new heuristic approach to obtain general initial results to optimize transportation problem and to demonstrate their method they use 35 examples. [9] they showed that expected cost minimization and chance constraint uncertain model can be converted into their deterministic form through standard optimization solver Gurobi. [10] suggested new approach to optimize solid transportation problem included multiple choices cost, supply as stochastic and demand. [11] formulated solid transportation problem with fixed charge under budget constraint by using genetic algorithm and swarm optimization. [12] hybrid intelligent algorithm to solve the constructed three models for solid transportation problem with fixed charge under uncertainty. [13] proposed different process to obtain optimal solution of transportation problem with mixed constraint. [14] proposed solution methodology to obtain minimum cost of transportation problem including special cases of fixed transportation problem along truck load constraint. [15] proposed constraint programming model and four mixed integer linear programming models to optimize shop scheduling problem with distributed flexible job. [16] suggested a better approach to optimize fuzzy transportation problem with cost, demand and supply as fuzzy parameters. [17] developed a new algorithm of fuzzy transportation

problem to obtain optimal solution. [18] suggested solution methodology to solve multi-objective stochastic solid transportation and obtain minimum cost and time. [19] their study aims to introduce a robust mechanism for addressing transportation challenges of solid transportation problem. [20] proposed a methodology to optimize intuitionistic transportation problem through specially multiply two fuzzy intuitionistic transportation issue.

The efficient allocation of resources in transportation systems is paramount for smooth functioning of various industries and economies. However, real world transportation scenarios often involve uncertainty and imprecision due to factors such as fluctuating demand, unreliable supply chains, and unpredictable traffic conditions. Addressing these complexities requires sophisticated mathematical models capable of handling stochasticity, fuzziness, and mixed constraints.

On the basis of above information, the main contribution of this paper is described in section wise. Section-2 presents assumptions and notations used in this research. Section-3 represents preliminaries which are relevant to research. Section-4 formulate the problem of SFSTPMC and transform the uncertain mathematical models into equivalent deterministic models. Section-5 described by the SFSTPMC Models. Section-6 develop an algorithm for optimization of generated models and also illustrate a numerical example to demonstrate the effectiveness of develop algorithm. Section-7 evaluates the computational results of sensitivity analysis. Section-8 summarize the research findings and in last section-9 shows the limitations and future scope of the research.

2. Assumptions and notations

SFSTPMC (Stochastic Fuzzy Solid Transportation Problem with Mixed Constraints) is associated with cost, supply, demand and conveyance capacity. Here source and destination are considered as mixed constraints. To establish mathematical models for SFSTPMC, subsequent symbols are presented as below:

m: number of sources for supply.

n: number of destinations for demands.

l: number of various different conveyance.

a_i: quantity of homogeneous items available at source i.

b_j: quantity of homogeneous items demands at destination j.

e_k: quantity of conveyance capacity for the kth modes of transport.

$\tilde{c}_{ijk}$: per unit cost of transportation of items from source i to destination j through kth modes of transportation.

x_{ijk}: quantity of items shipping from source i to destination j through kth modes of transportation.

P_{a_i}: possibilities associated with a_i.

P_{b_i}: possibilities associated with b_j.

P_{e_k}: possibilities associated with e_k.

γ_{a_i}: For a_i parameter of shape which following WD.

γ_{b_j}: For b_j parameter of shape which following WD.

γ_{e_k}: For e_k parameter of shape which following WD.

α_{a_i}: For a_i parameter of scale which following WD.

α_{b_j}: For b_j parameter of scale which following WD.

α_{e_k}: For e_k parameter of scale which following WD.

β_{a_i}: For a_i parameters of location which following WD.

β_{b_j}: For b_j parameters of location which following WD.

β_{e_k}: For e_k parameters of location which following WD.

3. Preliminaries

In 1965 Zadeh gives improved fuzzy theory. Mathematical techniques were proposed from fuzzy theory with fuzzy concepts and problems containing many possible optimal solutions.

- Let us consider S be a collection of sets, $\mu_S(x)$ is a function from R to [1,0]. A fuzzy set $\tilde{S}$ along with membership function $\mu_S(x)$ is defined by
$$\tilde{S} = \{(x, \mu_S(x); x \in A \; and \; \mu_S(x) \in [0,1]\}.$$
- Triangular Fuzzy number (TFN) is the type of fuzzy number, a continuous mapping: $\mu_{\tilde{s}}(x): R \to [0,1]$. Its membership function $\mu_{\tilde{s}}(x)$ in Figure 1 as

$$\mu_{\tilde{s}}(x) = \begin{cases} 0, & for -\infty < x < q_1, \\ \frac{x-q_1}{q_2-q_1}, & for\ q_1 \le x \le q_2, \\ \frac{q_3-x}{q_3-q_2}, & for\ q_2 \le x \le q_3, \\ 0, & for\ q_3 \le x \le \infty. \end{cases} \tag{1}$$

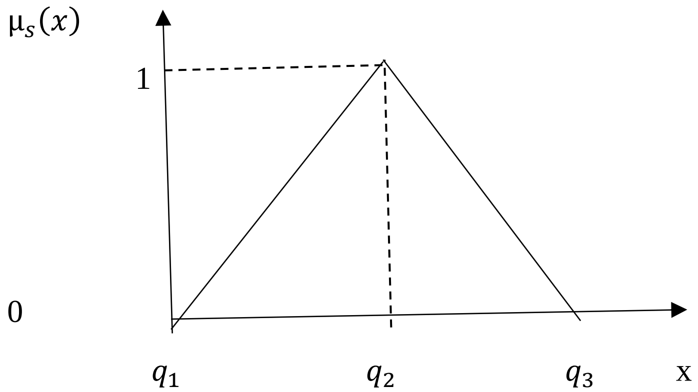

Figure 1. Triangular Fuzzy number (TFN)

- Here $\tilde{s}$ is a fuzzy number which is subset of the R real number with membership function $\mu_{\tilde{s}}$ fulfil the following conditions:

1. $\mu_{\tilde{S}}$ is a continuous function from $R \to [0,1]$.
2. $\mu_{\tilde{S}}$ is equal to zero $\forall\ (-\infty, q_1]$.
3. $\mu_{\tilde{S}}$ is equal to zero $\forall\ [q_3, \infty)$.
4. $\mu_{\tilde{S}}$ is strictly increasing on $[q_1, q_2]$.
5. $\mu_{\tilde{S}}$ is strictly decreasing on $[q_2, q_3]$.
6. $\mu_{\tilde{S}}$ is equal to one $\forall\ q \in q_2$ where $q_1 \le q_2 \le q_3$.

- A fuzzy set S is defined on X and $\alpha \in [0,1]$, then the alpha cut is defined as
  $S(\alpha) = \{x/\mu(\mathrm{x}) \ge \alpha\}$.
- A linear membership function can be defined as follows

$$\mu_S(X) = \begin{cases} 0, & if\ x_{ijk} \le \underline{x}_{ijk}, \\ \frac{\overline{x}_{ijk} - x_{ijk}}{\overline{x}_{ijk} - \underline{x}_{ijk}}, & if\ \underline{x}_{ijk} < x_{ijk} < \overline{x}_{ijk}, \\ 1, & if\ x_{ijk} > \overline{x}_{ijk}. \end{cases} \tag{2}$$

To obtain crisp set from the fuzzy system t, α-cut for linear membership function can be written as $\forall$ $\alpha \in [0,1], \left(\frac{\overline{x}_{ijk} - x_{ijk}}{\overline{x}_{ijk} - \underline{x}_{ijk}}\right) = \alpha$

Such that $x_{ijk} = (1-\alpha)\overline{x}_{ijk} + \underline{x}_{ijk}$ (3)

4. Problem's Formulation and Optimal Solution

Our main objective is minimization of cost of transportation for SFSTPMC problem. In practical scenarios, uncertainties arise in parameters like cost, cost, supply, demand, and conveyance capacity which creates challenges for decision makers in achieving optimal solutions. To address this, fuzzy and random variables are employed. Cost is treated as triangular fuzzy variables and constraints are consider as random variables in this model. On the basis of conditions of decision makers, occurrence of uncertainty in demand, supply and conveyance capacity may be occur. On the basis constraint's uncertainty formulate four SFSTPMC models.

4.1. Weibull Distribution

Weibull distribution of three parameter is followed by probability density function and cumulative density function of random variable t given as

$$f(t) = \frac{\gamma}{\delta}\left(\frac{t-\beta}{\delta}\right)^{\gamma-1} e^{-\left(\frac{t-\beta}{\delta}\right)^{\gamma}}, \tag{4}$$

and $F(t) = 1 - e^{-\left(\frac{t-\beta}{\delta}\right)^{\gamma}}$, (5)

Here $f(t) \ge 0, t \ge 0$ or γ. Note that shape parameter $\gamma > 0$, scale parameter $\delta > 0$ and location parameter $-\infty < \beta < \infty$.

4.2. Chance Constraint Programming

The closed form of quantiles for a probability distribution function can be achieved by applying the constraint from the proposed SP model to the deterministic constraints. The utilization of the Weibull distribution is also motivated by its closed form for quartiles. This paper outlines the CCP model for SFSTPMC as:

(R) Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \tilde{c}_{ijk} x_{ijk}$

Subject to

$$P(\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq a_i) \geq P_{a_i}, \qquad i \in r_1, \tag{6}$$

$$P(\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i) \geq P_{a_i}, \qquad i \in r_2, \tag{7}$$

$$P(\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq a_i) \geq P_{a_i}, \qquad i \in r_3, \tag{8}$$

$$P(\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq b_j) \geq P_{b_j}, \qquad j \in q_1, \tag{9}$$

$$P(\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j) \geq P_{b_j}, \qquad j \in q_2, \tag{10}$$

$$P(\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq b_j) \geq P_{b_j}, \qquad j \in q_3, \tag{11}$$

$$P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq e_k) \geq P_{e_k}, \qquad k \in u_1, \tag{12}$$

$$P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k) \geq P_{e_k}, \qquad k \in u_2, \tag{13}$$

$$P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_k) \geq P_{e_k}, \qquad k \in u_3, \tag{14}$$

$$x_{ijk} \geq 0, \quad \forall\, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \text{ and } k = 1, 2, \ldots, l \tag{15}$$

Where $P_{a_i}, P_{b_i}$and P_{e_k} are the given probabilities. Assumed that these random variables (a_i, b_j and e_k) follows the Weibull distribution. The Weibull distribution for a_i characterized by three parameters: γ_{a_i} as shape parameter, α_{a_i} as scale parameter, and β_{a_i} as location parameters. Similarly, for b_j and e_k Weibull distribution also have their respective specified parameters. Constraints (6) to (8) represents the probabilistic constraints for the quantity of supply at source i. These constraints establish, with a specified probability P_{a_i}, the total quantities of shipping of products from supply origin i must be distribute either be exactly a_i units, at least a_i units, or at most a_i units. Here r_1, r_2, and r_3 are partitions of i, where i takes values from 1 to m. Similarly, constraints (9) to (11) represents the demand at destination j must be receive either be exactly b_j units, at least b_j units, or at most b_j units. Here q_1, q_2, and $q_3$are partitions of j, where j takes values from 1 to n. Similarly, constraint (12) to (14) represents required conveyance capacity for k modes of transportation either be exactly e_k units, at least e_k units, or at most e_k units. Here are partitions of k, where k takes values from 1 to l. Fuzzy transportation cost for transportation of goods from i origins to j destinations, represented as $\tilde{c}_{ijk}$.

In order to satisfy mixed type of constraints such as supply and demand, the goal is to characterise the quantity x_{ijk} delivered from i origin to destination j while reducing the total transportation cost. In three cases we consider only one random variable as uncertain among a_i, b_j and e_k. In fourth case we consider all these random variables (a_i, b_j and e_k) are uncertain.

4.2.1. Case I: - Only a_i (i is equal to 1 to m) is uncertain

(i) Here proof for $P\left(\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq a_i\right) \geq P_{a_i},\ i \in r_1$ shown as follows.

It is considered that independent random variable a_i follows the WD along with three parameters γ_{a_i}, δ_{a_i}, and β_{a_i}. Equation (6) can be reconstructed as

$$P(a_i \leq \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}) \geq P_{a_i},\ \ i = r_1. \tag{16}$$

Assuming that $\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = \xi_{a_i}$, equation (16) can be represented as $P(a_i \leq \xi_{a_i}) \geq P_{a_i}$, $\mathrm{i} \in r_1$, for probability function we use Weibull distribution which can be written as

$$\int_{-\infty}^{\xi_{a_i}} \frac{\delta_{a_i}}{\gamma_{a_i}}\left(\frac{a_i-\beta_{a_i}}{\gamma_{a_i}}\right)^{\delta_{a_i}-1} e^{\left\{-\left(\frac{a_i-\beta_{a_i}}{\gamma_{a_i}}\right)^{\delta_{a_i}}\right\}} da_i \geq P_{a_i}, \qquad i \in r_1 \tag{17}$$

Simultaneously, $\beta_{a_i} \leq a_i, i \in r_1$, equation (17)'s integration yields the following form:

$$\int_{\beta_{a_i}}^{\xi_{a_i}} \frac{\delta_{a_i}}{\gamma_{a_i}}\left(\frac{a_i-\beta_{a_i}}{\gamma_{a_i}}\right)^{\delta_{a_i}-1} e^{\left\{-\left(\frac{a_i-\beta_{a_i}}{\gamma_{a_i}}\right)^{\delta_{a_i}}\right\}} da_i \geq P_{a_i}, \qquad i \in r_1 \tag{18}$$

After solving equation (18), we get

$$1 - e^{\left\{-\left(\frac{\xi_{a_i}-\beta_{a_i}}{\gamma_{a_i}}\right)^{\delta_{a_i}}\right\}} \geq P_{a_i}. \tag{19}$$

Logarithm applied on equation (19) then we get,

$\xi_{a_i} \geq \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(1-P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}$, $\quad i \in r_1$. Then, this can be expressed as deterministic constraint: $\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(1-P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}, i \in r_1$.

Here proof of $P\left(\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq a_i\right) \geq P_{a_i},\ i = r_3,\ \ (i = m_1+1, m_2+2, \ldots, m)$ for obtaining the deterministic values from probabilistic values through utilizing Weibull distribution was obtained in [18].

4.2.2. Case II: - Only b_j (i is equal to 1 to n) is uncertain

(i) Proof of $P\left(\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq b_j\right) \geq P_{b_j},\ j \in q_1, (j = 1, 2, 3, \dots, n)$ for obtaining the deterministic values from probabilistic values through utilizing Weibull distribution was obtained in [18].

(ii) Proof of $P\left(\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq b_j\right) \geq P_{b_j},\ j \in q_3$, shows as below

It is considered that independent random variable b_j follows the WD along with three parameters γ_{b_j}, δ_{b_j}, and β_{b_j}. Equation (11) can be reconstructed as

$$P(\mathrm{b}_j \geq \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}) \geq P_{b_j}, \qquad j = q_3. \tag{20}$$

Assuming that $\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = \xi_{b_j}$, equation (20) can be represented as $P(\mathrm{b}_j \geq \xi_{b_j}) \geq P_{b_j},\ j \in q_3$, for probability function we use Weibull distribution which can be written as

$$\int_{\xi_{b_j}}^{\infty} \frac{\delta_{b_j}}{\gamma_{b_j}} \left(\frac{b_j - \beta_{b_j}}{\gamma_{b_j}}\right)^{\delta_{b_j}-1} e^{\left\{-\left(\frac{b_j-\beta_{b_j}}{\gamma_{b_j}}\right)^{\delta_{b_j}}\right\}} db_j \geq P_{b_j}, \qquad j \in q_3 \tag{21}$$

After solving equation (21), we get

$$e^{\left\{-\left(\frac{\xi_{b_j}-\beta a_j}{\gamma_{b_j}}\right)^{\delta_{b_j}}\right\}} \geq P_{b_j}. \tag{22}$$

Logarithm applied on equation (22) then we get,

$\xi_{b_j} \leq \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}$, $\quad j \in q_3$. Then, this can be expressed as deterministic constraint: $\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}},\ j \in q_3$.

4.2.3. Case III: - Only e_k (k is equal to 1 to *l*) is uncertain

(i) Proof of $P\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq e_k\right) \geq P_{e_k}, k \in u_1$ shows as below

It is considered that independent random variable b_j follows the WD along with three parameters γ_{e_k}, δ_{e_k}, and β_{e_k}. Equation (12) can be reconstructed as

$$P(\mathrm{e}_k \leq \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}) \geq P_{e_k}, \qquad (k \in u_1) \tag{23}$$

Assuming that $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \xi_{e_k}$, equation (23) can be represented as $P(\mathrm{e}_k \leq \xi_{e_k}) \geq P_{e_k}, k \in u_1$ for probability function we use Weibull distribution which can be written as

$$\int_{-\infty}^{\xi_{e_k}} \frac{\delta_{e_k}}{\gamma_{e_k}}\left(\frac{e_k-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{b_k}-1} e^{\left\{-\left(\frac{e_k-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}}\right\}} de_k \geq P_{e_k}, \qquad k \in u_1 \tag{24}$$

Simultaneously, $\beta_{e_k} \leq e_k, k \in u_1$, equation (24)'s integration yields the following form:

$$\int_{\beta_{e_k}}^{\xi_{e_k}} \frac{\delta_{e_k}}{\gamma_{e_k}}\left(\frac{e_k-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}-1} e^{\left\{-\left(\frac{e_k-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}}\right\}} de_k \geq P_{e_k}, \qquad k \in u_1 \tag{25}$$

After solving equation (25), we get

$$1-e^{\left\{-\left(\frac{\xi_{e_k}-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}}\right\}} \geq P_{e_k}. \tag{26}$$

Logarithm applied on equation (26) then we get,

$\xi_{e_k} \geq \beta_{e_k} + \gamma_{e_k}\{-\ln(1-P_{e_k})\}^{\frac{1}{\delta_{e_k}}},\ k \in u_1$. Then, this can be expressed as deterministic constraint: $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq \beta_{e_k} + \gamma_{e_k}\{-\ln(1-P_{e_k})\}^{\frac{1}{\delta_{e_k}}},\ k \in u_1$. (27)

(ii) Proof of $P\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_k\right) \geq P_{e_k},\ k \in u_3$ shows as below

It is considered that independent random variable b_j follows the WD along with three parameters γ_{e_k}, δ_{e_k}, and β_{e_k}. Equation (14) can be reconstructed as

$$P(\mathrm{e}_k \geq \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}) \geq P_{e_k}, \qquad , \ k \in u_3 \tag{28}$$

Assuming that $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \xi_{e_k}$, equation (28) can be represented as $P(\mathrm{e}_k \geq \xi_{e_k}) \geq P_{e_k},\ k \in u_3$ for probability function we use Weibull distribution which can be written as

$$\int_{\xi_{e_k}}^{\infty} \frac{\delta_{e_k}}{\gamma_{e_k}}\left(\frac{e_k-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}-1} e^{\left\{-\left(\frac{e_k-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}}\right\}} de_k \geq P_{e_k}, \quad k \in u_3 \tag{29}$$

After solving equation (29), we get

$$e^{\left\{-\left(\frac{\xi_{e}-\beta_{e_k}}{\gamma_{e_k}}\right)^{\delta_{e_k}}\right\}} \geq P_{e_k}. \tag{30}$$

Logarithm applied on equation (30) then we get,

$\xi_{e_k} \leq \beta_{e_k} + \gamma_{e_k}\{-\ln(P_{e_k})\}^{\frac{1}{\delta_{e_k}}}$. Then, this can be expressed as deterministic constraint: $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq \beta_{e_k} + \gamma_{e_k}\{-\ln(P_{e_k})\}^{\frac{1}{\delta_{e_k}}},\ k \in u_3$. (31)

4.2.4. Case IV: Equality constraint

Assuming these types of equality supply, demand and conveyance capacity constraints $P(\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk}=a_i)\geq P_{a_i}$, $i\in r_2=m_1+1,m_1+2,...,m_2$, $P(\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}=b_j)\geq P_{b_j}$, $j\in q_2=n_1+1,\mathrm{n}_1+2,...,\mathrm{n}_2$ and $P(\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}=e_k)\geq P_{e_k}$, $k\in u_2=l_1+1,l_1+2,\dots,l_2$. Change these equality type constraints to inequality type of constraints, for supply constraints can be written as $P\left(\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk}\geq a_i\right)\geq P_{a_i}$ and $P\left(\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk}\leq a_i\right)\geq P_{a_i}$, for the demand constraints can be written as $P\left(\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}\geq b_j\right)\geq P_{b_j}$ and $P\left(\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}\leq b_j\right)\geq P_{b_j}$, for the conveyance capacity constraints can be written as $P\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\geq e_k\right)\geq P_{e_k}$ and $P\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\leq e_k\right)\geq P_{e_k}$. Selecting the probability value at the 50-percentage level within the supply and demand inequality constraint by utilizing Weibull distribution results in obtaining an identical deterministic value for both types of inequalities in supply and demand constraints. Then the equality constraints for supply can be written in deterministic form as $\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk}\geq\beta_{a_i}+\gamma_{a_i}\{-\ln(1-P_{a_i})\}^{\frac{1}{\delta_{a_i}}}, i\in r_2$ or $\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk}\geq\beta_{a_i}+\gamma_{a_i}\{-\ln(P_{a_i})\}^{\frac{1}{\delta_{a_i}}}, i\in r_2$, for demand constraints can be written as $\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}\leq\beta_{b_j}+\gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}$, $j\in q_2$ or $\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}\leq\beta_{b_j}+\gamma_{b_j}\left\{-\ln\left(1-P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}$, $j\in q_2$ and for the conveyance capacity constraints can be written as $\xi_{e_k}\geq\beta_{e_k}+\gamma_{e_k}\{-\ln(1-P_{e_k})\}^{\frac{1}{\delta_{e_k}}}$, $k\in u_2$ or $\xi_{e_k}\leq\beta_{e_k}+\gamma_{e_k}\{-\ln(P_{e_k})\}^{\frac{1}{\delta_{e_k}}}, k\in u_2$.

5. MODELLING

In this section, deterministic mathematical programming models for SFSTPMC are introduced. These models incorporate the quantiles of the Weibull distribution as constraints and utilize a fuzzy linear membership function for the cost function. In practical situations, only specific elements of supply and demand might be subject to uncertainty and other parameters as certain. Therefore, deterministic form of SFSTPMC can be adjusted on the basis of circumstances. There are three models in this section. In model 1, 2 and 3 includes any one of the parameters such as $a_i, b_j, or\ e_k$ as uncertain respectively. In model 3 introduces a general model in where all random variables are subject to uncertainty.

Model 1: Modelling for deterministic SFSTPMC can be converted to stochastic one when supply is uncertain, demand and conveyance capacity constraints as certain.

$$(\mathrm{R}_1)\ \mathrm{Min} z=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}c_{ijk}\left((1-\alpha)\overline{x}_{ijk}+\alpha\underline{x}_{ijk}\right). \tag{32}$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk}\geq\beta_{a_i}+\gamma_{a_i}\{-\ln(1-P_{a_i})\}^{\frac{1}{\delta_{a_i}}},\qquad i\in r_1, \tag{33}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(1-P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}, \quad i \in r_2, \tag{34}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}, \quad i \in r_3, \tag{35}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq b_j, \quad j \in q_1, \tag{36}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j, \quad j \in q_2, \tag{37}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq b_j, \quad j \in q_3, \tag{38}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq e_k, \quad k \in u_1, \tag{39}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k, \quad k \in u_2, \tag{40}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k \in u_3, \tag{41}$$

$$x_{ijk} \geq 0, i, j \text{ and } k, \tag{42}$$

Model 2: Modelling for deterministic SFSTPMC can be converted to stochastic one when demand is uncertain, supply and conveyance capacity constraints as certain.

$$(R_2)\ \text{Min}z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}\left((1-\alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk}\right). \tag{43}$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq a_i, \quad i \in r_1, \tag{44}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i, \quad i \in r_2, \tag{45}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq a_i, \quad i \in r_3, \tag{46}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(1-P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}, \quad j \in q_1, \tag{47}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_j}}, \quad j \in q_2, \tag{48}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_j}}, \quad j \in q_3, \tag{49}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq e_k, \quad k \in u_1 \tag{50}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k, \quad k \in u_2 \tag{51}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k \in u_3 \tag{52}$$

$$x_{ijk} \geq 0, i, j \text{ and } k, \tag{53}$$

Model 3: Modelling for deterministic SFSTPMC can be converted to stochastic one when conveyance capacity is uncertain, supply and demand constraints as certain.

$$(R_3)\ \text{Min}z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}\left((1-\alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk}\right). \tag{54}$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq a_i, \qquad i \in r_1, \tag{55}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i, \qquad i \in r_2, \tag{56}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq a_i, \qquad i \in r_3, \tag{57}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq b_j, \qquad j \in q_1, \tag{58}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j, \qquad j \in q_2, \tag{59}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq b_j, \qquad j \in q_3, \tag{60}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq \beta_{e_k} + \gamma_{e_k}\left\{-\ln\left(1-P_{e_k}\right)\right\}^{\frac{1}{\delta_{e_k}}},\ k \in u_1, \tag{61}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \beta_{e_k} + \gamma_{e_k}\left\{-\ln\left(1-P_{e_k}\right)\right\}^{\frac{1}{\delta_{e_k}}}, k \in u_2, \tag{62}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq \beta_{e_k} + \gamma_{e_k}\left\{-\ln\left(P_{e_k}\right)\right\}^{\frac{1}{\delta_{e_k}}}, \qquad k \in u_3, \tag{63}$$

$$x_{ijk} \geq 0, i, j\ and\ k, \tag{64}$$

Model 4: Modelling for deterministic SFSTPMC can be converted to stochastic one when supply, demand and conveyance capacity constraints as uncertain.

$$(\mathrm{R}_4)\ \mathrm{Min} z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}\left((1-\alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk}\right). \tag{65}$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(1-P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}, \qquad i \in r_1, \tag{66}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(1-P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}, \qquad i \in r_2, \tag{67}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq \beta_{a_i} + \gamma_{a_i}\left\{-\ln\left(P_{a_i}\right)\right\}^{\frac{1}{\delta_{a_i}}}, \qquad i \in r_3, \tag{68}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(1-P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}, \qquad j \in q_1, \tag{69}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}, \qquad j \in q_2, \tag{70}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq \beta_{b_j} + \gamma_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\delta_{b_j}}}, \qquad j \in q_3, \tag{71}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq \beta_{e_k} + \gamma_{e_k}\left\{-\ln\left(1-P_{e_k}\right)\right\}^{\frac{1}{\delta_{e_k}}},\ k \in u_1, \tag{72}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \beta_{e_k} + \gamma_{e_k}\left\{-\ln\left(1-P_{e_k}\right)\right\}^{\frac{1}{\delta_{e_k}}}, k \in u_2, \tag{73}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq \beta_{e_k} + \gamma_{e_k}\{-\ln(P_{e_k})\}^{\frac{1}{\delta_{e_k}}}, \qquad k \in u_3, \tag{74}$$

$$x_{ijk} \geq 0, i, j \text{ and } k, \tag{75}$$

6. Algorithm for Optimization

Method for optimize the SFSTPMC model is shows as below:

Step 1: - By utilizing alpha cut method, we convert cost which is given as triangular fuzzy problem into deterministic equivalent form.

Step 2: - Then, get 4 models by transformed problem and its constraints.

Step 3: - In model 1 we take supply constraint a_i as uncertain which is probabilistic, e_k & b_j as certain constrains. To obtain deterministic form of a_i supply constraints (33) to (35), we use Weibull distribution. With the minimum deterministic cost, deterministic supply constraints (33) to (35), certain demand constraint (36) to (38) and certain conveyance capacity constraint (39) to (41), formulate the problem (R_1). To get the optimal cost of transportation and flow of unit we optimize the problem (R_1) at zero level of alpha, and then use Lingo software.

Step 4: - In model 2 we take only demand constraint b_j as uncertain which is probabilistic and remaining constraints as certain. To obtain deterministic form of b_j (47) to (49), we use Weibull distribution. With the minimum deterministic cost, deterministic demand constraint (47) to (49), certain supply constraint (44) to (46) and certain conveyance capacity constraint (50) to (52), formulate the problem (R_2). To get the optimal cost of transportation and flow of unit we optimize the problem (R_2) at zero level of alpha, and then use Lingo software.

Step 5: - In model 3 we take conveyance capacity constraint e_k as uncertain which is probabilistic and remaining constraint as certain. To obtain deterministic form of e_k (61) to (63), we use Weibull distribution. With the minimum deterministic cost, deterministic conveyance capacity (61) to (63), certain supply constraint (55) to (57) and certain demand constraint (58) to (60), formulate the problem (R_3). To get the optimal cost of transportation and flow of unit we optimize the problem (R_3) at zero level of alpha, and then use Lingo software.

Step 6: - In model 4 we take a_i, b_j and e_k as probabilistic uncertain variables. To obtain deterministic form of supply, demand and conveyance capacity constraint (66) to (74), we use Weibull distribution. With the minimum deterministic cost, deterministic constraints (66) to (74), formulate the problem (R_4). To get the optimal cost of transportation and flow of unit we optimize the problem (R_4) at zero level of alpha, and then use Lingo software.

Step 7: - For varying alpha values, repeat above steps from 3 to 6, to determine the optimal cost of transportation and flow of unit.

6.1. Computational analysis

To show the applicability and effectiveness of SFSTPMC and its different versions, offers a numerical example in this section. The food factory supplies homogeneous items, and there are three food factories and four warehouses. Food factory A has capacity of production at least a_1 units, food factory

B has capacity of production exactly $a_2$units, and Food factory C has capacity of production at most a_3 units. Similarly, warehouse 1 has demand's capacity at most b_1 units, warehouse 2 has demand's capacity at least b_2 units, warehouse 3 has demand's capacity at exactly b_3 units, and warehouse 4 has demand's capacity at least b_4 units. Appropriate conveyance is accommodated by food factory supplier. Also, address two conveyances in this problem along with their different loading capacities which are denoted as for first conveyance has loading capacity at least e_1 units and for second conveyance has loading capacity at most e_2 units. The cost of transportation per unit from every food factory to each warehouse represented as triangular fuzzy numbers which is denoted as $\tilde{c}_{ijk}$, and cost for two conveyance is represented as $c_{ij1}$and c_{ij2} shown in table 1 and 2 respectively.

The supply of the fresh food is uncertain because of unpredictable circumstances at food factory such as availability of labour and severity of weather. It is easy to think of scenario in which the supply is uncertain, implementing probability on such availability of production. Therefore, P_{a_1} is the probability for food factory A that the needed quantity of goods is available. Similarly, P_{a_2} and P_{a_3} are the probabilities for food factories B and C respectively.

The demand for fresh food is also naturally unpredictable due to inaccurate demand forecast, erratic delivery schedules. Thus, P_{b_1} is the probability for warehouse 1 that the needed expected demand. Similarly, P_{b_2} and P_{b_3} are the probabilities for warehouses 2 and 3 respectively. Similarly, blockages of roads and traffic jams make the capacity of conveyance is unpredictable or uncertain. P_{e_1} and P_{e_2} are the probabilities that the availability of capacities of two conveyances.

In the numerical example, the nominal values of uncertain constants are taken as $a_1 = 19$, $a_2 = 17$, $a_3 = 23$, $b_1 = 12$, $b_2 = 13$, $b_3 = 18$, $b_4 = 15$, $e_1 = 27$ and $e_2 = 28$. Arbitrary probabilities are $P_{a_1} = 0.90$, $P_{a_2} = 0.50$, $P_{a_3} = 0.85$, $P_{b_1} = 0.25$, $P_{b_2} = 0.30$, $P_{b_3} = 0.28$, $P_{b_4} = 0.35$, $P_{e_1} = 0.07$ and $P_{e_2} = 0.06$. Here a_i, & b_j are uncertain and follows the Weibull distribution. So, consider different values for parameters of the Weibull distribution as $\beta_{a_1} = 25$, $\beta_{a_2} = 18$, $\beta_{a_3} = 13$, $\beta_{b_1} = 11$, $\beta_{b_2} = 10$, $\beta_{b_3} = 16$, $\beta_{b_4} = 14$, $\beta_{e_1} = 2$, $\beta_{e_2} = 3$, $\delta_{a_i} = \delta_{b_j} = \delta_{e_k} = 2$, $\gamma_{a_i} = \gamma_{b_j} = \gamma_{e_k} = 2$. Deterministic form of constraints which are probabilistic can be obtain by using equations (66) to (74).

Warehouse Food factory	A	B	C	D	a_i
1	(2.5,3,3.5)	(4,4.5,5)	(2.5,3,3.5)	(4.5,5,5.5)	$\geq a_1$
2	(0,0.5,1)	(1.5,2,3)	(2,2.5,3)	(2.5,3,3.5)	$= a_2$
3	(4.5,5,5.5)	(7,7.5,8)	(6.5,7,7.5)	(5.5,6,6.5)	$\leq a_3$
b_j	$\leq b_1$	$\geq b_2$	$= b_3$	$\geq b_4$	

Table 1: Cost of transportation for e_1 conveyance.

Warehouse Food factory	A	B	C	D	a_i
1	(4.5,5,5.5)	(8.5,9,9.5)	(6.5,7,7.5)	(8,8.5,9)	$\geq a_1$
2	(2.5,3,3.5)	(4.5,5,5.5)	(6.5,7,7.5,)	(5,5.5,6)	$= a_2$
3	(3.5,4,4.5)	(6.5,7,7.5)	(4.5,5,5.5)	(5,5.5,6)	$\leq a_3$
b_j	$\leq b_1$	$\geq b_2$	$= b_3$	$\geq b_4$	

Table 2: Cost of transportation for e_2 conveyance.

Through step 1, deterministic form of triangular fuzzy cost can be obtained by utilizing linear membership function. Cost of the problem shows as alpha cut representation for conveyance capacity e_1 & e_2 in table 3 & 4 respectively.

Warehouse Food factory	A	B	C	D	a_i
1	$(1-\alpha)3.5\bar{x}_{111} + 2.5\alpha\underline{x}_{111}$	$(1-\alpha)5\bar{x}_{121} + 4\alpha\underline{x}_{121}$	$(1-\alpha)3.5\bar{x}_{131} + 2.5\alpha\underline{x}_{131}$	$(1-\alpha)5.5\bar{x}_{141} + 4.5\alpha\underline{x}_{141}$	$\geq a_1$
2	$(1-\alpha)1\bar{x}_{211} + 0\alpha\underline{x}_{211}$	$(1-\alpha)3\bar{x}_{221} + 1.5\alpha\underline{x}_{221}$	$(1-\alpha)3\bar{x}_{231} + 2\alpha\underline{x}_{231}$	$(1-\alpha)3.5\bar{x}_{241} + 2.5\alpha\underline{x}_{241}$	$= a_2$
3	$(1-\alpha)5.5\bar{x}_{311} + 4.5\alpha\underline{x}_{311}$	$(1-\alpha)8\bar{x}_{321} + 7\alpha\underline{x}_{321}$	$(1-\alpha)7.5\bar{x}_{331} + 6.5\alpha\underline{x}_{331}$	$(1-\alpha)6.5\bar{x}_{341} + 5.5\alpha\underline{x}_{341}$	$\leq a_3$
b_j	$\leq b_1$	$\geq b_2$	$= b_3$	$\geq b_4$	

Table 3: Transportation cost c_{ij1} alpha cut representation for conveyance capacity e_1.

Warehouse Food factory	A	B	C	D	a_i
1	$(1-\alpha)5.5\bar{x}_{111} + 4.5\alpha\underline{x}_{111}$	$(1-\alpha)9.5\bar{x}_{121} + 8.5\alpha\underline{x}_{121}$	$(1-\alpha)7.5\bar{x}_{131} + 6.5\alpha\underline{x}_{131}$	$(1-\alpha)9\bar{x}_{141} + 8\alpha\underline{x}_{141}$	$\geq a_1$
2	$(1-\alpha)3.5\bar{x}_{211} + 2.5\alpha\underline{x}_{211}$	$(1-\alpha)5.5\bar{x}_{221} + 4.5\alpha\underline{x}_{221}$	$(1-\alpha)7.5\bar{x}_{231} + 6.5\alpha\underline{x}_{231}$	$(1-\alpha)6\bar{x}_{241} + 5\alpha\underline{x}_{241}$	$= a_2$
3	$(1-\alpha)4.5\bar{x}_{311} + 3.5\alpha\underline{x}_{311}$	$(1-\alpha)7.5\bar{x}_{321} + 6.5\alpha\underline{x}_{321}$	$(1-\alpha)5.5\bar{x}_{331} + 4.5\alpha\underline{x}_{331}$	$(1-\alpha)6\bar{x}_{341} + 5\alpha\underline{x}_{341}$	$\leq a_3$
b_j	$\leq b_1$	$\geq b_2$	$= b_3$	$\geq b_4$	

Table 4: Transportation cost c_{ij2} as alpha cut representation for conveyance capacity e_2.

Through step 2, obtain 4 models by transformed problem (R) and its constraints. Through step 3, in model 1 we take supply constraint a_i as uncertain which is probabilistic, e_k & b_j as certain constrains. Function of cost at zero level of alpha, utilizing Weibull distribution on supply constraint which is probabilistic due to uncertainty in supply. Now, supplies are $a_1 = 28.035$, $a_2 = 19.665$, $a_3 = 13.8063$, demands are $b_1 = 12$, $b_2 = 13$, $b_3 = 18$, $b_4 = 15$, and conveyance capacities are $e_1 = 27$, $e_2 = 28$. Through lingo software, obtain optimal cost of transportation as 176.27 and $x_{131} = 18.00, x_{141} = 10.035, x_{211} = 1.70, x_{221} = 13.00, x_{241} = 4.965$ & unit flow as 47.7.

Through step 4, in model 2 we take only demand constraint b_j as uncertain which is probabilistic and remaining constraints as certain. Function of cost at zero level of alpha, utilizing Weibull distribution on demand constraint which is probabilistic due to uncertainty in demand. Now, supplies are $a_1 = 19$, $a_2 = 17$, $a_3 = 23$, demands are $b_1 = 13.36$, $b_2 = 11.195$, $b_3 = 17.665$, $b_4 = 15.313$, and conveyance capacities are $e_1 = 27$, $e_2 = 28$. Through lingo software, obtain optimal cost of transportation as 168.024 and $x_{131} = 17.665, x_{141} = 9.508, x_{221} = 11.195, x_{241} = 5.805$ & unit flow as 44.173.

Through step 5, in model 3 we take conveyance capacity constraint e_k as uncertain which is probabilistic and remaining constraint as certain. Function of cost at zero level of alpha, utilizing Weibull distribution on conveyance capacity constraint which is probabilistic due to uncertainty in conveyance capacity. Now, supplies are $a_1 = 19$, $a_2 = 17$, $a_3 = 23$, demands are $b_1 = 12$, $b_2 = 13$,

$b_3 = 18$, $b_4 = 15$, and conveyance capacities are $e_1 = 2.539$, $e_2 = 6.3547$. Through lingo software, obtain optimal cost of transportation as 176.50 and $x_{131} = 18.00, x_{141} = 11.00, x_{221} = 13.00, x_{241} = 4.00$ & unit flow as 46.

Through step 6, in model 4 we take a_i, b_j and e_k as probabilistic uncertain variables. Now, supplies are $a_1 = 28.035$, $a_2 = 19.665$, $a_3 = 13.8063$, , demands are $b_1 = 13.36$, $b_2 = 11.195$, $b_3 = 17.665$, $b_4 = 15.313$, and conveyance capacities are $e_1 = 2.539$, $e_2 = 6.3547$. Through lingo software, obtain optimal cost of transportation as 173.275 and $x_{131} = 17.665, x_{141} = 10.370, x_{211} = 3.527, x_{221} = 11.195, x_{241} = 4.943$ & unit flow as 47.7. Similarly, solve four cases of constraint for varying alpha values. Table 5 shows the computational results for 4 models.

Constructed four models for SFSTPMC. Model 1 depicts supply constraint as probabilistic values, demand and conveyance capacity as certain values, model 2 depicts demand as probabilistic values and other constraints as certain values, model 3 depicts conveyance capacity as probabilistic value and other constraints as certain values, model 4 depicts supply, demand and conveyance capacity as probabilistic values. The objective function's cost coefficient converts to alpha cut representation and mixed constraints which are stochastic are converts into crisp form by utilize Weibull distribution to solve the models. It's notable that these models are constructed from different perspectives. As the choice of model depends on the decision-maker's preference, determining the superior model for decision-making is inconclusive.

	Optimal solution	Flow of units	Shipment x_{ijk}
Model 1	176.27	47.7	$x_{131} = 18, x_{141} = 10.035, x_{211} = 1.70, x_{221} = 13.00, x_{241} = 4.965$.
Model 2	168.024	44.173	$x_{131} = 17.665, x_{141} = 9.508, x_{221} = 11.195, x_{241} = 5.805$.
Model 3	176.50	46	$x_{131} = 18, x_{141} = 11, x_{221} = 13.00, x_{241} = 4.00$.
Model 4	173.275	47.7	$x_{131} = 17.665, x_{141} = 10.35, x_{211} = 11.195, x_{221} = 13.00, x_{241} = 4.943$.

Table 5: Computational results of 4 models.

7. Sensitivity analysis and Discussion

In this section, SFSTPMC conducted a sensitivity analysis to assess optimality concerning variation in probabilities related to uncertain parameters like source, demand and conveyance capacity. Model 4th has chosen for this analysis, with probabilities ranging from 0 to 1 for parameters a_i, b_j & e_k. The analysis involved, holding one probability parameter (P_{a_i}or P_{b_j} or P_{e_k}) as constant at 0.5 while varying the probabilities of other parameters (P_{a_i}or P_{b_j} or P_{e_k}). For equality constraints, the probabilities of that parameter remain 0.5 by using case IV. Each parameter which is stochastic and cost which is fuzzy in model 4 we obtained and listed optimal results as units of entire shipping (flow) and cost of transportation. The lingo software aided in solving the optimization problem. Results of sensitivity analysis for probabilistic demand b_j presented in Table 6. Graphical representation of cost of transportation & flow of units for probabilistic demand in model 4th shown in figures 1 and 2. It's important that change in the probability of demand requirements significantly affect cost of transportation. Figure 1 shows that the cost of transportation increase gradually for probabilistic demand parameter and obtain least cost of transportation when $P_{b_j} = 0.01$. It's important that change in the probability of demand requirements significantly affect cost of transportation and flow of unit. Figure 2 shows that the flow of unit largest when $P_{b_j} = 0.99$. When $0 \leq P_{b_j} \geq 0.70$, the flow of unit unchanged. When $P_{b_j} > 0.70$, the flow of unit increases.

The sensitivity analysis reveals that the cost of transportation cost and flow of unit in this problem are sensitive through changes in the probabilities of demand parameter b_j. This insight aids decision makers in selecting suitable probabilities for the availability of supply. Understanding these sensitivity patterns regarding uncertain parameters empowers decision maker with valuable insights and the ability to enhance the system of transportation.

Sr. No.	Probability for $a_i(P_{a_i})$	Probability for $b_j(P_{b_j})$	Probability for $e_k(P_{e_k})$	Cost of transportation	Flow of unit
1	0.50	0.99	0.50	194.562	**50.2488**
2		0.95		185.845	48.5883
3		0.90		181.363	47.7347
4		0.85		178.422	47.1744
5		0.80		176.139	46.7396
6		0.75		174.223	46.3746
7		0.70		173.368	46.33
8		0.65		172.714	46.33
9		0.60		172.108	46.33
10		0.55		171.535	46.33
11		0.50		170.985	46.33
12		0.45		170.415	46.33
13		0.40		169.925	46.33
14		0.35		169.400	46.33
15		0.30		168.868	46.33
16		0.25		168.32	46.33
17		0.20		167.744	46.33
18		0.15		167.121	46.33
19		0.10		166.141	46.33
20		0.05		165.531	46.33
21		0.01		**164.3948**	46.33

Table 6: Sensitivity analysis of SFSTPMC

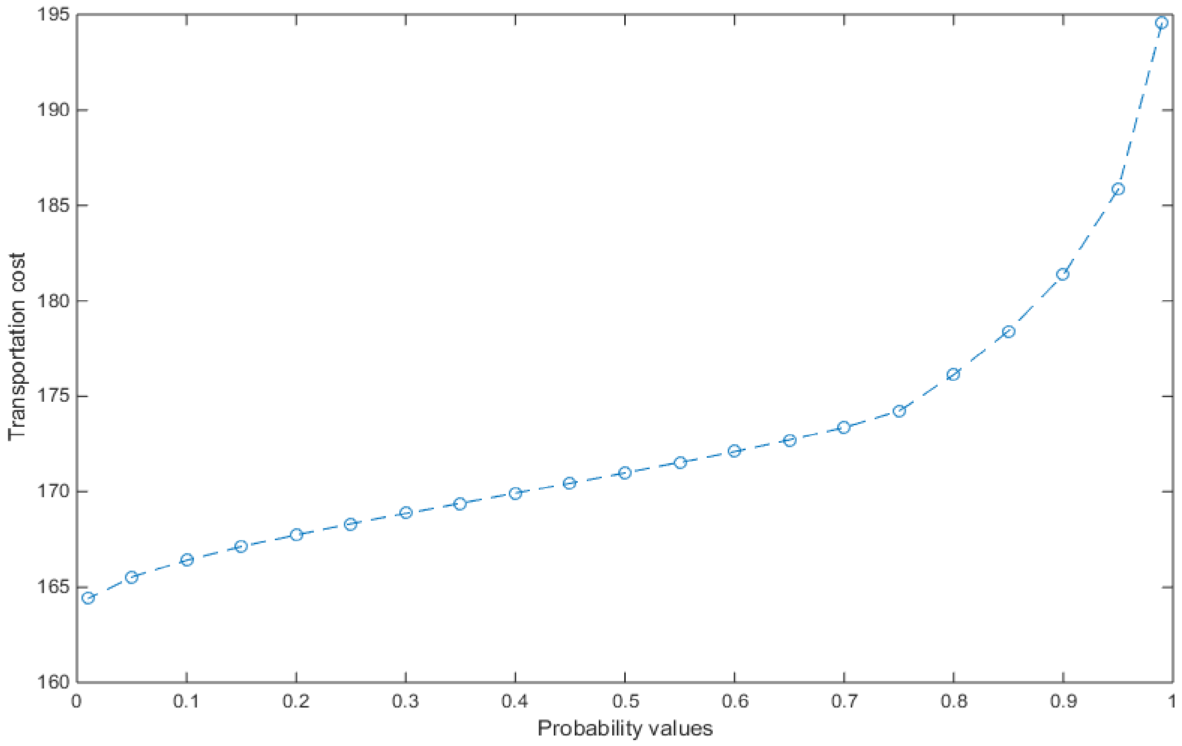

Figure1: Sensitivity analysis of transportation cost.

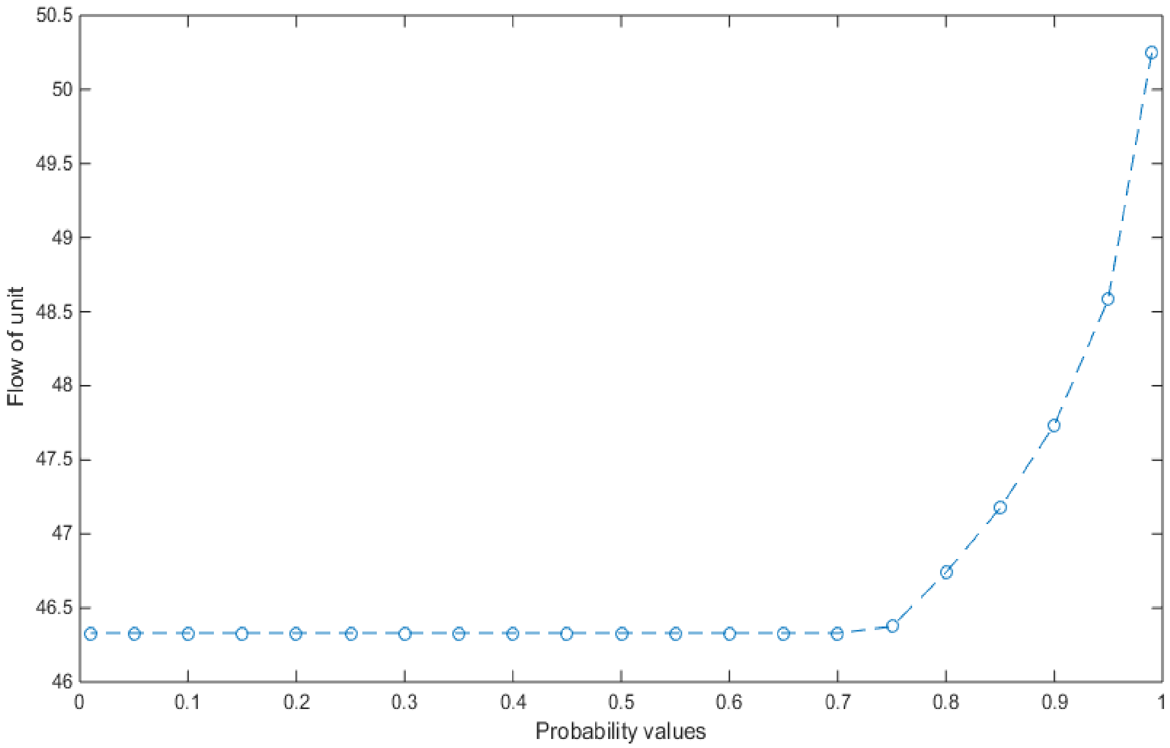

Figure2: Sensitivity analysis of flow of unit.

8. Conclusion

The purpose of this study was to introduced an approach for solving the SFSTPMC under probabilistic constraints such as supply, demand & conveyance capacity described as Weibull distribution and also includes fuzzy objective function such as cost of transportation. Obtain alpha cut representation from cost coefficient of the fuzzy objective function and through Weibull distribution, obtain deterministic form of stochastic constraints. Developed four models of SFSTPMC and optimize these models through Lingo software. In this study we address solid transportation problem which includes three mixed constraint stochastic parameters (supply, demand and conveyance). Its's noted that several factors significantly influence the overall cost of transportation and unit of shipping, when distributing harmful goods such as vaccines and fragile items. Consequently, its's crucial to incorporate these factors as parameters for STP's and implement the relevant constraints accordingly. An illustrative numerical example is provided to demonstrate the efficacy of these models. Results of sensitivity analysis for model 4 Supply, demand and conveyance capacity, presented. Figures 1 & 2 visually represents the impact of change in demand parameters on entire cost of transportation and flow of unit in model 4 as part of sensitivity analysis. Similarly, sensitivity analysis is conducted for the supply and conveyance capacity parameters. Moreover, this analysis underscores the importance of understanding the sensitivity of mixed constraint in the condition of increase uncertainty, aiding decision makers in determining the appropriate level of uncertainty for uncertain parameters. The computed results clearly demonstrate the validity of the designed model across various parameters. SFSTPMC problem shows important role in various decision makers managerial scenarios, particularly in problem including planning of resource allocation in industries production where demand, conveyance capacity and supply are inherently random variables. For optimization, this tool is effective on these types of problems.

9. Limitations and Future Scope

The current study's limitations are outlined, a SFTP with mixed constraints is examined within stochastic environment, focusing only on cost of transportation as a single objective. However, in real-world decision-making processes, dealing with complex organizational contexts cannot apply only single criterion. Therefore, it's necessary to recognize the presence of multiple criteria that can enhance multi criteria decision making. Consequently, our future research aims to explore SFSTPMC scenarios by incorporating multiple objectives including multiple item parameters. Employing the methodology introduces in the study, variations in supply, demand and conveyance can be factored into the economic order quantity model. Additionally, in forthcoming research, we intent to collect real world data from authoritative sources and utilize statistical regularity criteria to establish its probability distribution, thereby expanding the applicability of the Weibull distribution.

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