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Enhancing mathematical modelling education at agricultural universities: A comparative study of dynamic vector diagrams using GeoGebra

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Abstract. This research investigates the effectiveness of visualisation techniques in teaching mathematical modelling fundamentals to agricultural university students. We examine the hypothesis that dynamic vector diagrams representing mechanical motion characteristics (velocity, acceleration, and force) enhance student learning outcomes. The study compares three instructional approaches: using Excel spreadsheets, utilising GeoGebra dynamic geometry software, and employing both tools simultaneously. Our methodology involved 167 engineering students divided into three homogeneous groups, each completing identical modelling tasks concerning projectile motion under various conditions. Results demonstrate that students who simultaneously employed Excel and GeoGebra with dynamic vector diagram visualisation achieved significantly higher academic performance (mean score 78.25) compared to those using either Excel (73.36) or GeoGebra (73.85) exclusively. Statistical analysis through ANOVA confirms these differences are significant (p=0.010613). We observed that Excel users demonstrated stronger quantitative analytical skills, while GeoGebra users excelled in qualitative assessment tasks. This research extends previous findings on visualisation in mathematical education and provides practical insights for enhancing STEM education in agricultural universities through appropriate technology integration.

Keywords: mathematical modelling education, agricultural engineering education, visualisation techniques, dynamic vector diagrams, GeoGebra, Excel, educational technology, projectile motion modelling, STEM pedagogy, comparative instructional methods

1. Introduction

Contemporary education is witnessing profound technological transformations that reshape traditional teaching methodologies. One significant shift involves the transition from predominantly verbal communication channels in learning towards visually-oriented approaches. While researchers continue to investigate this phenomenon, today's students have already embraced visual learning methods.

The pedagogical approach this study examines aligns with the ancient wisdom: "I hear and forget. I see and remember. I do and understand" [13]. We investigate the impact of visualisation techniques on teaching the fundamentals of mathematical modelling (BMM) using GeoGebra, a dynamic geometry system [9]. This research builds upon established methodological principles for integrating information and communication technologies (ICT) into university-level mathematical education [1, 12].

Teaching mathematical modelling constitutes an essential component in training

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modern agricultural engineers, providing them with analytical skills necessary for addressing complex agricultural production challenges. The practical value of this training stems from its effectiveness as an educational strategy in contemporary contexts [2]. This study directly extends our earlier research, which demonstrated that visualising dynamic characteristics of mechanical processes using GeoGebra improved student learning outcomes in mathematical modelling courses.

Our previous experiment conducted in 2018 revealed promising results when students simultaneously used Excel and GeoGebra, achieving an average score of 77.4 compared to students using only Excel (73.4) or only GeoGebra (73.7). This suggested that visualisation of modelling results creates additional conditions for improving students' knowledge, taking into account the specifics of their professional training.

The main aim of this study is to further test the hypothesis that visual representation enhances educational achievements of agricultural university students when studying mathematical modelling fundamentals. Specifically, we investigate the effectiveness of dynamic vector diagrams as a visualisation method for depicting mechanical movement characteristics. We describe the methodology employed, compare the current research results with previous findings, and evaluate student educational achievements using different software tools.

2. Experiment description

2.1. General design

This study presents results from a pedagogical experiment conducted between September-November 2019 and September-November 2020. The experiment involved 167 students from the Faculty of Engineering and Technology at Poltava State Agrarian Academy. This sample size enabled us to establish significant differences between group means at the level of 1 point using ANOVA, with a significance level of 0.05, three comparison groups, and a statistical power of 80%.

At the experiment's commencement, students were categorised into three groups (E, G, and EG) using a methodology that ensured initial homogeneity based on academic performance criteria [5]. Group E utilised Excel spreadsheets during mathematical modelling instruction. Group G employed GeoGebra software. Group EG simultaneously used both Excel and GeoGebra. Throughout the study, students could migrate between experimental groups while remaining in their academic cohorts, with final group composition documented at the experiment's conclusion.

All groups received instruction following a single curriculum using a differentiated approach to teaching as described in our previous research [2]. The training duration was identical across all groups. Each group completed the same learning task directly relevant to agricultural engineering training: modelling the movement of a spherical object projected at an angle to the horizon, accounting for air resistance, gravitational interaction, electrostatic interaction, and magnetic interaction.

We implemented a differentiated approach through the gradual construction of mathematical models (MM) with increasing complexity levels, their computer implementation, and analysis through numerical calculations. The purpose was to familiarise students with mathematical model creation practices, computational algorithm development, computer experiment execution, and result interpretation.

Student learning outcomes were assessed on a 100-point scale based on performance in individual assignments [2].

2.2. Mathematical model development

Students were expected to employ competencies acquired from previous coursework in "Higher Mathematics", "Physics", "Theoretical Mechanics", and "Applied Mathematics" when constructing and analysing mathematical models.

Following an algorithm detailed in previous publications [2, 4], students developed three mathematical models of increasing complexity (MM I, MM II, and MM III). These models were constructed by analysing the vector equations of resultant forces acting on a projectile under different conditions.

MM I describes the motion of an object projected at an angle to the horizon without air resistance. Based on the mechanical interpretation of derivatives, all models included two first-order ordinary differential equations:

$$\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y. \tag{1}$$

When an object projected at an angle to the horizon is affected only by gravitational force $\overline{F}_T = (F_1^x, F_1^y)$ where $F_1^x = 0$ and $F_1^y = -mg$, two additional equations are added to MM I:

$$\frac{dv_x}{dt} = 0, \quad \frac{dv_y}{dt} = -g. \tag{2}$$

MM I, therefore, comprises four differential equations (1) and (2), representing the simplest model describing projectile motion in a gravitational field without air resistance. This corresponds to the first (lowest) difficulty level in mathematical modelling instruction.

MM II introduces medium resistance, representing the second complexity level. Here, the equivalent force acting on the object is $\overline{F}_2 = \overline{F}_T + \overline{F}_r$, where $\overline{F}_r = -k_2 v^2 \frac{\overline{v}}{v}$ represents air resistance force, $v = \sqrt{v_x^2 + v_y^2}$ is object velocity, and k_2 is the medium resistance coefficient. From this relationship: $F_2^x = -k_2 v_x v$, $F_2^y = -g - k_2 v_y v$. Therefore, in addition to equation (1), MM II includes:

$$\frac{dv_x}{dt} = -\frac{k_2}{m} v_x \sqrt{v_x^2 + v_y^2}, \quad \frac{dv_y}{dt} = -g - \frac{k_2}{m} v_y \sqrt{v_x^2 + v_y^2}. \tag{3}$$

MM III represents the highest complexity level, incorporating the Magnus effect. The equivalent force becomes $\overline{F_3} = \overline{F}_T + \overline{F}_r + \overline{F}_M$, where $\overline{F}_M = -k_3 v^2 (\overline{\omega} \times \overline{v})$ and k_3 is the Magnus effect coefficient. This yields: $F_3^x = -k_2 v_x v \pm k_3 v_y v$, $F_3^y = -g - k_2 v_y v \mp k_3 v_x v$ (the positive sign applies for clockwise rotation, the negative sign for counterclockwise rotation). In addition to equation (1), MM III includes:

$$\frac{dv_x}{dt} = \left(-\frac{k_2}{m} v_x \pm \frac{k_3}{m} v_y\right) \sqrt{v_x^2 + v_y^2}, \quad \frac{dv_y}{dt} = -g + \left(-\frac{k_2}{m} v_y \mp \frac{k_3}{m} v_x\right) \sqrt{v_x^2 + v_y^2}. \tag{4}$$

In these mathematical models, we employ the following notation:

- $x = x(t)$, $y = y(t)$ – coordinates of the object's centre at time t (m)
- $v_x = v_x(t)$, $v_y = v_y(t)$ – object velocity projections on coordinate axes at time t (m/s)
- g – gravitational acceleration (m/s^2)
- m – object mass ($m \neq 0$) (kg)
- $k_2 = \frac{1}{2} C_D \rho S$ – medium resistance factor (kg/m)
- $k_3 = \frac{1}{2} C_L \rho S$ – Magnus force factor (kg/m)
- C_D – drag coefficient, dependent on object shape and medium characteristics (dimensionless; for a sphere in air, $C_D = 0.47$)
- C_L – Magnus effect coefficient, dependent on object shape, surface quality, and medium properties (dimensionless; for a sphere, $0.1 \leq C_L \leq 0.6$)
- ρ – medium density (for air, $\rho = 1.213$ kg/m^3)
- S – object's cross-sectional area perpendicular to motion direction (m^2); for a spherical object, $S = \pi r^2$, where r is the sphere's radius (m)

2.3. Implementation in Excel (Group E)

We present this section in condensed form, with detailed methodological descriptions available in our previous publication [4].

The computational scheme for implementing MM I (equations 1 and 2) in Excel is derived directly from its analytical solution [8, 11]:

$$v_{xi} = v_{0x}, \quad v_{yi} = v_{0y} - gt_i, \quad x_i = x_0 + v_{0x}t_i, \quad y_i = y_0 + v_{0y}t_i - \frac{gt_i^2}{2}, \quad i = \overline{0,\ n} \tag{5}$$

where:

- x_0, y_0 – initial coordinates of the object's centre (m)
- v_0 – initial object velocity (m/s)
- α_0 – initial projection angle to the horizon (radians)
- $v_{0x} = v_0 \cos\alpha_0$, $v_{0y} = v_0 \sin\alpha_0$ – initial velocity projections
- $t_i = i \cdot \Delta t, t_i \in [0,\ t_M], i = \overline{0,\ n}$ – observation time points
- i – position observation index
- n – number of observation points
- $\Delta t = t_M/n$ – time interval between observations (s)
- t_M – simulation duration (s)
- $x_i = x(t_i)$, $y_i = y(t_i)$ – object centre coordinates at time t_i
- $v_{xi} = v_x(t_i)$, $v_{yi} = v_y(t_i)$ – velocity projections at time t_i
- $v_i = \sqrt{v_{xi}^2 + v_{yi}^2}$ – object velocity magnitude at time t_i

Models MM II (equations 1 and 3) and MM III (equations 1 and 4), unlike MM I, lack analytical solutions. Therefore, we implemented computational schemes based on numerical methods.

The computational scheme for MM II using the Euler method for first-order ODE systems is [10]:

$$\begin{aligned} v_{x\ i+1} &= v_{xi} - \frac{k_2}{m}v_{xi}v_i\Delta t, \\ v_{y\ i+1} &= v_{yi} - \left(g + \frac{k_2}{m}v_{yi}v_i\right)\Delta t, \\ x_{i+1} &= x_i + v_{xi}\Delta t, \\ y_{i+1} &= y_i + v_{yi}\Delta t. \end{aligned} \tag{6}$$

For MM III, the computational scheme is similar, but the first two equations in (6) are replaced by:

$$\begin{aligned} v_{x\ i+1} &= v_{xi} - \left(\frac{k_2}{m}v_{xi}v_i - \text{rot}\frac{k_3}{m}v_{yi}v_i\right)\Delta t, \\ v_{y\ i+1} &= v_{yi} - \left(g + \frac{k_2}{m}v_{yi}v_i + \text{rot}\frac{k_3}{m}v_{xi}v_i\right)\Delta t. \end{aligned} \tag{7}$$

The initial parameters for calculations using equations (5), (6), and (7) include g, x_0, y_0, v_0, α_0, r, m, ρ, C_D, C_L, t_0, t_M, n, and rot. The parameter rot can take three fixed values: -1 for counterclockwise rotation, $+1$ for clockwise rotation, and 0 for no rotation. MM III with rot $= -1$ is denoted as MM III−, and with rot $= +1$ as MM III+. With rot $= 0$, MM III becomes equivalent to MM II.

Numerical calculations using equations (5), (6), and (7) were performed using standard Excel functions. The results provided tables of values including i, t_i, x_i, y_i, v_{xi}, v_{yi}, v_i, α_i, E_{ki}, E_{pi}, where $\alpha_i = \arctan\frac{v_{yi}}{v_{xi}}$ represents the trajectory angle to the horizon, $E_{ki} = \frac{mv_i^2}{2}$ is the translational kinetic energy, and $E_{pi} = mgy_i$ is the potential energy.

Figure 1 demonstrates the data input interface for models MM I, MM II, and MM III in Excel. Figure 2 displays a portion of the calculation table for MM III with the input parameters shown in figure 1.

	A	B	C	D	E	F
1	**Model of dynamics of translational-rotational motion**					
2	**of a body in a dense medium**					
3						
4	g=	9,81	m/s2	d=	0,24	m
5	x0=	0	m	m=	0,68	kg
6	y0=	0	m	CD=	0,47	
7	alpha0=	45	degrees	ro=	0	kg/m3
8	v0=	9,3	m/s	S=	0,045239	m2
9	v0x=	6,576093	m/s	k2=	0	kg/m
10	v0y=	6,576093	m/s	k2/m=	0	1/m
11	Time of modeling:			CL=	0,35	
12	Start t0=	0,0000	s	Direction of rotation:		
13	End t=	1,3914	s		0	
14	TM=	1,3914	s	k3=	0	kg/m
15	deltaT=	0,013914	s	k3/m=	0	1/m

Figure 1: Interface for parameter input for mathematical models MM I-MM III in Excel.

	A	B	C	D	E	F	G
17	**i**	**ti, s**	**vxi, m/s**	**vyi, m/s**	**vi, m/s**	**xi, m**	**MM I**
18	0	0	6,576093	6,576093	9,3000	0,00000	0
19	1	0,013914	6,576093	6,439597	9,2040	0,09150	0,0915
20	2	0,027828	6,576093	6,3031	9,1090	0,18300	0,1811
21	3	0,041742	6,576093	6,166604	9,0151	0,27450	0,268802
22	4	0,055656	6,576093	6,030108	8,9223	0,36600	0,354604
23	5	0,06957	6,576093	5,893611	8,8306	0,45750	0,438507
111	93	1,294002	6,576093	-6,11807	8,9820	8,50948	0,384657
112	94	1,307916	6,576093	-6,25456	9,0755	8,60098	0,29953
113	95	1,32183	6,576093	-6,39106	9,1701	8,69248	0,212504
114	96	1,335744	6,576093	-6,52756	9,2657	8,78398	0,123579
115	97	1,349658	6,576093	-6,66405	9,3624	8,87548	0,032755
116	98	1,363572	6,576093	-6,80055	9,4600	8,96698	-0,05997
117	99	1,377486	6,576093	-6,93704	9,5586	9,05848	-0,15459
118	100	1,3914	6,576093	-7,07354	9,6582	9,14998	-0,25111

Figure 2: Fragment of the calculation table for MM I in Excel.

Figures 3 and 4 display the trajectories and velocities calculated for all models. Visual comparison and analysis of these graphs allowed students to formulate substantive conclusions about movement characteristics under various conditions.

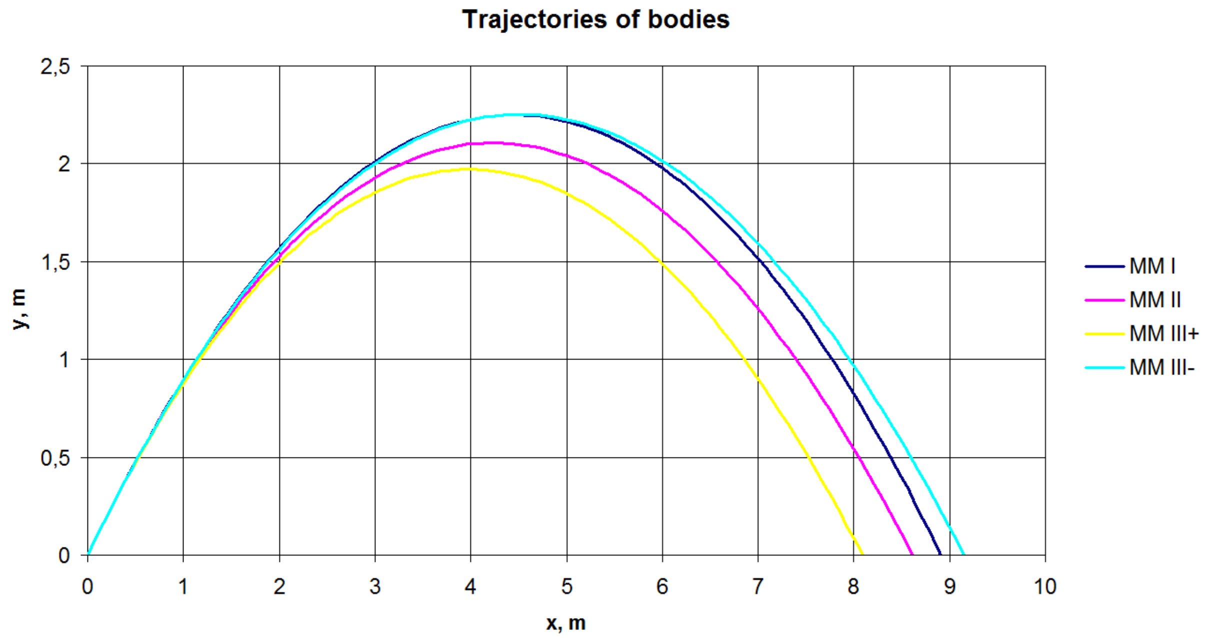

Figure 3: Trajectories of projectiles for models MM I-MM III in Excel.

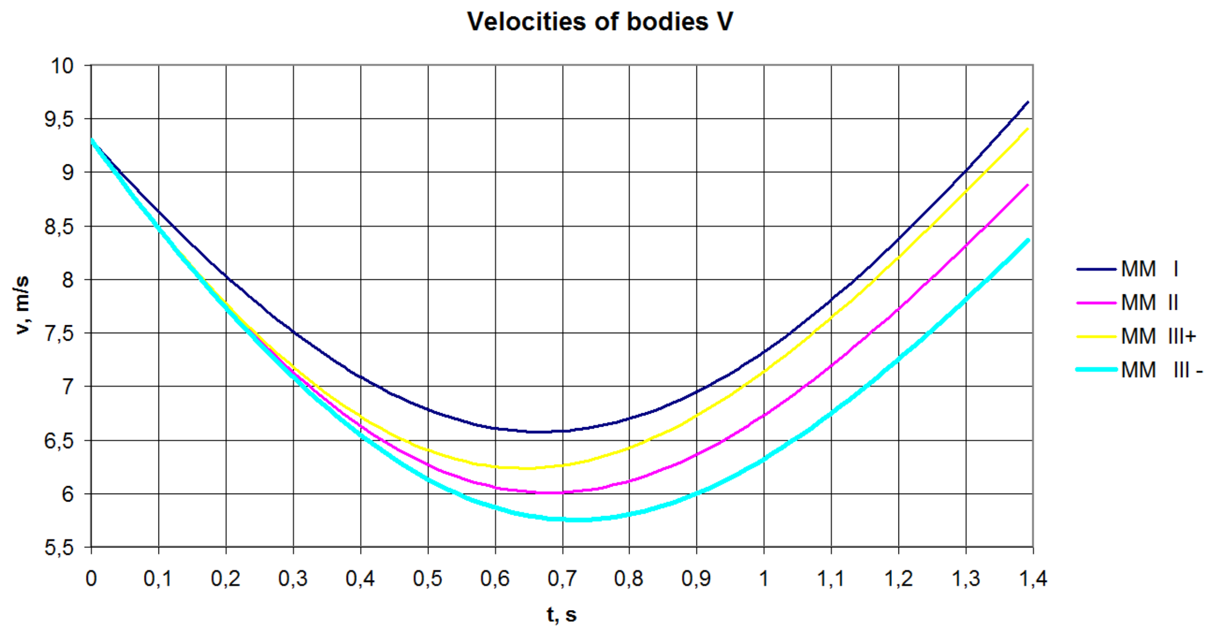

Figure 4: Velocity graphs for models MM I-MM III in Excel.

Students using Excel analysed the numerical results and graphs by varying model input parameters, enabling them to study projectile motion under different conditions, answer self-assessment questions, and prepare for examinations.

2.4. Implementation in GeoGebra (Group G)

Implementing mathematical models of mechanical motion in GeoGebra involves distinct approaches as previously noted in [2].

The primary characteristic of GeoGebra is its algebraic-geometric approach to mathematical object description. Solving MM I, MM II, and MM III in GeoGebra is considerably more straightforward than in Excel. GeoGebra eliminates the need to construct detailed calculation schemes for solving differential equation systems. Instead, models can be entered using GeoGebra syntax and solved using the built-in NSolveODE command for numerical solutions of first-order differential equations [6].

MM I is represented in GeoGebra based on equations (1) and (2) as follows (all commands are entered through the command line):

```
x1'(t,x1,y1,vx1,vy1) = vx1
y1'(t,x1,y1,vx1,vy1) = vy1
vx1'(t,x1,y1,vx1,vy1) = 0
vy1'(t,x1,y1,vx1,vy1) = -g
```

The following command solves this differential equation system:

```
NSolveODE({x1',y1',vx1',vy1'},0,{x0,y0,vx0,vy0},TM)
```

The NSolveODE command implements the 4th-order Runge-Kutta method [6, 7, 9], generating a numerical solution to the Cauchy problem over the interval $t \in [0,\ t_M]$ with initial conditions `{x0, y0, vx0, vy0}` and other input parameters. This tabulates four functions: $x_1 = x_1(t)$, $y_1 = y_1(t)$, $v_{x1} = v_{x1}(t)$, $v_{y1} = v_{y1}(t)$, which are assigned identifiers:

```
numericalIntegral1 = x1(t)
numericalIntegral2 = y1(t)
numericalIntegral3 = vx1(t)
numericalIntegral4 = vy1(t)
```

Similarly, MM II based on equations (1) and (3) is represented as:

```
x2'(t,x2,y2,vx2,vy2)=vx2
y2'(t,x2,y2,vx2,vy2)=vy2
vx2'(t,x2,y2,vx2,vy2)=-k_2*vx2*sqrt(vx2^2 + vy2^2)/m
vy2'(t,x2,y2,vx2,vy2)=-g - k_2*vy2*sqrt(vx2^2 + vy2^2)/m
```

MM II is solved using:

```
NSolveODE({x2',y2',vx2',vy2'},0,{x0,y0,vx0,vy0},TM)
```

This yields four functions as the MM II solution:

```
numericalIntegral5 = x2(t)
numericalIntegral6 = y2(t)
numericalIntegral7 = vx2(t)
numericalIntegral8 = vy2(t)
```

Likewise, MM III based on equations (1) and (4) is represented as:

```
x3'(t,x3,y3,vx3,vy3)=vx3
y3'(t,x3,y3,vx3,vy3)=vy3
vx3'(t,x3,y3,vx3,vy3)=-k_2*vx3*sqrt(vx3^2 + vy3^2)/m +
+ rot*k_3*vy3*sqrt(vx3^2 + vy3^2)/m
vy3'(t,x3,y3,vx3,vy3)=-g - k_2*vy3*sqrt(vx3^2 + vy3^2)/m -
- rot*k_3*vx3*sqrt(vx3^2 + vy3^2)/m
```

MM III is solved using:

```
NSolveODE({x3',y3',vx3',vy3'},0,{x0,y0,vx0,vy0},TM)
```

This produces the MM III solution:

```
numericalIntegral9 = x3(t)
numericalIntegral10 = y3(t)
numericalIntegral11 = vx3(t)
numericalIntegral12 = vy3(t)
```

Due to the uniformity of operations, solutions for all three models were obtained in GeoGebra significantly faster than in Excel. Once solutions were available, students could readily visualise the dynamic characteristics of the mathematical models as shown in figure 5.

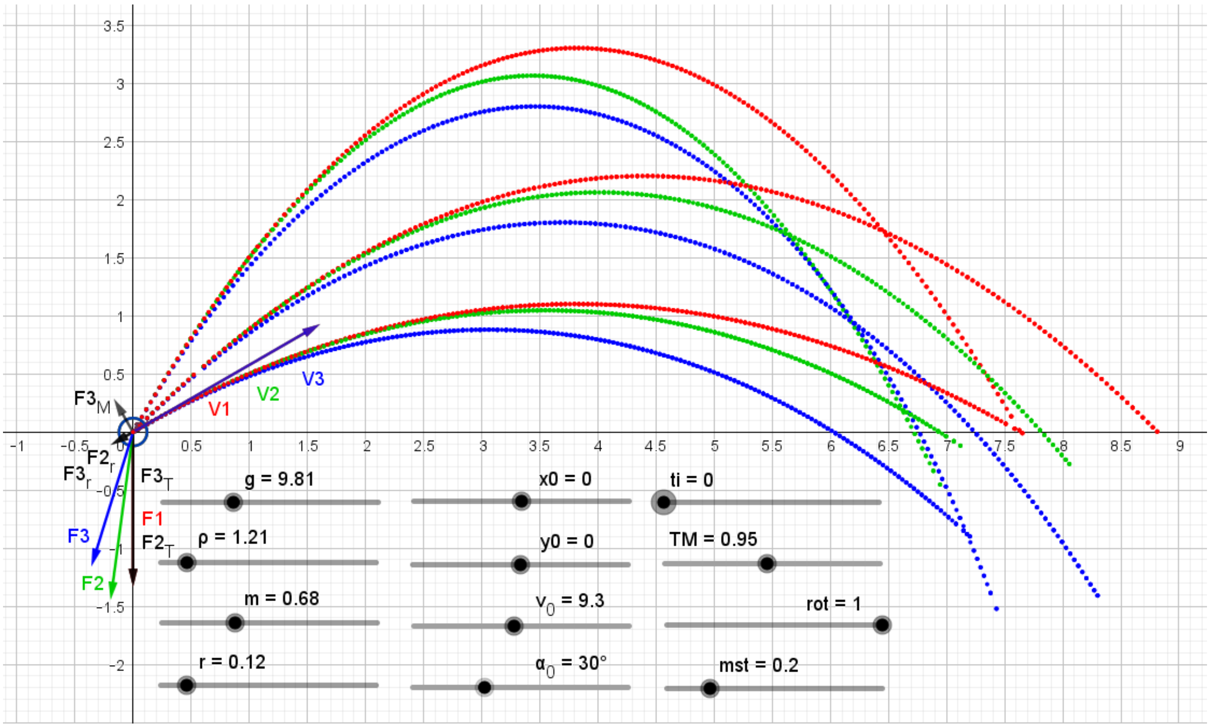

Figure 5: Trajectories of projectiles for models MM I-MM III in GeoGebra for different initial angles α_0: $30°$, $45°$, and $60°$.

The trajectories visualised in GeoGebra for models MM I, MM II, and MM III used the function values `x1(t)`, `y1(t)`, `x2(t)`, `y2(t)`, `x3(t)`, `y3(t)` as coordinates for three moving points `A=(x1, y1)`, `B=(x2, y2)`, and `C=(x3, y3)` respectively. Figure 5 shows projectile trajectories (for three different initial angles) for models: MM I (red line), MM II (green line), and MM III (blue line). The arrows in figure 5 represent velocity vectors and force vectors acting on the projectiles during motion.

GeoGebra provides superior graphics compared to Excel and offers additional visualisation capabilities. Using the animation feature, students could observe dynamic function graphing—effectively simulating the mechanical motion process. Figures 6 and 7 show trajectory fragments for opposite rotation directions controlled in MM III by the parameter rot.

Velocity vectors in figures 6 and 7 are defined in GeoGebra as follows:

```
V1=Vector(A, A3), where A3=(x(A1), y(A2)),
A1=(x(A) + mst*y(Point(numericalIntegral3, c)), y(A))
or A1=(x(A) + mst*vx1, y(A)),
A2=(x(A), y(A) + mst*y(Point(numericalIntegral4, c)))
or A2=(x(A), y(A) + mst* vy1);
```

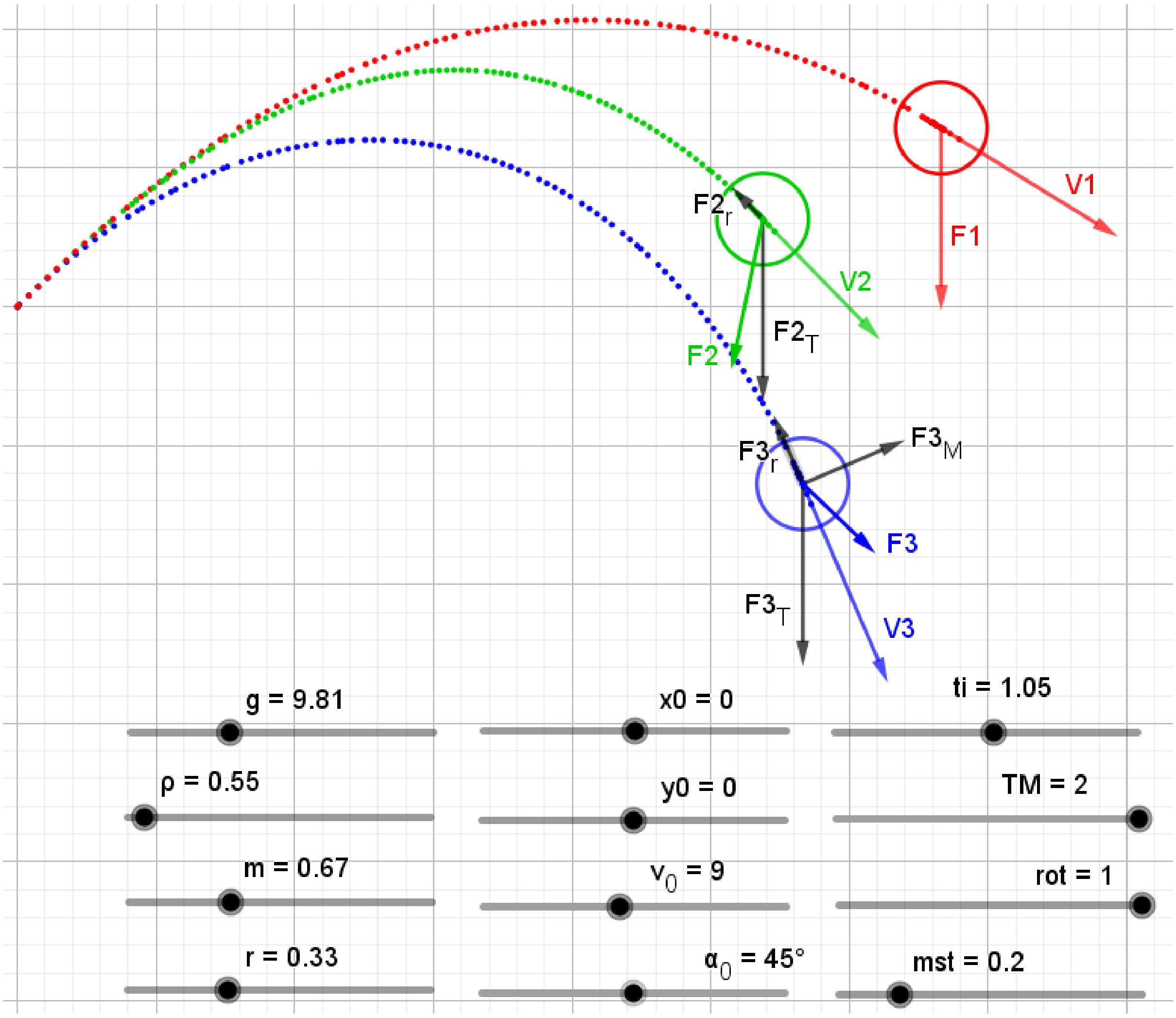

Figure 6: Trajectory fragments plotted in GeoGebra for models MM I-MM III with `rot=+1` (clockwise rotation).

```
V2=Vector(B, B3), where B3=(x(B1), y(B2)),
B1=(x(B) + mst*y(Point(numericalIntegral7, c)), y(B))
or B1=(x(B) + mst*vx2, y(B)),
B2=(x(B), y(B) + mst*y(Point(numericalIntegral8, c)))
or B2=(x(B), y(B) + mst* vy2);

V3=Vector(C, C3), where C3=(x(C1), y(C2)),
C1=(x(C) + mst*y(Point(numericalIntegral11, c)), y(C))
or C1=(x(C) + mst*vx3, y(C)),
C2=(x(C), y(C) + mst*y(Point(numericalIntegral12, c)))
or C2=(x(C), y(C) + mst* vy3);
```

where `mst` is a scale factor for vector display (in figures 5-7, `mst = 0.1`).

Force vectors acting on the projectiles and their resultants for the three models are represented in figures 6 and 7. Indices 1, 2, and 3 refer to models MM I, MM II, and MM III respectively:

```
F1=Vector(A,F1_1), where F1_1=(x(A),y(A) - mst*m*g);
```

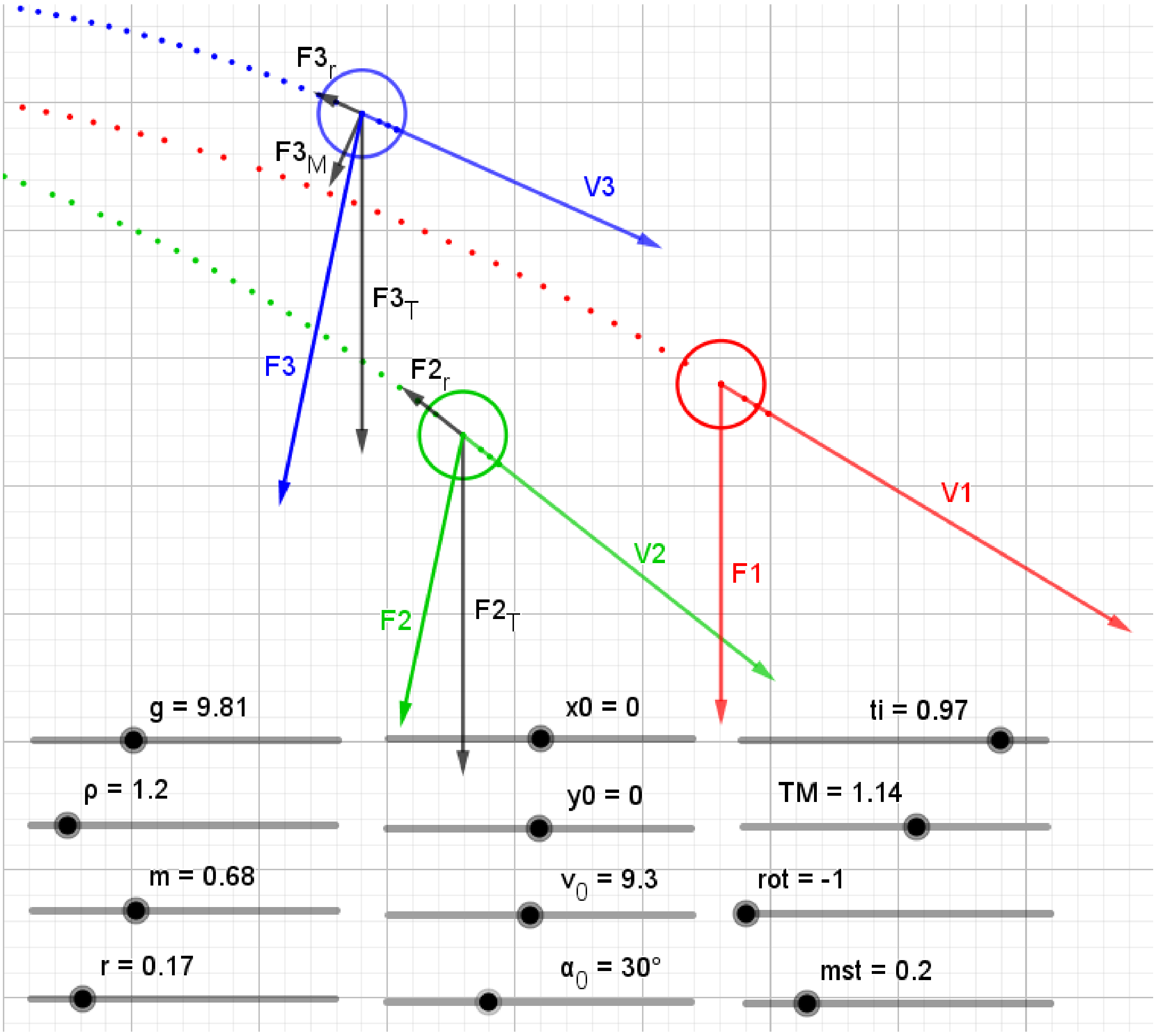

Figure 7: Trajectory fragments plotted in GeoGebra for models MM I-MM III with `rot=-1` (counterclockwise rotation).

```
F2=Vector(B,F2_3), where F2_3=F2_1 + F2_2 - B,
F2_1=(x(B),y(B) - mst*m*g), F2_2=(x(B) + mst*F2x_r, y(B) + mst*F2y_r),
F2x_r= -k_2*vx2*sqrt(vx2^2 + vy2^2), F2y_r= -k_2*vy2*sqrt(vx2^2 + vy2^2);
F2_T= Vector(B,F2_1); F2_r= Vector(B,F2_2);

F3= Vector(C, F3_5), where F3_5= F3_3 + F3_4 - C, F3_3= F3_1 + F3_2 - C,
F3_4= (x(C) + mst*F3x_M, y(C) + mst*F3y_M), F3_1= (x(C), y(C) - mst*m*g),
F3_2= (x(C) + mst*F3x_r, y(C) + mst*F3y_r),
F3x_r=-k_2*vx3*sqrt(vx3^2 + vy3^2),
F3y_r=-k_2*vy3*sqrt(vx3^2 + vy3^2), F3x_M=rot*(-k_3)*vy3*sqrt(vx3^2 + vy3^2),
F3y_M=rot*k_3*vx3*sqrt(vx3^2 + vy3^2).
```

The vectors of gravitational force $F3\_T$, air resistance force $F3\_r$, and Magnus force $F3\_M$ for model MM III are represented as:

```
F3_T=Vector(C,F3_1), F3_r=Vector(C,F3_2), F3_M=Vector(C,F3_4)
```

Through this implementation, students created mathematical models MM I, MM II, and MM III in GeoGebra using coordinate methods and elementary vector algebra.

Figures 8, 9, and 10 show the dynamic vector diagrams that students plotted and studied in GeoGebra. These diagrams display velocity vectors and their projections in different motion phases according to models MM I, MM II, and MM III. These diagrams are dynamic and interactive, automatically updating when model parameters are adjusted using the sliders shown in the figures. The figures represent results at time `ti = 1.04` s with the following initial parameters: `g=9.81`, `ro=1.213`, `m=0.68`, `r=0.12`, `x0=0`, `y0=0`, `v0=9.3`, `alpha0=45`, `rot=1`, `mst=0.1`.

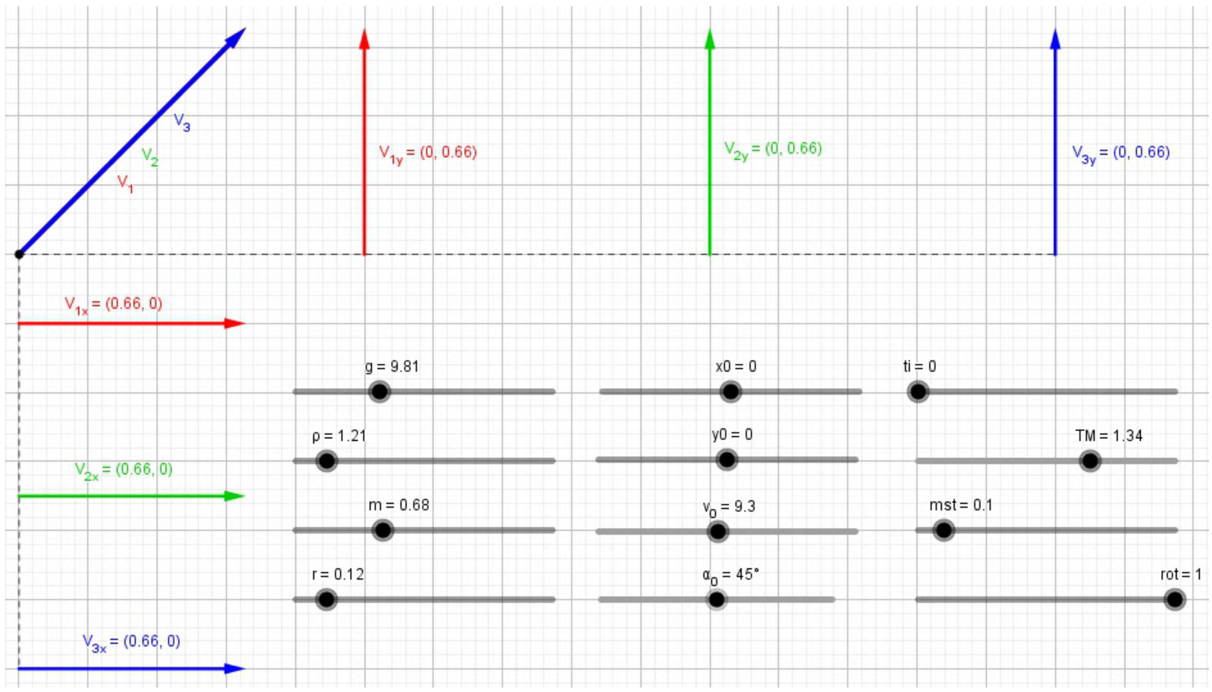

Figure 8: Dynamic vector diagram of velocities plotted in GeoGebra for models MM I-MM III+ (clockwise rotation) at movement initiation: `ti=0`, `rot=+1`.

The use of dynamic vector diagrams for visualising mechanical movement characteristics represents the primary innovation in this study compared to our previous research. Computer simulation and analysis of dynamic vector diagrams allowed students to interactively observe velocity vector changes in response to parameter adjustments, and to compare dynamic motion characteristics at different phases.

Students simultaneously analysed velocity plots and their projections on coordinate axes, similar to those shown in figures 11 and 12. While these plots effectively illustrate velocity magnitude changes, dynamic vector diagrams provide clearer visual representations of both velocity magnitude (indicated by vector length) and motion direction (indicated by vector orientation).

Figure 11 compares horizontal velocity projection plots `vx1(t)`, `vx2(t)`, `vx3(t)` for the three models with clockwise rotation (`rot=+1`) and counterclockwise rotation (`rot=-1`). These plots clearly demonstrate that with clockwise rotation, the projectile initially accelerates horizontally before decelerating, whereas with counterclockwise rotation (`rot=-1`), it initially decelerates but slightly accelerates towards the end of its trajectory.

GeoGebra facilitates the construction of similar plots and diagrams for all model solution functions. Interactive comparison with various parameter values enables rapid investigation to establish facts or identify patterns. For example, Figure 12 displays plots of functions `y1(t)`, `y2(t)`, `y3(t)` for models MM I, MM II, and MM III with both rotation directions. By adjusting the simulation time TM, students could quickly determine when the projectile contacts the ground (flight time). As

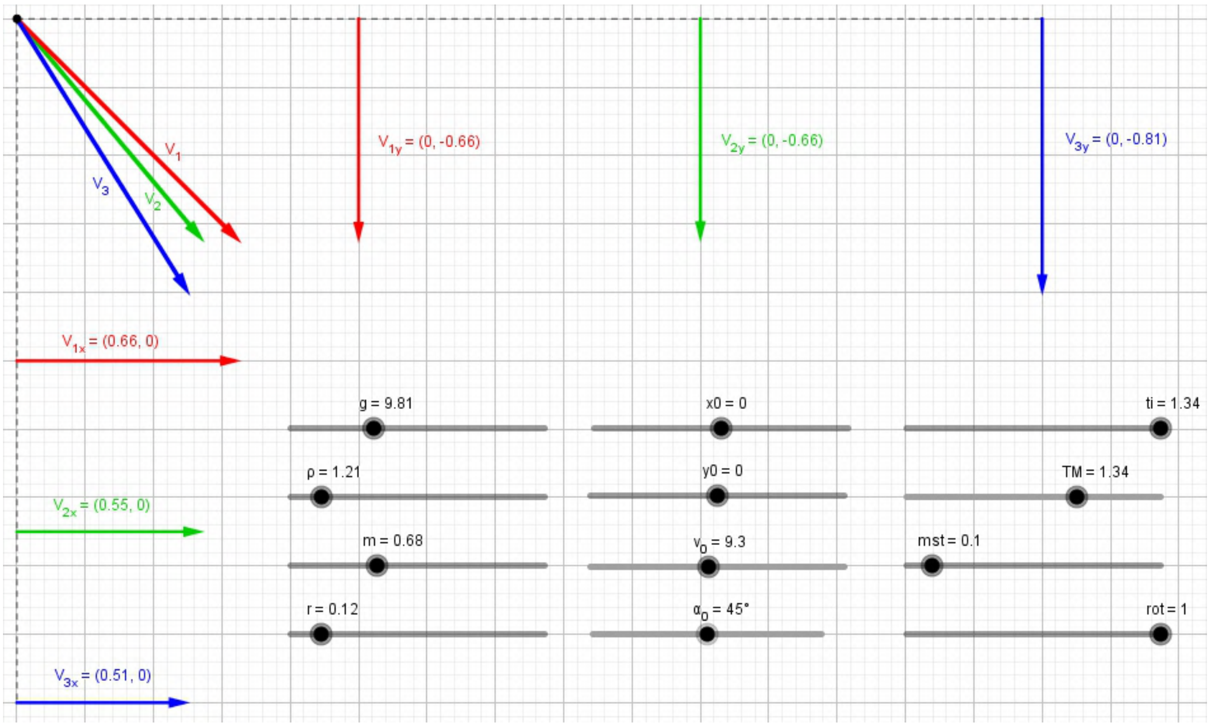

Figure 9: Dynamic vector diagram of velocities plotted in GeoGebra for models MM I-MM III+ (clockwise rotation) at motion termination: `ti=1.34`, `rot=+1`.

demonstrated, with identical input parameters, a clockwise-rotating projectile falls to the ground in 1.14 s, while a counterclockwise-rotating projectile remains airborne for 1.46 s. This approach effectively develops students' research competencies.

Similarly, students studied and analysed forces acting on the projectile during motion, including resultant forces and their projections. Through computer simulations in Excel and GeoGebra, they observed and analysed how these vector quantities changed over time, identifying trends and their significance. This approach contributed to the development of analytical abilities and graphic analysis skills.

2.5. Combined implementation with Excel and GeoGebra (Group EG)

The experimental group EG completed identical learning exercises using both Excel and GeoGebra simultaneously, following the methodology described above.

Participants in this group used Excel when necessary for numerical calculations or tabular presentation of results, and GeoGebra for visual representation and analysis of dynamic characteristics through plots and dynamic vector diagrams. Figures 13 and 14 show examples of vector diagrams of forces acting on the projectile according to models MM I, MM II, and MM III.

3. Results

Table 1 presents the final learning outcomes of engineering students after completing the experiment on teaching mathematical modelling fundamentals using dynamic vector diagrams to visualise mechanical movement characteristics. These results are presented on a 100-point scale [2].

Primary statistical analysis of experimental data (table 2) revealed differing average scores across all groups. The means for groups E and G were similar (73.36 and 73.85, respectively) but both lower than the mean for group EG (78.25). The mean values for groups E and G closely corresponded to similar indicators from previous years: $Mean_E$=73.36 (2019) vs. 73.4 (2018); $Mean_G$=73.85 (2019) vs. 73.7 (2018). Conversely,

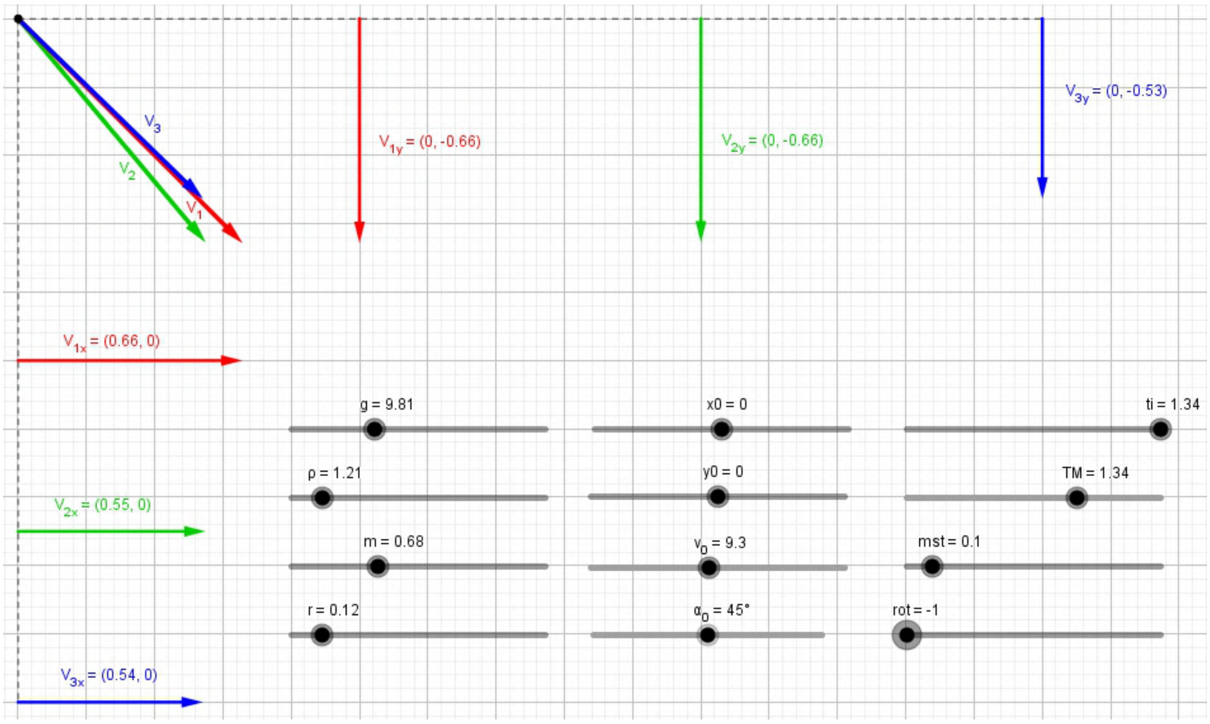

Figure 10: Dynamic vector diagram of velocities plotted in GeoGebra for models MM I-MM III- (counterclockwise rotation) at motion termination: `ti=1.34`, `rot=-1`.

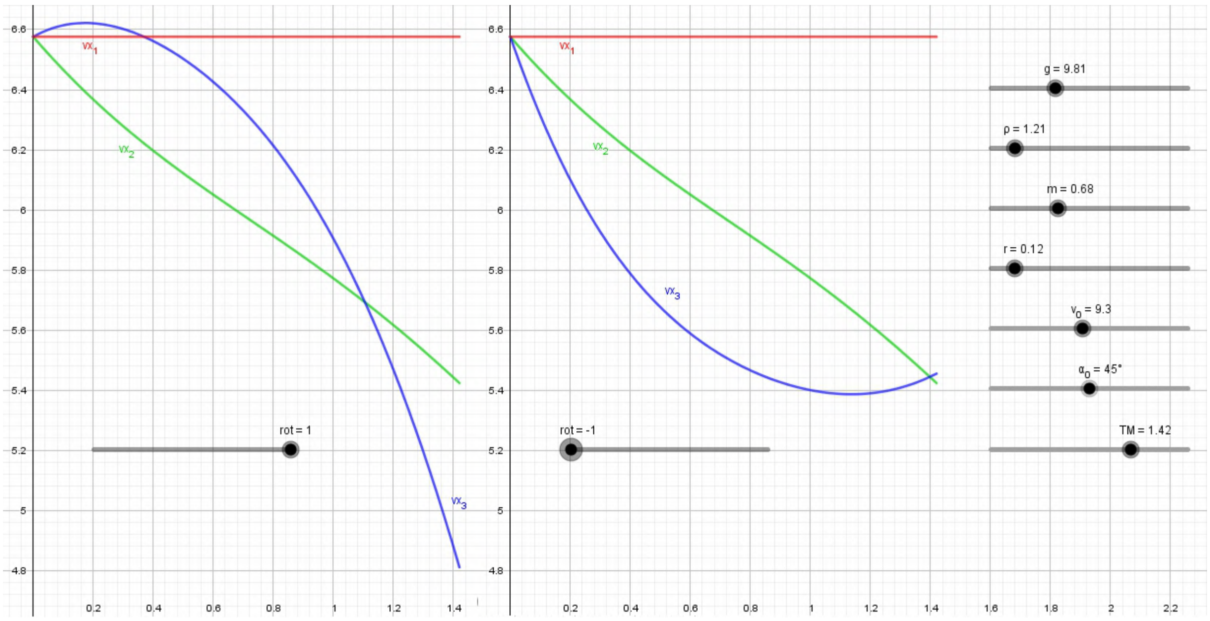

Figure 11: Visual analysis in GeoGebra: v_x projection changes for models MM I, MM II, MM III+ (`rot=+1`, clockwise rotation) and MM III- (`rot=-1`, counterclockwise rotation).

the mean value for group EG-2019 exceeded that of group EG-2018: Mean_{EG}=78.25 (2019) vs. 77.4 (2018). The Shapiro-Wilk test results supported the hypothesis of normal data distribution across all groups.

Analysis of variance (ANOVA) demonstrated a statistically significant difference in average learning outcome values across all groups (F = 4.678693; p = 0.010613) (table 3). Post-hoc comparison of means for groups E vs. G, E vs. EG, and G vs. EG indicated that the difference between means for groups E and G fell within statistical error margins (table 4). Pairwise post-hoc comparison results confirmed statistical

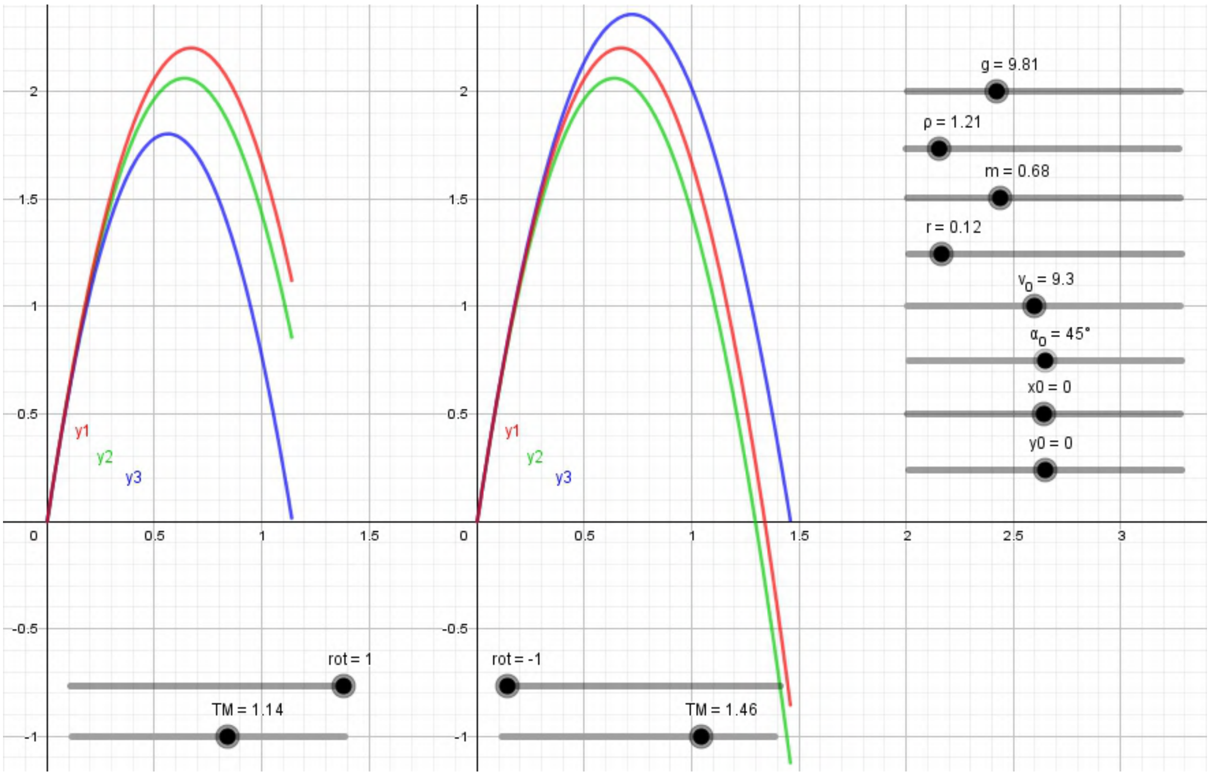

Figure 12: Visual analysis in GeoGebra: Flight duration comparison for different mathematical models with clockwise and counterclockwise rotation (`rot=+1` and `rot=-1`).

Table 1
Final learning outcomes of students after the experiment.

Score	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-95
Group E	1	2	6	9	14	10	7	4	-
Group G	-	3	6	10	13	15	8	3	1
Group EG	-	1	3	5	10	15	9	7	5

Table 2
Primary statistical data processing results.

Group	Valid N	Mean	Conf.-95%	Conf.+95%	Median	Mode
E	53	73.36	71.16	75.55	74	Multiple
G	59	73.85	71.74	75.95	74	76
EG	55	78.25	75.94	80.57	77	76

Group	Freq. Mode	Min	Max	SD	Shapiro–Wilk	test
E	5	55	90	7.96	W=0.98823	p=0.87840
G	5	56	91	8.08	W=0.98749	p=0.80486
EG	6	60	95	8.56	W=0.98158	p=0.55809

significance in the difference between the mean for group EG and the means for groups E and G.

4. Discussion

This study compared our earlier findings when students initially used Excel spreadsheets for learning exercises before incorporating GeoGebra. Throughout the training,

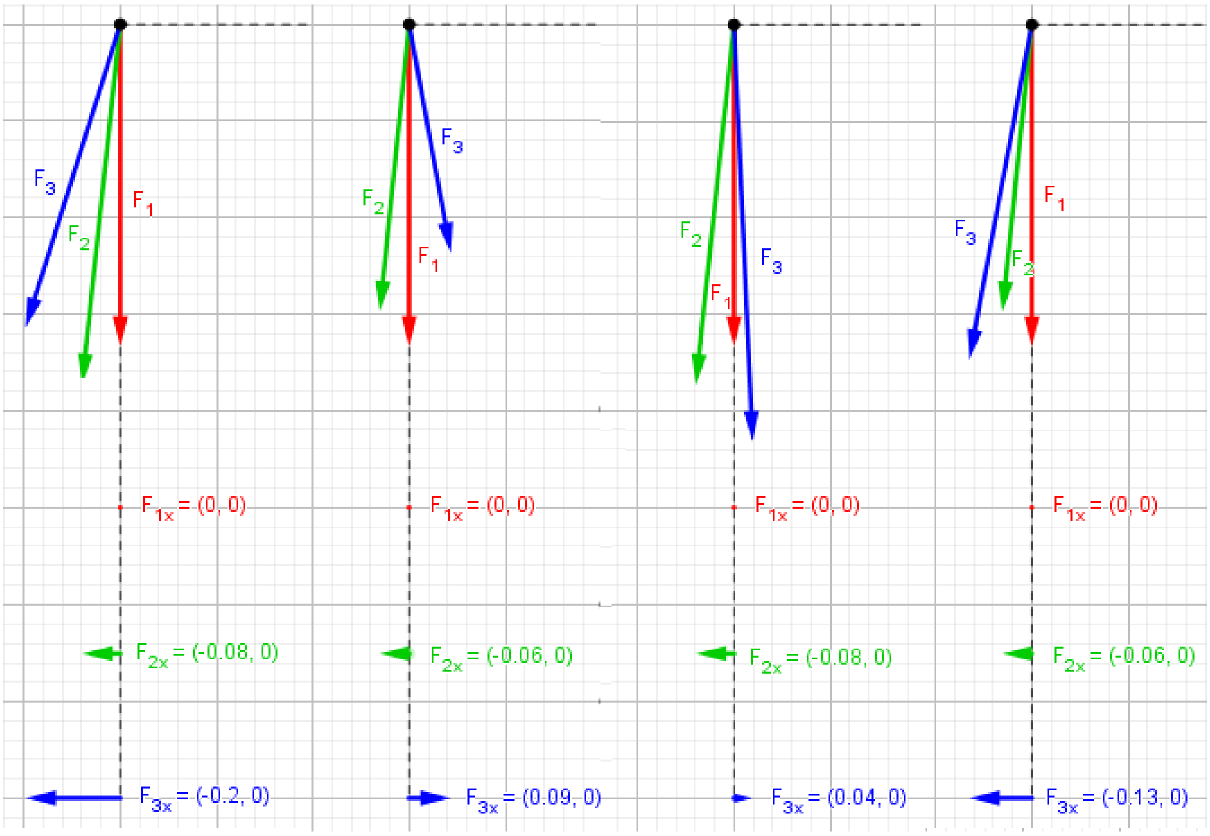

Figure 13: Dynamic vector diagram of X-component F_x of the resultant forces acting on the projectile in four flight cases: `ti = 0, rot = 1`; `ti = TM, rot = 1`; `ti = 0, rot = -1`; `ti = TM, rot = -1`.

Table 3
ANOVA results.

	SS	df	MS	SS_Err	df_Err	MS_Err	F	p
Point	1249.818	2	624.9091	10588.73	164	64.56544	4.678693	0.010613

Table 4
Pairwise post-hoc comparison results.

Pairwise post-hoc comparisons of means	E vs. G	E vs. EG	G vs. EG
LSD-test	p>0.9468	p<0.0262	p<0.0120
Duncan-test	p>0.9475	p<0.0135	p<0.0151
Tukey HSD for unequal N test	p>0.9977	p<0.0061	0.0048

we systematically examined how various visualisation methods affected educational outcomes for agricultural university students.

Our findings indicate that visualising dynamic characteristics of mechanical movement using dynamic vector diagrams in GeoGebra improved educational achievements of agricultural university students studying mathematical modelling fundamentals.

Several factors may explain these results. Firstly, implementing mathematical models in Excel essentially involves stepwise reproduction of mathematical operations, which enhances students' understanding of the technical aspects of mathematical modelling. This approach allows students to self-diagnose calculations and provides instructors with additional didactic advantages, such as the ability to identify and

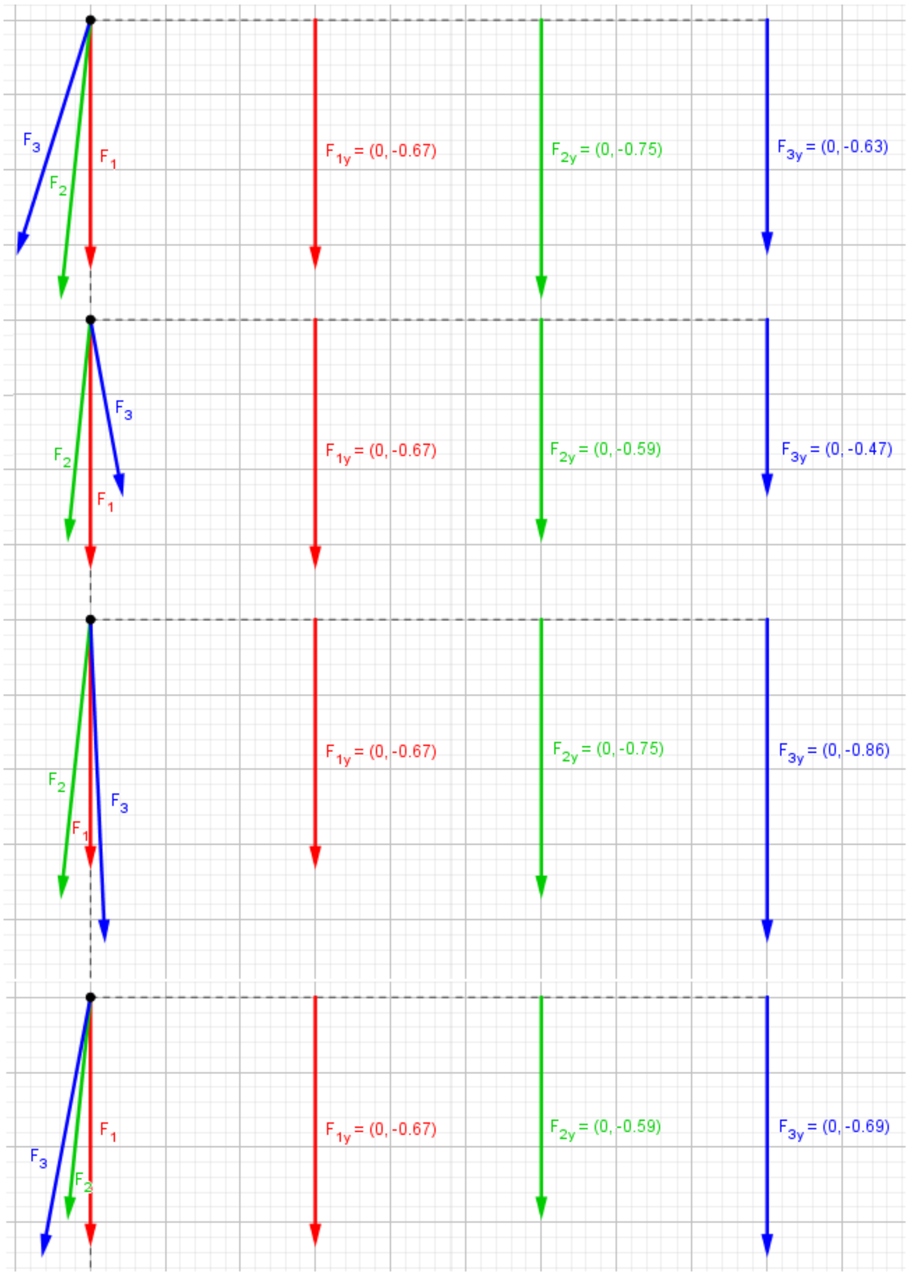

Figure 14: Dynamic vector diagram of Y-component F_y of the resultant forces acting on the projectile in four cases (from top to right): `ti = 0`, `rot = 1`; `ti = TM`, `rot = 1`; `ti = 0`, `rot = -1`; `ti = TM`, `rot = -1`.

discuss intermediate effects and simulation outcomes that might remain hidden when using professional computer mathematics systems like Mathcad [3], Mathematica, MATLAB, or Maple.

Additionally, visualising mechanical motion trajectories through Excel charts cultivates intuitive understanding of how movement characteristics change in response to initial conditions and other parameters. This enables students to consciously adjust input parameters to achieve desired simulation results. However, standard Excel features do not facilitate visualisation of instantaneous velocity changes and motion direction along trajectories. Consequently, Excel usage alone does not create sufficient conditions for developing skills in analysing dynamic movement characteristics based on numerical data.

Conversely, GeoGebra enables students to develop intuitive spatial perceptions more rapidly through visual analysis of dynamic motion characteristics, primarily through dynamic visualisation of vector characteristics. We observed that after completing learning exercises using GeoGebra, students more readily formulated meaningful responses to qualitative evaluation questions. Interactive study of dynamic plots, such as Figure 11, allowed them to quickly and accurately answer questions like "How does rotation direction affect linear velocity?" and "At what points will the velocity in airless space equal the velocity in air?" However, students working exclusively in Excel demonstrated superior performance in addressing questions about quantitative model characteristics.

As in our previous research [2, 4], we maintain that using GeoGebra alone for studying mathematical modelling fundamentals creates difficulties in numerically evaluating simulation results. Consequently, it does not develop sufficient skills for numerically assessing phenomenon or process characteristics. Concurrently, extensive GeoGebra usage contributes to effective formation of intuitive spatial representations, which are vital for engineering professionals. Since no significant difference was observed between final learning outcomes in groups E and G, the simultaneous use of Excel and GeoGebra appears to compensate for individual limitations, thus yielding superior educational achievements. Moreover, incorporating dynamic vector diagrams in GeoGebra further enhanced these results.

5. Conclusion

Analysis of our collected data demonstrates that students who simultaneously utilised GeoGebra and Excel at the highest learning difficulty level with dynamic vector diagram visualisation displayed enhanced understanding of mathematical modelling problems and improved ability to apply mathematical modelling in problem-solving. We found no statistically significant difference in learning outcomes between student groups who exclusively used either Excel or GeoGebra during mathematical modelling instruction. Furthermore, we observed that students who employed Excel for computer modelling responded more effectively to quantitative examination questions, while students who exclusively used GeoGebra demonstrated superior performance on qualitative questions.

Our study concludes that the simultaneous use of Excel and GeoGebra with dynamic vector diagram visualisation enhanced students' academic achievement in mathematical modelling. This is evidenced by the statistically significant difference between average academic achievement results shown in table 2 and confirmed by the results in tables 3 and 4.

This research demonstrates that appropriate software utilisation for visualising dynamic characteristics of mathematical models in teaching mathematical modelling at agricultural universities effectively improves student performance. We confirm that visualising mechanical movement characteristics using dynamic vector diagrams enhances student educational achievements. However, the hypothesis that combining GeoGebra visualisation with a differentiated learning approach creates additional conditions for improving students' knowledge, accounting for their vocational training specifics, requires further verification.

Declaration on generative AI: The authors have not employed any generative AI tools.

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