CHEMICAL ENGINEERING TRANSACTIONS

VOL. 120, 2025

A publication of
AIDIC
The Italian Association
of Chemical Engineering
Online at www.cetjournal.it

Guest Editors: Bing Shen How, Viknesh Andiappan, Denny K.S. Ng, Hon Loong Lam, Petar S. Varbanov

ISBN 979-12-81206-21-2; **ISSN** 2283-9216

DOI: 10.3303/CET25120079

Application of Game Theory to Interplant Heat Exchanger Networks Involving Multiple Unequal Period Durations

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The integration of heat exchange networks across multiple plants, known as interplant heat integration, offers significant opportunities for energy savings and cost reduction. However, most existing studies have focused on plants with single operational periods, with only a few addressing multiperiod scenarios. In cases where these periods vary in duration, there is a need for methods to assess the economic benefits accruing to each plant participating in the integrated network. This study explores the application of game theory to evaluate the economic advantages of coalition formation among plants in a multiperiod interplant network, where integration may occur through an energy hub and operational periods are unequal. In the three co-located process plant case study considered in this paper, each plant has a 2-period operational profile consisting of daytime and nighttime with unequal durations across plants. The three plants can form a coalition involving interplant process heat exchange and heat exchange with hot and cold utilities at the energy hub. To determine the benefits that accrue to each plant for participating in the coalition, Shapley values are computed to establish the marginal contribution of each plant to the total annual cost of the integrated network. The result obtained for the total annual cost for the grand coalition comprising the three plants is $ 418,348 with marginal contributions for plants 1, 2 and 3 being $ 369,997, -$ 30,339, and $ 78,690.

1. Introduction

Heat integration in process plants can be used to reduce greenhouse gas emissions. Two key methods for process heat integration are pinch technology and mathematical programming. For complex heat integration problems involving multiple process plants, each operating over several periods, applying pinch technology can become tedious. In such cases, mathematical programming offers a more practical and efficient alternative. Although most Heat Exchanger Network (HEN) synthesis problems to which mathematical programming has been applied have mostly involved single periods of operation, real-life heat integration problems may involve multiple periods of operation. In such cases, stream supply and target temperatures and flow rates may vary periodically due to upstream process upsets, changes in feedstock quality or environmental conditions, plant start-ups and shutdowns, etc. Problems of this nature have been addressed by Aaltola (2002) using an average heat exchanger area approach and by Verheyen and Zhang (2006) using a maximum heat exchanger area approach. In some problems, variations in stream parameters across periods may be unequal, necessitating the use of a correct weighting approach in the objective function, as done by Isafiade (2023).

Heat integration problems involving multiple plants, known as Interplant Heat Integration (IPHI), have received less attention in the literature despite their potential benefits in providing opportunities for heat sharing among co-located process plants. Song et al. (2016) used an interplant shifted composite curve with an intermediate fluid to integrate heat among multiple plants, where each plant has a single period of operation for its process parameters. Song et al. (2017) used a pinch technology-based screening algorithm combined with the theoretical maximum heat recovery potential for heat integration among multiple plants. The energy hub approach for IPHI, where only interplant process heat exchange can occur at the energy hub, was adopted by Chang et al. (2018). Isafiade et al. (2022) modified the energy hub approach for IPHI problems involving multiple periods of operations by restricting intra-plant process heat exchange to the individual plant levels, while the energy hub is reserved for both inter-plant process heat exchange and heat exchange with utilities. Isafiade

Paper Received: 03 July 2025; Revised: 29 August 2025; Accepted: 21 October 2025
Please cite this article as: Isafiade A.J., 2025, Application of Game Theory to Interplant Heat Exchanger Networks Involving Multiple Unequal Period Durations, Chemical Engineering Transactions, 120, 469-474 DOI:10.3303/CET25120079

(2023) extended the work of Isafiade et al. (2022) for IPHI problems involving multiple periods of operations to the case where periodic durations across plants are unequal.

In IPHI problems, where individual plants form coalitions to share energy, game theory helps guide strategic decisions about coalition formation. The Shapley value (Shapley, 1953), a concept from game theory, can be used to determine each plant's marginal contribution to the coalition's total annual cost (TAC). Cheng et al. (2014) applied Game Theory and a four-step approach to design an interplant heat-integrated network that maximizes the financial benefit for each participating plant. Jin et al. (2017) utilized cooperative Game Theory and a risk-based Shapley value to allocate co-construction costs among multiple plants in a coalition. For IPHI problems involving plants with multiperiod process parameter profiles and unequal periodic durations, it is essential to ensure fair cost allocation, an area where Game Theory offers valuable tools. To the best of the author's knowledge, this aspect has not been addressed in existing literature. This paper advances the state of the art by applying Game Theory to determine equitable cost distribution in IPHI scenarios where both the process parameters and the durations of their variations differ across plants and periods.

2. Problem statement

The problem addressed in this paper involves a set of co-located process plants, P, where each plant contains a set of hot, H, and cold, C, process streams. Each stream is characterized by a supply temperature (T^s), a target temperature (T^t), and a heat capacity flow rate (*FCPH* for hot streams and *FCPC* for cold streams). The process parameters in each plant follow a multiperiod profile, represented by set T, with unequal durations across the different plants. Additional parameters include interplant layout data, such as distances between plants and between each plant and f as well as stream-specific heat transfer coefficients and cost-related data, including cost functions for heat exchangers and for hot and cold utilities available at the energy hub. The objective is to synthesize an interplant heat integration network that minimizes the TAC by optimally integrating the heat demands of the multiple co-located plants with each other and with the energy hub. This must be achieved while accounting for variations in process parameters and unequal period durations across plants and ensuring a fair allocation of both annual operating and annual capital costs among the coalition members.

3. Methodology

The methodology adopted and the case study investigated in this paper is based on the work of Isafiade (2023) with the superstructure for the multi-plant network shown in Figure 1. In the figure, a stream within any plant can exchange process heat with another stream from the same plant (intra-plant heat exchange). At the energy hub, it can also exchange heat with a stream from a different plant (inter-plant heat exchange) or with a utility. The dashed lines indicate that the streams originate from process units, enter the heat exchange network to transfer heat, and then proceed to other process units. Since the case study includes multiperiod profile with unequal period duration within each plant and across plants, Isafiade (2023) developed the mapping approach shown in Figure 2. In Figure 2, although each plant has daytime and nighttime periods, it's only in plant 1 that the duration of daytime period (0 – 12 h), represented by green arrow, is equal to that of nighttime (12 – 24 h), represented by orange arrow. For plants 2 and 3, the daytime period duration is less than nighttime. This necessitates the use of the Time Slice periodic mapping approach developed by Isafiade (2023) for IPHI with unequal period durations across plants. The objective function based on the maximum area approach used by Isafiade (2023) which is also used in this paper is shown in Eq(1). In this equation, AF (0.2 /y) represents annualization factor for heat exchangers and pipes, CF ($ 5,500) represents installation cost for heat exchangers, AC (700 $/m^2) represents cost per heat exchanger area, AE represents cost exponent for heat exchanger areas, binary variable $y_{p,i,j,k}$ indicates whether a match exists between streams i and j as intra-plant heat exchanger in plant p or as interplant heat exchanger at the energy hub. In Eq(1), the average interplant distance and distance between each plant and the energy hub is represented by $Dist_{i,j}$ (m), while $PC_{p,i}$ ($/m) and $PC_{p,j}$ ($/m) represent the per unit costs of pipes connecting each plant to the energy hub (same parameters used by Chang et al. (2018)), DOP_{ts} represents the duration of each periodic time slice ts, while the number of periodic time slices is represented by NOP. The unit costs of cold and hot utilities are represented by CUC ($/(kW·y)) and HUC ($/(kW·y)) while heat exchanger heat load is represented by $q_{p,I,j,k,ts}$ (kW) and area of the representative heat exchanger is represented as $A_{p,I,j,k}$ (m^2). To adequately account for the fractional contributions of quantity of utilities consumed per period per plant in Eq(1), Isafiade (2023) applied the weighting terms that are multiplied by annual operating costs of cold and hot utilities and summed over the time slices.

This paper builds on the work of Isafiade (2023) by regarding the 3-plant network as players in a coalition and then applying Shapley values to determine the economic benefits gained by each plant in participating in the coalition. For a detailed description of the Time Slice periodic mapping approach, the reader is referred to Isafiade (2023). For the case considered in this paper, the computation of the Shapley value of plant p is as shown in Eq(2). In the equation, $\emptyset_p$ denotes payoff for plant p in the game defined by the value function v, s represents the size of coalition S while N represents the set of all players and n the total number of players in N. Eq(1) is first applied to all possible coalitions to determine their TAC, after which the Shapley values are computed using Eq(2).

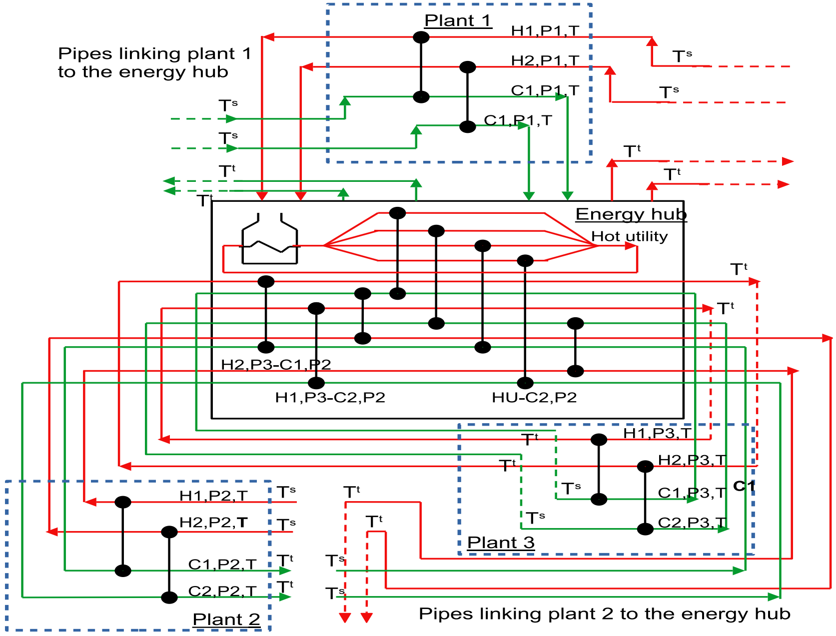


Figure 1: Interplant heat integration superstructure (Isafiade, 2023)

4. Case study

The case investigated is taken from Isafiade (2023). Tables 1 and 2 show the process hot stream data for plants 1, 2 and 3 for daytime and nighttime periods, while the equivalents for process cold streams are shown in Tables 3 and 4. For data used for hot and cold utilities and average distances between plants and the energy hub, the reader is referred to Isafiade et al. (2022).

$$
\begin{aligned}
\text{Min}\, TAC = AF\Bigg\{ & CF.\sum_{p\in P}\sum_{i\in H}\sum_{j\in C}\sum_{k\in K} y_{p,i,j,k} + AC\,.\sum_{p\in P}\sum_{i\in H}\sum_{j\in C}\sum_{k\in K}\left[A_{p,i,j,k}\right]^{AE} \\
& + \sum_{p\in P}\sum_{i\in H}\sum_{j\in C}\sum_{k\in K}\left(y_{p,i,j,k}\cdot\left(Dist_{i,j}\cdot PC_{p,i} + Dist_{i,j}\cdot PC_{p,j}\right)\right)\Bigg\} \\
& + \sum_{ts\in TS}\left(\frac{DOP_{ts}}{\sum_{ts=1}^{NOP} DOP_{ts}}\cdot\sum_{p\in P}\sum_{i\in H}\sum_{j\in C}\sum_{k\in K} CUC\cdot q_{p,i,j,k,ts}\right) \\
& + \sum_{ts\in TS}\left(\frac{DOP_{ts}}{\sum_{ts=1}^{NOP} DOP_{ts}}\cdot\sum_{p\in P}\sum_{i\in H}\sum_{j\in C}\sum_{k\in K} HUC\cdot q_{p,i,j,k,ts}\right)
\end{aligned}
\tag{1}
$$

$$
\emptyset_p^*(v) = \sum_{s\subseteq N\backslash p}\frac{s!\,(n-s-1)!}{n!}\left[v(S\cup p) - v(S)\right] \tag{2}
$$

Since the case study comprises 3 plants, there are 7 possible coalitions. Coalition 1 comprises only plant 1 (P1), coalition 2 comprises only plant 2 (P2) and coalition 3 comprises only plant 3 (P3). The other four possible coalitions, which are P1-P2, P1-P3, P2-P3 and P1-P2-P3 as well as their TACs and heat exchange profiles, are shown in Table 5. The Time Slice model of Isafiade (2023) was modified and solved for each of the possible coalitions. For the coalition involving only plant 1, the TAC is $414,764. This involves two intra-plant heat exchangers and three utility exchangers. The equivalent TACs for P2 only coalition and P3 only coalition are $51,039 and $106,027. The P2 only coalition involves two intra-plant heat exchangers and two utility heat exchangers at the hub. The P3 only coalition involves three intra-plant heat exchangers and one utility exchanger at the hub. Coalitions P1-P2, P2-P3 and P1-P2-P3 all involve inter-plant heat exchange at the energy hub as shown in Table 5.

Table 1: Process hot stream data for plants 1, 2 and 3 in daytime period (Isafiade et al., 2022)

Plants	Streams	T^s (K)	T^t (K)	FCPH (kg/s)	h (kW/(m^2·K))	Cp (kJ/(kg·K))	ρ (kg/m^3)	DOP_{ts} (h)
Plant 1	H1	660	370	2.0	1.0	3.2	780	12
	H2	590	320	1.6	1.0	3.5	820	
Plant 2	H1	553	333	2.0	1.6	3.7	790	10
	H2	493	310	2.5	1.6	3.3	810	
Plant 3	H1	520	390	2.0	2.1	3.2	782	8
	H2	473	370	1.0	4.1	3.0	760	

Table 2: Process hot stream data for plants 1, 2 and 3 in nighttime period (Isafiade et al., 2022)

Plants	Streams	T^s (K)	T^t (K)	FCPH (kg/s)	h (kW/(m^2·K))	Cp (kJ/(kg·K))	ρ (kg/m^3)	DOP_{ts} (h)
Plant 1	H1	640	350	2.1	1.0	3.2	780	12
	H2	600	310	1.8	1.0	3.5	820	
Plant 2	H1	543	333	1.9	1.6	3.7	790	14
	H2	483	300	2.4	1.6	3.3	810	
Plant 3	H1	522	395	2.1	2.1	3.2	782	16
	H2	479	380	1.8	4.1	3.0	760	

Table 3: Process cold stream data for plants 1, 2 and 3 in daytime period (Isafiade et al., 2022)

Plants	Streams	T^s (K)	T^t (K)	FCPC (kg/s)	h (kW/(m^2·K))	Cp (kJ/(kg·K))	ρ (kg/m^3)	DOP_{ts} (h)
Plant 1	C1	320	650	3.5	2.0	4.0	790	12
	C2	360	500	3.3	2.0	3.3	770	
Plant 2	C1	303	430	2.0	2.6	4.1	810	10
	C2	363	413	2.5	2.6	3.4	790	
Plant 3	C1	303	398	3.5	2.1	3.9	790	8
	C2	308	373	3.9	2.1	3.3	780	

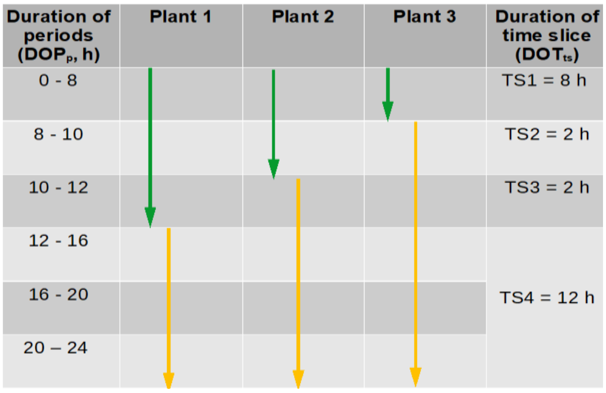
Table 4: Process cold stream data for plants 1, 2 and 3 in night-time period (Isafiade et al., 2022)

Plants	Streams	T^s (K)	T^t (K)	FCPC (kg/s)	h (kW/(m^2·K))	Cp (kJ/(kg·K))	ρ (kg/m^3)	DOP_{ts} (h)
Plant 1	C1	310	640	3.4	2.0	4.0	790	12
	C2	350	500	3.5	2.0	3.3	770	
Plant 2	C1	293	420	2.1	2.6	4.1	810	14
	C2	353	415	2.2	2.6	3.4	790	
Plant 3	C1	299	390	3.4	2.1	3.9	790	16
	C2	296	373	3.9	2.1	3.3	780	

For P1-P2 coalition, exchanger H1,P2-C2,P1 transfers process heat at the hub between hot stream 1 in plant 2 and cold stream 2 in plant 1. This heat exchange takes place in time slice TS3. In this time slice, plant 1 is in the daytime period while plant 2 is in the nighttime period. For the P2-P3 coalition, exchanger H1,P2-C1,P3 is involved in interplant heat exchange at the energy hub. For the grand coalition involving P1-P2-P3, there are three exchangers involved in interplant heat exchange at the energy hub. The marginal contributions of each plant to the TAC of the grand coalition are: P1 = $ 369,997, P2 = -$ 30,339, P3 = $ 78,690. This implies that by entering the grand coalition, plant 1 saves 44,767 $/y while plant 3 saves 27,337 $/y. On the other hand, plant 2 will receive 30,339 $/y to join the coalition, with 25,018 $/y contributed by plant 1 and 5,321 $/y by plant 3. From Table 5, the 2 two-plant coalitions, i.e., P1-P2 and P2-P3, having interplant heat exchange at the hub involves plant 2 while for the grand coalition, two of the three interplant heat exchange at the hub include plant 2.

Table 5: TAC, heat exchanger area and heat load distribution among the seven possible coalitions

Coalitions		Exchanger	Area (m^2)	Heat load (kW)			
TAC ($/y)				TS1	TS2	TS3	TS4
				(0 - 8 h)	(8 – 10 h)	(10 – 12 h)	(12 – 24 h)
P1-P2	Intra-plant heat exchange						
335,634		H1,P1-C1,P1	17	1,856	1,856	1,856	1,856
		H2,P1-C1,P1	72.4	1,486.4	1,486.4	1,486.4	1,486.4
		H1,P2-C1,P2	23.8	252	252	347.9	347.9
		H2,P2-C1,P2	29	535.7	535.7	353	353
		H2,P2-C1,P2	45.6	253.8	253.8	392.5	392.5
		H2,P2-C2,P2	11.4	425	425	463.8	463.8
	Interplant heat exchange at the hub						
		H1,P2-C2,P1	54.6	1,376	1,376	1,376	1,128.4
	Heat exchange with utilities at the energy hub						
		HU1-C1,P1	28.1	1,277.7	1,277.7	1,277.7	1,277.7
		H2,P1-CU1	1.2	25.7	25.7	25.7	25.7
		HU2-C2,P1	3.8	148.6	148.6	148.6	396.2
		H2,P2-CU2	13	295.3	295.3	240	240
P1-P3	Intra-plant heat exchange						
498,703		H1,P1-C1,P1	27	1,856	1,856	1,856	1,948.8
		H2,P1-C1,P1	87.8	1,496	1,496	1,496	1,762.5
		H1,P3-C1.P3	4.1	497.5	397.1	397.1	397.1
		H1,P3-C2,P3	3.9	334.5	456.4	456.4	456.4
		H2,P3-C2,P3	7.9	309	534.6	534.6	534.6
	Heat exchange with utilities at the energy hub						
		HU1-C1,P1	27.95	1,268	1,268	1,268	776.7
		H2,P1-CU3	1.1	16.04	16.04	16.04	64.5
		HU2-C2,P1	16	1,524.6	1,524.6	1,524.6	1,732.5
		HU2-C1,P3	5.2	799.3	809.6	809.6	809.6
P2-P3	Intra-plant heat exchange						
61,756		H1,P2-C1,P2	39	828.8	819	666.7	666.7
		H1,P2-C1,P3	5.9	799.3	809.6	809.6	809.6
		H2,P2-C1,P2	6	212.7	223	426.8	426.8
		H2,P2-C2,P2	12.4	425	425	463.8	463.8
		H1,P3-C1,P3	4.1	497.5	497.5	397.1	397.1
		H1,P3-C2,P3	3.4	334.5	334.5	456.4	456.4
		H2,P3-C2,P3	10.1	309	309	534.6	534.6
	Heat exchange with utilities at the energy hub						
		H2,P2-CU2	14.6	872.1	861.7	558.8	558.8
P1-P2-P3	Intra-plant heat exchange						
418,348		H1,P1-C1,P1	27	1,856	1,856	1,856	1,948.8
		H2,P1-C1,P1	87.8	1,496	1,496	1,496	1,762.5
		H1,P2-C1,P2	34.7	540.9	385	348.8	140.9
		H2,P2-C1,P2	43.6	213	303.1	242.7	242.7
		H1,P3-C2,P3	5.1	334.5	456.4	456.4	4564
		H2,P3-C2,P3	7.9	309	534.6	534.6	534.6
	Interplant heat exchange at the hub						
		H1,P2-C2,P1	82.3	1,027.1	1,127.6	1,127.6	1,335.5
		H2,P2-C1,P3	20.2	1,296.8	1,206.7	1,206.7	1,206.7
		H1,P3-C2,P1	11.6	497.5	397.1	397.1	397.1
	Heat exchange with utilities at the energy hub						
		HU1-C1,P1	27.95	1,268	1,268	1,268	776.7
		H2,P1-CU3	1.1	16	16	16	64.5
		HU2-C1,P2	5.5	287.5	353.3	502	710
		HU2-C2,P2	3.7	425	425	463.8	463.8
		H1,P2-CU1	2.3	60	115.5	-	-



Duration of periods (DOP_p, h)	Plant 1	Plant 2	Plant 3	Duration of time slice (DOT_{ts})
0 - 8				TS1 = 8 h
8 - 10				TS2 = 2 h
10 - 12				TS3 = 2 h
12 - 16				TS4 = 12 h
16 - 20				
20 – 24				

Figure 2: Duration of time slices for case study investigated (Isafiade, 2023).

5. Conclusions

This paper has demonstrated the integration of Game theory, through Shapley values computation, with IPHI problems involving multiple periods of operations with unequal durations. The method adopted builds on the energy hub approach where both heat exchange with utilities and interplant process heat exchange can take place at the energy hub. The results obtained show that heat sharing among multiple plants through interplant heat exchange at the energy hub has the potential to reduce the TAC that a plant would incur if it were not involved in a coalition with other plants. Future work will include detailed calculations of costs of pumping between plants and the energy hub and detailed heat losses along pipes transporting fluids. Also, the marginal contributions of each plant to environmental impacts of the grand coalition will be investigated simultaneously with the marginal contributions to the TAC through multi-objective optimisation.

Acknowledgments

The financial support of the University of Cape Town is acknowledged.

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