

# Thermal Effectiveness Modelling for Parabolic Trough Solar Collector Systems: Incorporating Load and Seasonal Variability

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The thermal effectiveness approach, widely used for steady-state heat exchangers, has limitations when applied directly to solar thermal plants, where operating conditions vary continuously. In particular, the requirement to calculate instantaneous effectiveness reduces its usefulness in design tasks. This study proposes a simplified expression that relates thermal effectiveness ( $\epsilon$ ) to the number of transfer units (Ntu) and the process thermal load ( $Q_p$ ) for solar thermal plants using parabolic trough collectors. The expression was derived from a dataset of a total of 168 optimal plant designs—previously obtained by minimising total cost under varying thermal loads, inlet and outlet temperatures, and seasonal conditions (summer and winter). The resulting correlation reproduces the expected  $\epsilon$ -Ntu behaviour observed in conventional heat exchangers, while also accounting for the influence of thermal load and seasonal effects. In the case study, plants operating at lower loads achieved higher effectiveness, with seasonal differences being significant: maximum values reached about 0.60 in summer and 0.50 in winter. The proposed relationship can support preliminary sizing of solar thermal networks, providing a fast alternative to detailed simulations, with future work aimed at extending its validation to different climatic and operational contexts.

## 1. Introduction

The urgent need to transition from fossil fuels to clean energy in industrial processes has positioned solar thermal systems as a promising and sustainable alternative. Among available technologies, the parabolic trough collector (PTC) remains the most commercially mature, accounting for nearly 90 % of the global collector solar thermal capacity in 2022 (Adib and Zervos, 2023). Its ability to operate across a wide temperature range (50 °C to 400 °C) offers considerable flexibility for medium- and high-temperature applications (Kalogirou, 2019).

Recent studies have explored various configurations and optimisation strategies to improve the techno-economic performance of PTC-based systems. Rodríguez et al. (2024) conducted a comparative analysis of PTC and linear Fresnel collectors in the Spanish dairy industry, highlighting the importance of load profile, plant size, and location in determining the cost-effectiveness of solar thermal energy. Gharzi et al. (2023) proposed a hybrid PTC-thermoelectric generator system, achieving a 15.75 % efficiency increase through combined thermal and electrical generation strategies. Similarly, Erikgenoğlu et al. (2025) applied multi-criteria optimisation methods to a two-stage solar power plant, showing the viability of cost-optimised systems in low-irradiance regions. Other works, such as that of Wang et al. (2024), have focused on hybrid systems integrating solar input with waste heat sources, optimized using artificial neural networks and evolutionary algorithms. Additionally, Buonomano et al. (2023) demonstrated the potential of dynamic simulation tools in real-world solar networks, achieving significant reductions in primary energy consumption and CO<sub>2</sub> emissions.

The application of artificial intelligence and machine learning has also gained traction. Ghalati et al. (2024) employed artificial neural networks to study the effect of collector tilt and plate type on thermal performance, finding that porous plates yielded significantly higher thermal power and efficiency, especially when modeled using radial basis function networks. Collectively, these studies reflect a growing interest in improving the design, control, and integration of solar thermal systems. However, they generally rely on detailed simulations or system-

level optimisation, and do not address the use of simplified, design-oriented performance models such as those based on thermal effectiveness.

The Ntu–effectiveness method, built on the relationship between number of heat transfer units (Ntu) and thermal effectiveness ( $\epsilon$ ), has long been a standard tool in the design of conventional heat exchangers (Kays and London, 1984). Yet, its application to solar thermal networks has received little attention, likely due to the unsteady and time-dependent nature of solar input. The direct use of instantaneous effectiveness values under dynamic conditions is of limited practical value, particularly for design purposes, where aggregated performance measures are more relevant. To the best of our knowledge, there are no published studies that adapt the Ntu–effectiveness methodology to solar thermal systems in a way that supports design decision-making. This represents a notable gap in the open literature, especially given the method’s simplicity and proven value in conventional thermal engineering. The present study addresses this gap by proposing a generalised thermal effectiveness expression for solar thermal networks using parabolic trough collectors. The expression is based on a previously developed dataset of a total of 168 cost-optimised system designs under varying operating conditions, including seasonal variations. By correlating effectiveness with Ntu through multivariate regression, the model offers a practical and scalable tool for system design, laying the foundation for its integration in future solar field sizing methodologies.

## 2. Methodology

The methodology used in this study is based on a dataset of  $\epsilon$ –Ntu values derived from a previous optimisation study by Lizárraga-Morazán and Picón-Núñez (2024). That work focused on the design of flexible solar thermal networks using parabolic trough collector (PTC) technology. The optimisation was formulated as a Mixed-Integer Nonlinear Programming (MINLP) problem and solved using a one-dimensional transient thermo-hydraulic model. A Particle Swarm Optimization (PSO) algorithm was applied to identify cost-optimal designs under varying conditions. Specifically, the thermal load of the process  $Q_p$ , the required outlet temperature of the heat transfer fluid  $T_{obj}$ , and the inlet temperature  $T_{in}$  were varied across broad ranges, representing typical operating conditions for both summer and winter. Table 1 summarizes the range of values used for these input variables.

*Table 1: Range of operating variables*

Variables	Value
$Q_p$ (GWh·y <sup>-1</sup> )	1.056 – 39.6
$T_{obj}$ (°C)	70 - 400
$T_{in}$ (°C)	(0.7-0.9)· $T_{obj}$

A structured sampling procedure was applied across the entire range of input variables, and the optimization problem was solved for each resulting combination. This process produced 168 data points for each season, with each data point corresponding to an optimal system configuration. The outputs included design variables such as collector area, network layout, and PTC dimensions, along with operating parameters like inlet temperature, mass flow rate, and the type of heat transfer fluid. For the thermal load  $Q_p$ , eight discrete values were considered: 1.056, 4.224, 7.392, 10.56, 13.2, 19.8, 33, and 39.6 GWh·y<sup>-1</sup>. The thermal effectiveness ( $\epsilon$ ) for each case was calculated using the following expression:

$$\epsilon = \frac{\bar{Q}}{Q_{max}} = \frac{(\bar{T}_{out} - T_{in})}{(T_{out,max} - T_{amb,min})} \quad (1)$$

Here,  $\bar{Q}$  and  $Q_{max}$  refer to the average and maximum daily thermal energy collected by the solar thermal network. The temperatures  $\bar{T}_{out}$ ,  $T_{in}$ ,  $T_{out,max}$  and  $T_{amb,min}$  correspond to the average daily outlet temperature of the heat transfer fluid (HTF), its inlet temperature, the maximum daily outlet temperature, and the minimum ambient temperature recorded during the day, respectively. Notably, the expression for thermal effectiveness incorporates  $T_{in}$ , one of the key variables in the optimization process. The outlet temperature profile of the HTF over the course of the day was obtained from the PSO-based optimization of each solar network design. Ambient temperature data between 9:00 and 18:00 h were collected from the meteorological station at the University of Guanajuato, Mexico, for representative winter and summer days. The number of transfer units (Ntu) was calculated using the following expression:

$$Ntu = \frac{N_r A h_f}{\dot{m}_f c_{pf}} \quad (2)$$

In this expression,  $Ntu$  represents the number of heat transfer units in the network, and  $N_r$  is the number of collector rows. The parameters  $A$  and  $\dot{m}_f$  denote the heat transfer area per row and the mass flow rate of the heat transfer fluid (HTF), respectively. The terms  $h_f$  and  $C_{p,f}$  refer to the average heat transfer coefficient and the specific heat capacity of the HTF. Once the  $\varepsilon$ - $Ntu$  data were calculated, several functional forms were tested to identify the best statistical fit for both seasonal datasets. To support the analysis, two-dimensional and three-dimensional plots were generated to visualize the relationships between effectiveness,  $Ntu$ , and the key influencing variables.

### 3. Results

The  $\varepsilon$ - $Ntu$  data were analyzed using different regression models to identify the most suitable mathematical expression. Several functional forms were tested, each incorporating different combinations of variables. The model that showed the best statistical fit included both the number of transfer units ( $Ntu$ ) and the process thermal load  $Q_p$ . The resulting equation is shown below.

$$\varepsilon_{estimated} = b_1 Ntu / (1 + b_1 Ntu) \quad (3)$$

$$b_1 = b_2 Q_p^{b_3} \quad (4)$$

Table 2 presents the statistical performance of the fitted model given by Eq(3). Figure 1 illustrates how the parameter  $b_1$  varies with the process thermal load ( $Q_p$ ), while Figure 2 compares the calculated effectiveness values with those predicted using Eq(3).

Table 2: Results of the regression analysis

Season	$b_2$	$b_3$	$R^2$
Winter	0.00071991	-0.855392	0.997
Summer	0.00103116	-0.793455	0.987

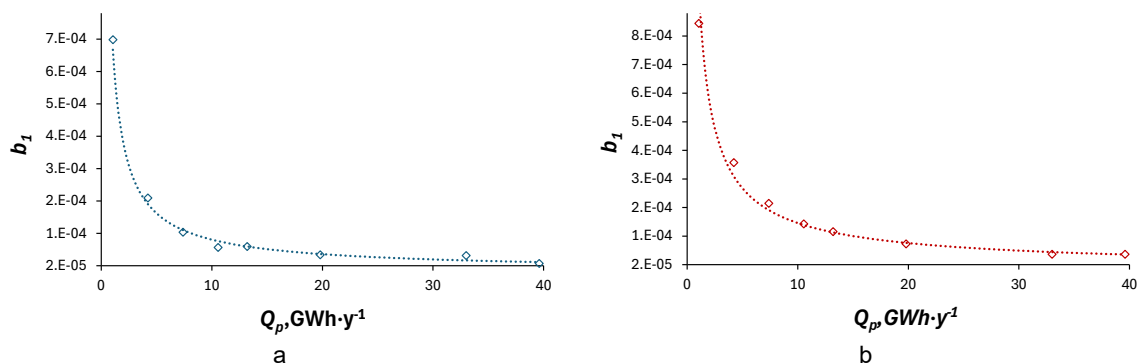


Figure 1: Relationship between  $b_1$  and  $Q_p$ : (a) winter and (b) summer

Figure 3 presents the effectiveness profiles for the thermal load levels analysed in both seasons. Overall, for a given  $Ntu$ , thermal effectiveness is higher in summer than in winter, with the difference becoming more pronounced as the thermal load increases. For instance, at  $Ntu=10$  and  $Q_{p5} = 13.2 \text{ GWh}\cdot\text{y}^{-1}$ , the network effectiveness is 0.57 in summer and 0.33 in winter, indicating greater thermal benefits for the same collector area. This difference is primarily due to higher solar irradiance in summer, which enhances heat absorption and increases the average temperature of the heat transfer fluid. At lower heat loads, such as  $Q_{p1} = 1.056 \text{ GWh}\cdot\text{y}^{-1}$ ,  $Ntu$  values remain below 3, leading to relatively high and similar effectiveness in both seasons.

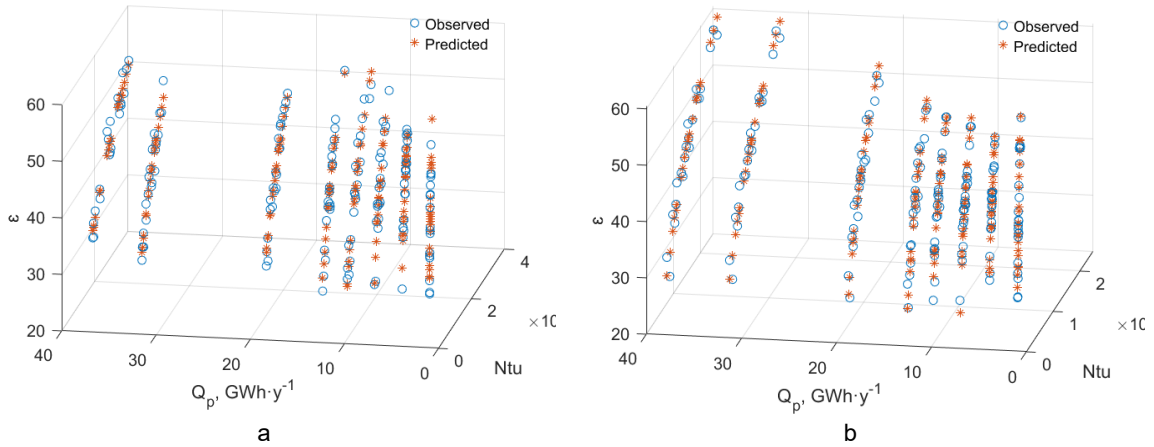


Figure 2: Fit of the  $\epsilon - Q_p$  regression model: (a) winter and (b) summer

As both the thermal load and the required collector area increase, the operational differences between summer and winter become more evident. A larger collector area also translates into greater space requirements. For example, at  $Q_{p7} = 39.6 \text{ GWh}\cdot\text{y}^{-1}$  and a summer thermal effectiveness of 60 %, the network requires  $Ntu=25$ . In winter, achieving the same effectiveness demands  $Ntu=60$ , which corresponds to an area 2.4 times larger. Figure 4 shows 2D  $\epsilon - Ntu$  plots for the selected thermal load levels. The results indicate that as thermal load increases, thermal effectiveness decreases, making it necessary to expand the collector area to maintain performance.

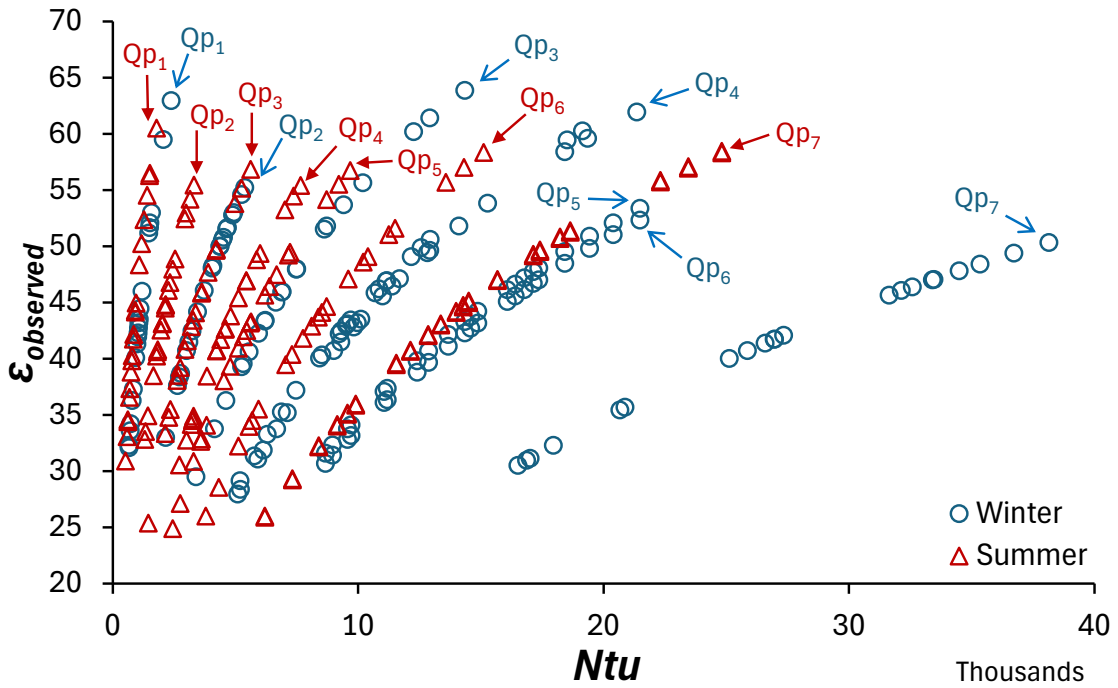


Figure 3: Comparison of thermal effectiveness ( $\epsilon$ ) profiles for the analysed seasons

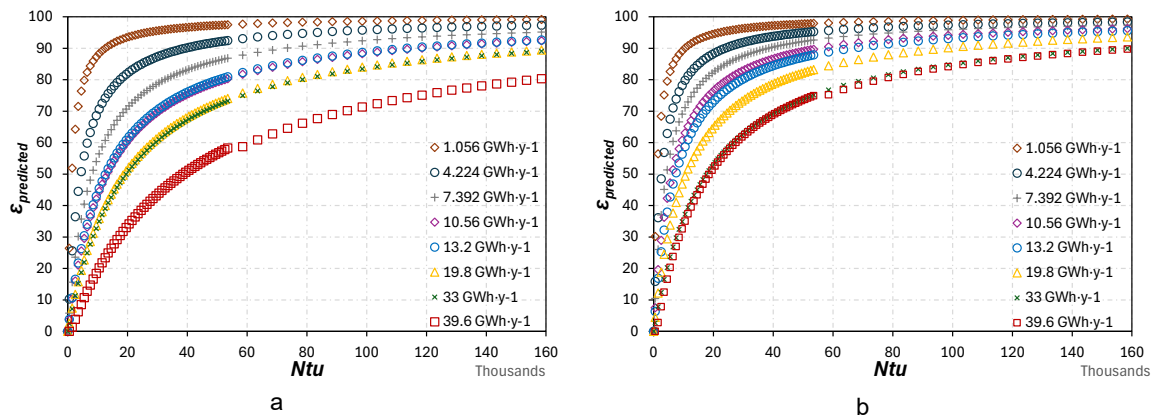


Figure 4:  $\varepsilon$  -  $Ntu$  relationship: (a) Winter, and (b) Summer

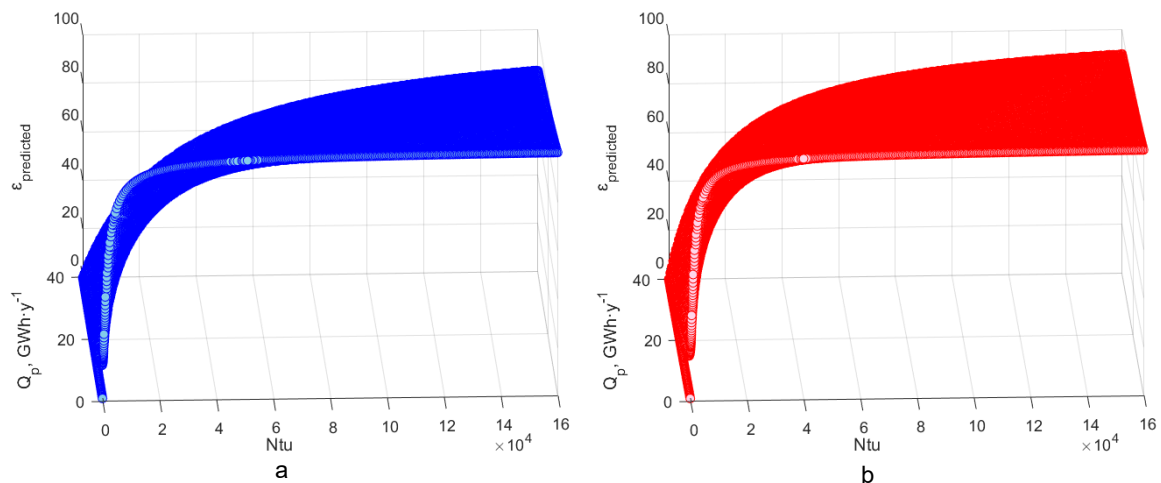


Figure 5: Three-dimensional  $\varepsilon$  -  $Ntu$  -  $Q_p$  relationship: (a) winter and (b) summer

Finally, Figure 5 presents 3D plots of the  $\varepsilon$ - $Ntu$ - $Q_p$  relationships. The overall trends are similar for both seasons, showing greater thermal benefits in summer and higher thermal effectiveness at lower thermal loads. Thermal effectiveness is a design parameter that indicates how closely the average temperature rise ( $\bar{T}_{out} - T_{in}$ ) approaches the maximum possible temperature rise in the network ( $T_{out,max} - T_{amb,min}$ ). A key limitation of this study is that the maximum effectiveness is influenced by the site's solar radiation, meaning the derived expressions apply only to the location where the system was evaluated. For design purposes, the achievable maximum effectiveness values are 0.6 in summer and 0.5 in winter.

#### 4. Conclusions

This study developed a simplified expression relating thermal effectiveness ( $\varepsilon$ ) to the number of transfer units ( $Ntu$ ) and the process thermal load ( $Q_p$ ) for solar thermal plants using parabolic trough collectors. The analysis shows that the variation of  $\varepsilon$  with  $Ntu$  follows the trends observed in conventional heat exchangers. For a given collector area, plants operating at lower thermal loads achieve higher effectiveness, while higher loads reduce performance. Seasonal effects are significant: in the case study, maximum effectiveness values reached approximately 0.60 in summer and 0.50 in winter. In winter, the impact of  $Q_p$  is more pronounced, and achieving higher  $\varepsilon$  requires proportionally greater increases in  $Ntu$ , especially at higher loads. The proposed relationships can support the preliminary sizing of solar thermal networks, offering a rapid alternative to detailed simulation during early design stages. However, their applicability is subject to the operational and climatic conditions under which they were derived. Future work will focus on validating the model across different locations, load profiles, and collector configurations.

### Nomenclature

$A$ – heat transfer row area, $m^2$	$T_{in}$ – HTF inlet temperature, $^{\circ}C$
$C_{pf}$ – HTF average specific heat capacity, $J/kg \cdot ^{\circ}C$	$T_{obj}$ – HTF target outlet temperature, $^{\circ}C$
$h_f$ – HTF average heat transfer coefficient, $W/m^2 \cdot ^{\circ}C$	$T_{out}$ – HTF outlet temperature, $^{\circ}C$
$\dot{m}_f$ – HTF mass flow rate, $Kg/s$	$\bar{Q}$ – average heat harvested, $GWh \cdot y^{-1}$
$N_r$ – number of collector rows, -	$Q_{max}$ – maximum heat harvested, $GWh \cdot y^{-1}$
$N_{tu}$ – number of transfer units, -	$Q_p$ – process thermal load, $GWh \cdot y^{-1}$
$T_{amb}$ – ambient temperature, $^{\circ}C$	$\varepsilon$ – effectiveness, -

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### References

- Adib R., Zervos A., 2023, Renewables 2023 global status report. International Atomic Energy Agency, <inis.iaea.org/records/b9y79-qmm30>, accessed 12.12.2024.
- Buonomano A., Forzano C., Palombo A., Russo G., 2023, Solar-assisted district heating networks: Development and experimental validation of a novel simulation tool for the energy optimization. *Energy Conversion and Management*, 288, 117133.
- Erikgenoğlu D.K., Arslan O., Arslan A.E., 2025, multi-criteria decision-making optimization of a two-staged solar power plant for low radiation zone through the social decision units. *Energy Conversion and Management*, 323, 119263.
- Ghalati A., Maleki A., Besharati S., Zarein M., 2024, Prediction and Optimization of Performance Parameters of Solar Collectors with Flat and Porous Plates using ANN and RSM: Case Study of Shahrekord, Iran. *Case Studies in Thermal Engineering*, 60, 104719.
- Gharzi M., Kermani A.M., Tash Shamsabadi H., 2023, Experimental investigation of a parabolic trough collector-thermoelectric generator (PTC-TEG) hybrid solar system with a pressurized heat transfer fluid. *Renewable Energy*, 202, 270–279.
- Kalogirou S., 2019, *Solar energy engineering: processes and systems*. Academic Press, London, UK.
- Kays W.M., London A.L., 1984, *Compact heat exchangers*. McGraw-Hill, New York, USA.
- Lizárraga-Morazán J.R., Picón-Núñez M., 2024, Optimal design of parabolic through solar collector networks: A design approach for year-round operation. *Energy*, 306, 132434.
- Rodríguez Rodrigo R., Díaz Martín R., Baranda Fernández M., Román Gallego J.Á., Mayo del Río C., 2024, Technical and economic study of solar energy concentration technologies (linear Fresnel and parabolic trough collectors) to generate process heat at medium temperature for the dairy industry of Spain. *Solar Energy*, 271, 112420.
- Wang L., Yang J., Qu B., Pang C., 2024, Multi-Objective Optimization of an Organic Rankine Cycle (ORC) for a Hybrid Solar–Waste Energy Plant. *Energies*, 17, 1810.