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A Mixed-Integer Linear Programming Model for Enhanced Weathering Networks Considering Logistical Emissions

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Carbon dioxide removal (CDR) must be deployed at the scale of multiple gigatons per year to achieve the mid-century carbon neutrality target. Among the available CDR techniques, enhanced rock weathering or simply, enhanced weathering (EW) relies on processes that are technologically mature and result in durable carbon removal. It relies on the engineered acceleration of geochemical reactions between alkaline minerals and carbonic acid in water to permanently sequester carbon as bicarbonate ions in runoff. Ex situ EW involves spreading pulverized rock on the ground and allowing it to slowly dissolve in rainwater; in practice, the extent of completion of the reaction is dependent on local conditions and affects the “stoichiometric efficiency” of CDR. The CDR rate of suitable rocks is between 0.3-1.1 Gt CO_2 per Gt rock without considering yet the CO_2 penalties from processing the rocks. Scaling up EW to the required annual gigaton scale presents a massive logistical challenge that involves quarrying suitable rocks, grinding them into a suitably fine powder, transporting the rock powder to application sites, and finally putting it in the soil at a rate calibrated to match local conditions. In this work, we develop a mixed-integer linear programming model to optimize EW networks to maximize CDR for a given set of sources (rock-crushing plants) and sinks (application sites). The model determines both topology (source-sink matches) and physical flow rates; alternative solutions can be explored using integer cut techniques. The model is demonstrated using an illustrative case study of three sources and seven sinks. The optimal results utilized 76 % of the total available rock. The CO_2 footprint is mainly due to production (rock crushing activities) at 82 %, transportation at 16 %, and application at 2 %. The introduction of carbon footprint (CFP) from various steps reduces the total CDR by 45.62 % highlighting that the CDR potential of EW networks is affected not only by the CDR rate and stoichiometric efficiency but also the CO_2 penalties throughout the supply chain, with rock production giving the highest CO_2 footprint. The sensitivity analysis shows that it might be worthwhile to focus on improving the production CFP along with the source material CDR rate.

1. Introduction

The latest report from the Intergovernmental Panel on Climate Change (IPCC) has now included carbon dioxide removal technologies (CDR) in the portfolio of climate change mitigation solutions (IPCC, 2022). CDR technologies, also known as Negative Emission Technologies (NETs), work by transferring CO_2 from the atmosphere to a different environmental compartment such as in land, biomass, or in the ocean. Examples of NETs include biological solutions such as afforestation/reforestation and soil carbon sequestration, or engineered solutions such as direct air carbon capture and storage (Mac Dowell et al., 2022). Different NETs can also generate environmental impacts that should be evaluated for cost-benefit analysis (Deprez et al., 2024). Enhanced weathering (EW) is another NET that accelerates the natural weathering of rocks or minerals (Renforth, 2012). By spreading finely ground silicate rocks such as basalt and dunite on land, in contact with air and water, this process enhances the rate of conversion of CO_2 into stable forms that can be stored in soils

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and oceans. Aside from carbon removal, EW has other co-benefits like increasing soil nutrient availability when applied to land and decreasing ocean acidification when applied to shores (Beerling et al., 2018).

Enhanced weathering using alkaline materials was first conceptualized by Seifritz (1990). Land-based EW requires crushing, grinding, and spreading the rocks on soil (Strefler et al., 2018). Silicate rocks react with atmospheric CO_2 in the presence of water, and the resulting bicarbonate ions are carried by runoff into the ocean (Renforth, 2012). Many component technologies for EW are commercially mature (Campbell et al., 2022), although there are still challenges in monitoring actual CDR generated (Calabrese et al., 2022).

The large-scale implementation of EW faces resource and logistical challenges (Maesano et al., 2022). The crushing and grinding of rocks require energy and therefore carbon footprint penalties (CFP) must be considered (Strefler et al., 2018). Since crushing and grinding are critical steps in EW, a study on the effect of different particle sizes on EW have been conducted (Rinder and von Hagke, 2021). Emissions from the transportation and application of rocks into the soil using heavy equipment and trucks must also be considered in calculating the net CO_2 removal. Modelling studies have been conducted to evaluate the economics and feasibility of EW. The cost and energy requirements of global and regional EW have been assessed by Strefler et al. (2018). Mathematical programming has been used to design EW networks considering uncertainties in the rock flow rates (Aviso and Tan, 2020). Through life cycle analysis (LCA), Eufrasio et al. (2022) determined that the energy mix will impact the effectiveness of EW as a CDR technology. Lefebvre et al. (2019) also employed LCA to evaluate the feasibility of basalt-based enhanced weathering. A fuzzy optimization model was developed to consider techno-economic uncertainties in EW networks using industrial wastes (Aviso et al., 2022). Jerden et al. (2024) created an integrated model combining geochemical analysis with LCA to study EW-CDR systems. Oppon et al. (2023) evaluated the economic and environmental impacts of basalt EW using input-output analysis; Aviso et al. (2024) developed a related modelling approach that represented EW as a new economic sector. The existing studies have focused on the economics and feasibility analysis of EW. However, no work to date has developed any model to optimize the detailed operational aspects of EW.

To bridge this research gap, this work formulates a new model that maximizes the net CDR of an EW system considering the production, transportation, and application of rocks into sinks in multiple locations with different source lifespans. The model considers the CFP penalty at each stage to ensure that the net CDR is accounted for carefully. This work is important because it gives new insights into EW literature, regarding the design of optimal EW networks considering the geographical distances, CFP penalties, and operating lifespans of sources and sinks. The rest of the paper is organized as follows. Section 2 gives the formal problem statement. Section 3 presents the optimization model. Section 4 describes the case study and results. Section 5 presents the sensitivity analysis. Finally, Section 6 gives the conclusion of this study.

2. Problem Statement

The formal problem statement is stated as follows.

- Given sources, each source i is characterized by its lower and upper annual rock crushing flow rate ($\mathrm{S_i^L}, \mathrm{S_i^U}$), operating lifespan ($\mathrm{L_i}$), CDR rate based on the material ($\mathrm{M_i}$), and production CO_2 penalty ($\mathrm{P_i}$);
- Given sinks, each sink j is characterized by its annual rock application limit to ensure that the application rate is within the sites' projected weathering rate as determined by average precipitation level ($\mathrm{D_j}$), cumulative rock application limit over the entire project lifetime to ensure that the deposition of the silica co-product from weathering reactions is within sustainability limits to maintain soil quality ($\mathrm{G_j}$). dimensionless CDR efficiency compared to the stoichiometric ratio ($\mathrm{E_j}$), and application CO_2 penalty ($\mathrm{R_j}$);
- Given the transportation distance $\mathrm{T_{ij}}$ between source-sink pairs and transportation CO_2 penalty or emissions per unit distance ($\mathrm{N_{ij}}$);
- Assuming all sources and sinks are available at the start of the planning period.

The problem is to match sources and sinks to maximize net CDR, which is obtained by calculating the total CDR and subtracting the CO_2 footprint (CFP) from production, transportation, and application. The distances between sources and sinks are considered in the transportation CFP evaluation. Each possible connection is represented by the variable $\mathrm{x_{ij}}$, which is the physical flow rate from source i to sink j. The result of the model should give the optimal allocation $\mathrm{x_{ij}}$.

3. Optimization model

The model is described by Eq(1) to Eq(7). The model optimizes the allocation of crushed rock from sources to application sites by maximizing the net CO_2 removal against the CO_2 penalties from production, transport, and application while considering capacity and operational constraints. The objective function is to maximize the net CDR throughout the source lifespans as depicted in Eq(1). The first term in Eq(1) calculates the total CDR of the network by multiplying the lifespan (L_i), material CDR rate (M_i), sink CDR efficiency (E_j) and flow rate (x_{ij}). Terms 2 to 4 of Eq(1) calculate the CFP from the production, transportation, and application steps. The production CFP is the product of the lifespan (L_i), production CO_2 penalty (P_i), and flow rate (x_{ij}). The production CO_2 penalty (P_i) is primarily due to rock-crushing activities. The transportation CFP is the product of the lifespan (L_i), distance (T_{ij}), CO_2 emissions (N_{ij}), and flow rate (x_{ij}). The application CFP is obtained by multiplying the lifespan (L_i), application CO_2 penalty (R_j), and flow rate (x_{ij}). The CFP of each step is subtracted from the total CDR, resulting in the maximization of the net CDR in Eq(1).

The source balance in Eq(2) and Eq(3) ensures that the sum of the flow rates for each source (S_i) is between the lower (S_i^L) and upper (S_i^U) limits. S_i^L represents the smallest economically feasible flow rate while S_i^U is the limiting flow rate. The binary variable b_i takes a value of 1 when source i is active (flow rate exists) and takes a value of 0 when source i is inactive (no flow rate exists). Eq(5) and Eq(6) represent the demand balance. Eq(5) ensures that the total flow rate for each demand j does not exceed the annual application limit (D_j), while Eq(6) ensures that the cumulative total flow rate does not exceed the cumulative limit (G_j). Lastly, Eq(7) is the nonnegativity constraint for variable x_{ij}.

$$\max \sum_i \sum_j L_i M_i E_j x_{ij} - \sum_i \sum_j L_i P_i x_{ij} - \sum_i \sum_j L_i T_{ij} N_{ij} x_{ij} - \sum_i \sum_j L_i R_i x_{ij} \quad (1)$$

$$\sum_j x_{ij} = S_i \qquad \forall\, i \quad (2)$$

$$b_i S_i^L \leq S_i \leq b_i S_i^U \qquad \forall\, i \quad (3)$$

$$b_i \in \{0,1\} \qquad \forall\, i \quad (4)$$

$$\sum_i x_{ij} \leq D_j \qquad \forall\, j \quad (5)$$

$$\sum_i L_i x_{ij} \leq G_j \qquad \forall\, j \quad (6)$$

$$x_{ij} \geq 0 \qquad \forall\, i, j \quad (7)$$

Eq(1) to Eq(7) gives rise to a mixed integer linear programming model (MILP). Like the previously developed linear program by Tan and Aviso (2019), time intervals are not explicitly represented, but are implied by the presence of annual and cumulative flow constraints. The model is solved using the optimization software LINGO v19 in a device operating on Windows 11 with a processor AMD Ryzen 7 and 16.0 GB RAM.

4. Case study

The case study presents a hypothetical case of three sources and seven sinks with data presented in Table 1 and Table 2. The characteristics mentioned in Section 2 are presented in the said tables. The distances between sources and sinks are reported in Table 3. The assumed CDR rate (M_i) is 0.3 t CO_2/t rock for all sources (Tan and Aviso, 2019). The transportation CO_2 penalty (N_{ij}) is assumed to be 0.0001 t CO_2/t/km for all connections, and the application CO_2 penalty (R_j) is 0.001 t CO_2/t rock for all sinks based on estimates from Renforth (2012) and Strefler et al. (2018). The production CO_2 penalty (P_i) is site-specific, depending on the local energy mix. Based on the literature, the CO_2 penalty for crushing rocks is 20 t CO_2/kt rock if fossil fuel-based electricity is used (Strefler et al., 2018). In general, the CO_2 penalty will largely depend on the energy mix at the crushing plant site.

Table 1: Limiting data for sources

Sources	Flow rate, S_i^L, S_i^U (kt rock/y)	Lifespan, L_i (y)	Production CO_2 penalty P_i (t CO_2/kt rock)
A	4-40	25	30
B	10-100	20	50
C	6-60	15	75

Table 2: Limiting data for sinks

Sinks	Application limit, D_j (kt rock/y)	Cumulative application limit, G_j (kt rock)	Stoichiometric efficiency, E_j
1	22	400	0.3
2	30	300	0.2
3	30	600	0.2
4	20	400	0.3
5	60	250	0.2
6	50	1,000	0.4
7	40	1,500	0.5
Total		4,450	

Table 3: Distances (km)

Sinks	Source A	Source B	Source C
1	50	220	280
2	60	160	150
3	130	200	150
4	200	170	50
5	100	70	30
6	70	50	100
7	120	100	300

After solving for Eq(1) subject to the constraints found in Eq(2) to Eq(7) using the data above, the optimal results of the case study are presented in Table 4. The numbers inside the parenthesis show the cumulative values. Only sources A and B are activated. Only sinks 1, and 4 to 7 are utilized. From Table 4, the total rock flow rate from sources to sinks is 2,970 kt as reflected in the sum of both sinks and sources. This implies that out of the 3,900 kt rock cumulative limit of the sources, only 76 % is utilized; and out of the 4,450 kt rock cumulative limit of the sinks, only 66.7 % is utilized. All other constraints are met in the optimized results.

The total CDR is 345 kt CO_2 for the entire planning horizon of 25 years. To get the total CDR, each cumulative sink balance is multiplied by the CDR rate of the rock (M_i) of 0.3 t CO_2/t rock and the CDR efficiency of each sink (E_j) in Table 2. The production CFP is 128.5 kt, the transport CFP is 25.95 kt, and the application CFP is 2.97 kt for a total of 157.42 kt. This amount reduces the total CDR by 45.6 %, resulting in a net CDR of 187.58 kt CO_2 for the entire planning horizon. Figure 1(a) reports the CO_2 accounting of the optimal results. Breaking down the CFP, the highest CFP goes to the crushing of rock which accounts for 82 % of the total CFP, followed by transport at 16 %. Rock application is only at 2 %, as illustrated in Figure 1(b).

The case study results show that the CDR potential of EW networks is affected not only by the CDR rate and the stoichiometric efficiency, but also by the CFPs in the production, transport, and application of rocks. Among the CFPs, the production stage has the highest contribution, primarily due to the crushing of the rocks. The results agree that emissions from mining, transport, and crushing decrease the CDR efficiency of EW (Jerden et al., 2024); however, this is the first study to quantify the impact of each individual logistical step on emissions using an optimization model.

Table 4: Optimal results in kt rock/y. Numbers inside the parenthesis show the cumulative values in kt rock.

Sinks	Source A	Source B	Source C	Total for Sinks
1	16 (400)	0	0	16 (400)
2	0	0	0	0
3	0	0	0	0
4	0	20 (400)	0	20 (400)
5	0	12.5 (250)	0	12.5 (250)
6	0	50 (1,000)	0	50 (1,000)
7	24 (600)	16 (320)	0	40 (920)
Total for Sources	40 (1,000)	98.5 (1,970)	0	138.5 (2,970)

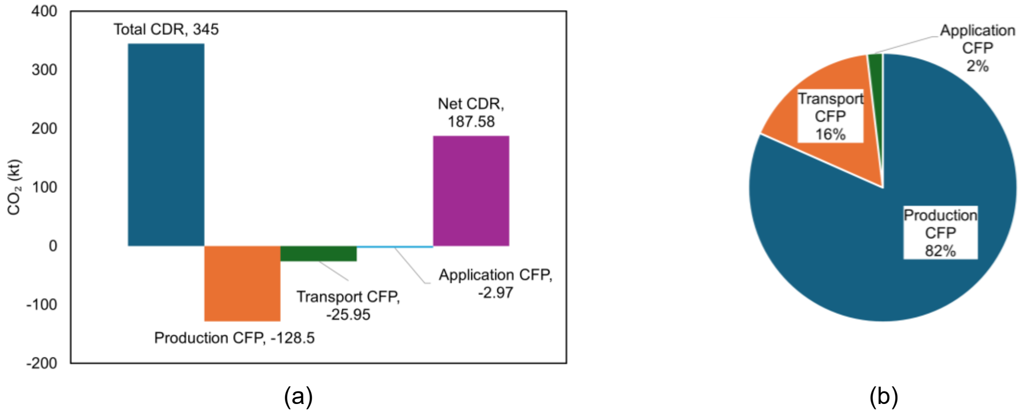

Figure 1: (a) Accounting of CO2 and (b) breakdown of CFP

5. Sensitivity analysis

Sensitivity analysis was performed by varying the CDR rate from 0.1 to 0.5 t CO_2/t rock as shown in Figure 2. While the increase in net CDR appears linear within the studied range, the rate of increase slows down relative to the total CDR due to the growing impact of the production CFP. This is evident in the widening gap between total CDR and net CDR, as illustrated by the orange bars in Figure 2. This implies that although it is worthwhile to improve the CDR rate of the materials, a focus on improving the production CFP is also beneficial overall. The point at which further improvement in the CDR rate becomes less beneficial cannot be precisely identified from the current range as the trend remains mostly linear up to a CDR rate of 0.5 t CO_2/t rock. However, the analysis suggests diminishing returns as each incremental gain in CDR rate results in a smaller increase in net CDR because the associated production emissions also increase.

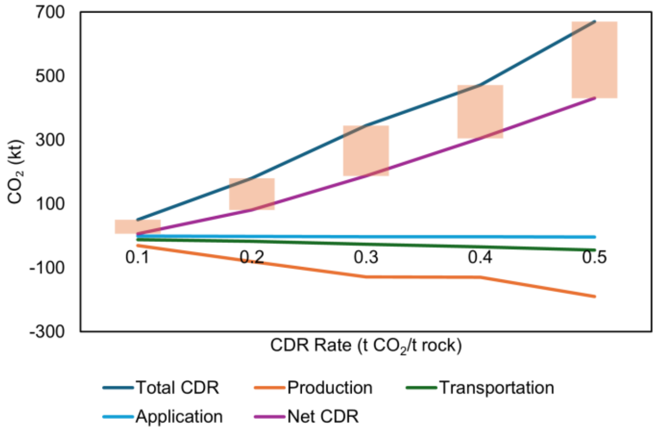

Figure 2: Effect of varying the CDR rate on the total and net CDR, and stepwise CO2 footprints.

6. Conclusions

This study proposes a novel mathematical model for maximizing the CDR of EW networks across multiple locations and periods considering the carbon footprints (CFP) from the crushing, transportation, and application of rocks into the soil. The model considers the lifespans of the sources, and the cumulative limits of the sinks, and ensures that once a source is activated, the flow rate meets the minimum and maximum flow rate limits of the source. A case study of three sources and seven sinks with varying lifespans and distances between each other demonstrates the capabilities of the model. The results show that the model was able to meet all the numerical and design constraints. A key insight from the results is that rock crushing is the dominant contributor in the CO_2 penalties, reducing the net CDR by 37.2 %. This highlights the importance of improving the energy intensity of the rock crushing process. The use of renewable energy at crushing sites can also significantly reduce the production CFP. Policies such as incentivizing low carbon production, carbon pricing, and clean energy transition can further improve the CDR efficiency of EW projects. However, these interventions may have tradeoffs such as high upfront capital and infrastructure costs. Policymakers must balance these tradeoffs against long-term climate benefits. Future work can focus on developing multi-objective and multi-level (i.e., game theoretic) variants of this formulation. Detailed operational decisions (e.g., rock or mineral types, transport modes) can also be integrated into model extensions.

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