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Research Article

Estimating Hazard Function through Reliability Function and Empirical Methods

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ABSTRACT

In this research, the reliability functions are applied to estimate the hazard function of four used car components such as (tires, brakes, lights, and engine), which are inspected by aperiodic vehicle inspection (PVI) established in Erbil city, a specialized company that conducts the annual technical inspection of vehicles to detect the failure component, that either require repair or replace it with a new one. For our purpose, the data about the failure components of a sample of size (50,000) cars are obtained from the Erbil traffic directorate, which are annually inspected for 11 years (2010–2020) by a (PVI) company. From the available data, the reliability function, hazard function, and probability density function of the failure time of each component are found by the non-parametric method and the estimated Rayleigh distribution since the failure rates of the components are the linear functions of time, also the comparison between their reliability values have made by the mean absolute error method.

Keywords: Basics of reliability function, empirical reliability, Rayleigh distribution, hazard function, mean absolute error

INTRODUCTION

Reliability theory played a great role in various areas of life such as medicine, mechanical, electrical, and electronic engineering. It has been increasingly applied after a wide expansion of industry and become an independent entity since the fast technological developments and the increasing complexity of the equipment parts in the last century. The reliability is mainly conserved with the determination of the probability that a system consisting possible of several components will operate adequately for a given period of time, thus the reliability of the system is through attention to the internal relations of the system components and the impact of these relations in the system reliability, so it is necessary to know the pattern of behavior of these components and then impact on the behavior of the system. In our cities, the vehicles are inspected annually by a (PVI) company, to reduce road accidents and preserve the safety of the passengers, which began to record high numbers of deaths and injuries due to the increase in the number of vehicles and the lack of control systems.

Smith[1] the scale is said to be highly reliable if it produces similar results under constant conditions. It is the characteristics of a set of test scores that relate to the amount of random error of the measurement process that may be included in the scores. Highly reliable scores are accurate, reproduced, and consistent when the test is repeated. That is, if you repeat the test process with a group of test takers, we will get essentially the same results, usually using different types of reliability scores, ranging from 0.00 (big error) to 1.00 (without error), indicating the amount of error in the scores.

Tanner and Wong[2] presented the smoothing of the empirical hazards, a kernel estimate of the hazard function from censored data is obtained and criteria for asymptotic normality are considered. The mean and variance of the estimator are presented in small and large sample expressions.

Hubbard[3] proposed an empirical examination of moral hazard in the vehicle inspection market, when vendors have an incentive to misrepresent a buyer's condition in "diagnosis-cure" sellers such as auto repair and health care, moral hazard occurs. This article looks at the California automobile emission inspection market to determine if there are any incentives for inspectors to assist vehicles pass. Consumers, in my experience,

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can typically give companies and inspectors incentives to assist them pass.

Thomas *et al*.[4] predicted the reliability of automotive components by the study of fatigue. Although calculations yield a wealth of data, an experimental investigation is always required to establish this reliability. The "Stress-Strength interference analysis" approach is used for results exploitation. The "Stresses" describe the severity of the distribution of the automobile owner's stresses, whereas the "Strength" depicts the strength of the components' dispersed fatigue resistance.

Klyatis[5] studied why existing accelerated reliability testing results for passenger automobiles frequently fail to provide appropriate information for field evaluation and prediction of reliability, fatigue, and durability. The fundamental ideas of strategy that can aid in the elimination of these factors will be discussed, when a simultaneous combination of fundamental environmental elements (temperature, humidity, pollution, radiation, etc.) is employed, how may expedited environmental testing be improved, how can accelerated corrosion testing of automotive components be improved when chemical, mechanical, motion, and other variables are taken into account.

Przybysz[6] studied the reliability and tests of a sample (37) in the operation military vehicles, during 2-year observation period to determine their reliability of them, using operational data, an empirical reliability function. The reliability function of mileage to damage of military units has a logarithmic distribution.

METHODOLOGY

This paper includes the basic concepts of reliability function, empirical reliability functions, and estimated Rayleigh distribution where its scale parameter is estimated by the maximum likelihood estimator, also it includes the mean and variance time to component failure and the mean absolute error (MAE) for different reliability values of components.

Basics of Reliability Function

The reliability function is defined as the probability that a system or device will operate for a given period of time (t) under given operating conditions, denoted by R(t),[7]

Let T be the lifetime of an item, then:

$$R(t) = Pr(T > t)$$

$$R(t) = \int_t^\infty f(t)\,dt \tag{1}$$

Where $R(0) = 1$, $R(\infty) = 0$ and $f(t)$ is the failure p. d. f in a time interval $(t, t + \Delta t)$

$$f(t) = Pr(t < T < t + \Delta t), \quad when \quad t \ge 0$$

With a cumulative distribution function (CDF)

$$F(t) = Pr(T \le t)$$

$$= \int_0^t f(t)\,dt$$

$$= 1 - R(t)$$

$$f(t) = -\frac{dR(t)}{dt} \tag{2}$$

Reliability declines over time, suggesting that the likelihood of failure will rise as the entire system or component matures as shown in Figure 1.

Failure (hazard) rate function

Is the conditional probability of a component or an item to fail in the interval $(t, t + \Delta t)$ given that is operated until time (t),[8]

$$h(t) = Pr(t < T < t + \Delta t | T > t)$$

$$= \lim_{\Delta t \to 0} \frac{Pr(t < T < t + \Delta t | T > t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{Pr(t < T < t + \Delta t)}{\Delta t \, Pr(T > t)}$$

$$= \frac{1}{R(t)} \lim_{\Delta t \to 0} \frac{F(t + \Delta t) - F(t)}{\Delta t}$$

The failure (hazard) rate h(t) can be written as:

$$h(t) = \frac{f(t)}{R(t)} \tag{3}$$

$$h(t) = -\frac{1}{R(t)} \frac{dR(t)}{dt}$$

$$= -\frac{dlnR(t)}{dt}$$

$$-\int_0^t h(t)\,dt = lnR(t)$$

Then the reliability function can be written in terms of failure rate as:

$$R(t) = e^{-\int_0^t h(t)dt}$$

$$= e^{-H(t)} \tag{4}$$

The probability density function (PDF) of failure can be written as

$$f(t) = h(t).e^{-H(t)}$$

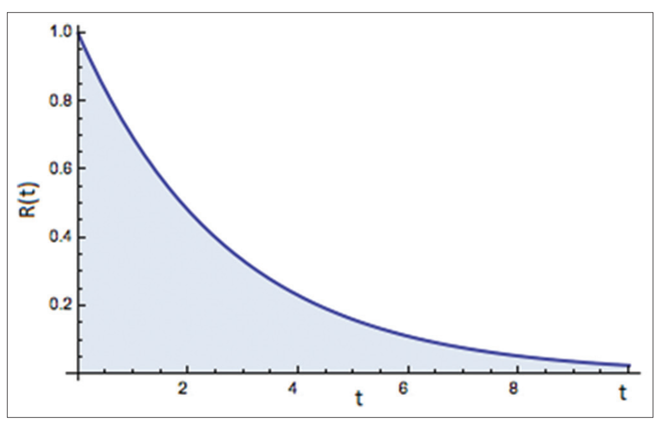

Figure 1: Reliability function declines over time

Such that,

$h(t) \geq 0$

$$\lim_{t\to\infty} \int_0^{\infty} h(t)\,dt = \infty$$

The relationship between failure functions is in the following planned (Hashimoto, 1998)

Mean and variance time to failure

Mean $E(t)$ Time to Failure:

$$E(t) = \int_0^{\infty} tf(t)\,dt$$

$$= \int_0^{\infty} R(t)\,dt \quad (5)$$

Variance σ_t^2 Time to Failure:

$$\sigma_t^2 = \int_0^{\infty} t^2 f(t)\,dt - [E(t)]^2 \quad (6)$$

Estimate Empirical Reliability

The reliability and hazard function can be estimated from the failure times $t_1 < t_2 < \cdots < t_n$. Let N_T be the total number of components, N_s be the number of survival components, and N_F be the number of failures components at time (t),[9]

$$\hat{R}(t) = \frac{N_S}{N_T} \quad (7)$$

$$\hat{F}(t) = \frac{N_F}{N_T} \quad (8)$$

$$\hat{R}(t) = \frac{N_T - N_F}{N_T}$$

$$\hat{R}(t) = 1 - \frac{N_F}{N_T}$$

$$\frac{d\hat{R}(t)}{dt} = -\frac{1}{N_T}\frac{dN_F}{dt} = -\hat{f}(t)$$

The estimated *p.d.f* of the failure function is

$$\hat{f}(t) = \frac{-d\hat{R}(t)}{dt} \quad (9)$$

$$\hat{f}(t) = \frac{-(\hat{R}(t_{i+1}) - \hat{R}(t_i))}{t_{i+1} - t_i} \quad when \;\; t_i < t < t_{i+1}$$

$$\hat{f}(t) = \frac{-\left(N_S(t_{i+1}) - N_S(t_i)\right)}{N_T(t_{i+1} - t_i)}$$

$$\hat{f}(t) = \frac{N_S(t_i) - N_S(t_{i+1})}{N_T(t_{i+1} - t_i)} \quad (10)$$

The estimated hazard function is

$$R'(t) = -\frac{dN_F}{N_T dt}$$

$$N_T R'(t) = -\frac{dN_F}{dt}$$

Dividing both sides by N_S

$$\frac{dN_F}{N_S dt} = -\frac{N_T R'(t)}{N_S} = -\frac{R'(t)}{R(t)} = \frac{\hat{f}(t)}{\hat{R}(t)}$$

$$\hat{h}(t) = \frac{\hat{f}(t)}{\hat{R}(t)}$$

$$\hat{h}(t) = \frac{N_S(t_i) - N_S(t_{i+1})}{N_S(t_i)(t_{i+1} - t_i)}, \quad when \;\; t_i < t < t_{i+1} \quad (11)$$

The estimated mean $\bar{t}$ and variance $\hat{S}_t^2$ time to failure are obtained as:

$$\bar{t} = \frac{\sum_{i=1}^{n} t_i}{n} \quad (12)$$

$$\hat{S}_t^2 = \frac{\sum_{i=1}^{n} t_i^2 - n\bar{t}^2}{n-1} \quad (13)$$

Rayleigh Distribution

The Rayleigh distribution is an applicable distribution in reliability and survival theory to model the distribution of the failure time of systems, whenever the failure rates are the linear functions of time. The Rayleigh distribution is a special case of the two parameters Weibull distribution when the shape parameter is equal to (2), its PDF is defined as:[10]

$$f(t) = \begin{cases} \dfrac{2}{\beta^2} t \, e^{-\left(\frac{t}{\beta}\right)2} & t, \beta > 0 \\ 0 & o.w \end{cases} \quad (14)$$

Where;
t: Variable of time
β: Scale parameter

With mean

$$E(t) = \sqrt{\frac{\pi\beta}{4}} \quad (15)$$

And variance

$$\sigma_T^2 = \beta\left(1 - \frac{\pi}{4}\right) \quad (16)$$

Where the failure rate (hazard function) is a linear function of time (t):

$$h(t) = \frac{2}{\beta^2} t \quad (17)$$

Moreover, the reliability function is:

$$R(t) = e^{-\left(\frac{t}{\beta}\right)2} \quad (18)$$

Figure 2 displays reliability over time. In the Rayleigh distribution when time increases, reliability decreases.

Figure 3 illustrates the hazard function over time. In the Rayleigh distribution when time increases, also hazard function increases.

Figure 4 shows how the distribution grows more dispersed and the peak of the Rayleigh function moves to the right as β grows. Accordingly, the probability mass is dispersed across a wider range of values for bigger values of β, suggesting a higher degree of variability in the underlying randomized process. The function's representation of a time-dependent process with a Rayleigh distribution, maybe in engineering or physics, is indicated by the time axis.

Estimate the scale parameter of Rayleigh distribution

This is one important way of method to estimate the parameters of any distribution, and this method relies on the use of possible functions. The following steps represent the estimation of the shape and the scale parameters of the Rayleigh distribution, as the maximum likelihood function will be as follows: [11,12]

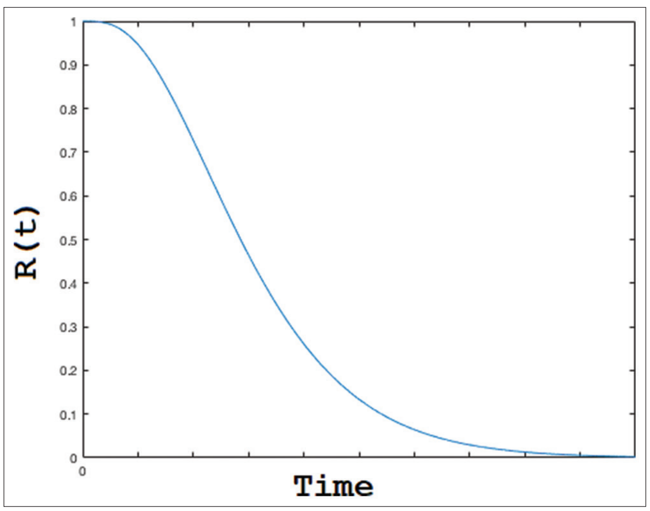

Figure 2: Reliability function

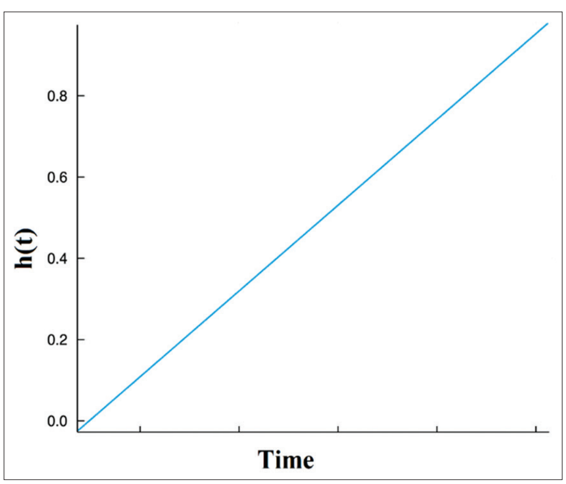

Figure 3: Hazard function

$$L(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n) = \prod_{i=1}^{n} \frac{2}{\beta^2} t_i\ e^{-\left(\frac{t_i}{\beta}\right)^2} \quad i > 1$$

$$L(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n) = \left(\frac{2}{\beta^2}\right)^n \prod_{i=1}^{n} t_i\ e^{-\frac{t_i^2}{\beta^2}}$$

Taking *ln* to both sides

$$lnL(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n) = \ln\left(\left(\frac{2}{\beta^2}\right)^n \prod_{i=1}^{n} t_i\ e^{-\frac{t_i^2}{\beta^2}}\right)$$

$$lnL(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n) = \ln\left(\frac{2^n}{\left(\beta^2\right)^n}\right) + \ln\left(\prod_{i=1}^{n} t_i\ e^{-\frac{t_i^2}{\beta^2}}\right)$$

$$lnL(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n) = nln2 - 2nln(\beta) + \sum_{i=1}^{n} lnt_i - \frac{\sum_{i=1}^{n} t_i^2}{\beta^2}$$

Taking partial derivative for scale parameter β is going to be:

$$\frac{\partial lnL(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n)}{\partial \beta} = \frac{\partial\left(nln2 - 2nln(\beta) + \sum_{i=1}^{n} lnt_i - \frac{\sum_{i=1}^{n} t_i^2}{\beta^2}\right)}{\partial \beta}$$

$$\frac{\partial lnL(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n)}{\partial \beta} = -2n\frac{\partial}{\partial \beta} ln(\beta) - \sum_{i=1}^{n} t_i^2 \frac{\partial}{\partial \beta}\frac{1}{\beta^2}$$

$$\frac{\partial lnL(\beta,\ t_1,\ t_2\ t_3, \ldots, t_n)}{\partial \beta} = -\frac{2n}{\beta} + \frac{2\sum_{i=1}^{n} t_i^2}{\beta^3} \quad (19)$$

Then, equaling to zero, we get

$$-\frac{2n}{\beta} + \frac{2\sum_{i=1}^{n} t_i^2}{\beta^3} = 0$$

$$\frac{n}{\beta} = \frac{\sum_{i=1}^{n} t_i^2}{\beta^3}$$

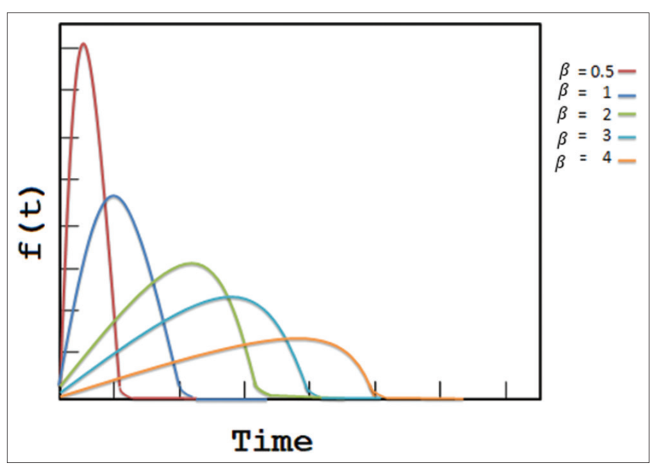

Figure 4: Rayleigh function over time

$$n\beta^2 = \sum_{i=1}^{n} t_i^2$$

$$\beta^2 = \frac{\sum_{i=1}^{n} t_i^2}{n}$$

Taking the second partial derivative of equation (19) for scale parameter β is going to be:

$$\frac{\partial^2 lnL(\beta,\, t_1,\, t_2\; t_3, \ldots, t_n)}{\partial^2 \beta} = \frac{\partial^2}{\partial^2 \beta}\left(-\frac{2n}{\beta}\right) + \frac{\partial^2}{\partial^2 \beta}\left(\frac{2\sum_{i=1}^{n} t_i^2}{\beta^3}\right)$$

$$\frac{\partial^2 lnL(\beta,\, t_1,\, t_2\; t_3, \ldots, t_n)}{\partial^2 \beta} = \frac{2n}{\beta^2} - \frac{6\sum_{i=1}^{n} t_i^2}{\beta^4}$$

$$\frac{\partial^2 lnL(\beta,\, t_1,\, t_2\; t_3, \ldots, t_n)}{\partial^2 \beta} = \frac{2n\beta^2 - 6\sum_{i=1}^{n} t_i^2}{\beta^4}$$

Since

$$n\beta^2 = \sum_{i=1}^{n} t_i^2$$

So

$$\frac{\partial^2 lnL(\beta,\, t_1,\, t_2\; t_3, \ldots, t_n)}{\partial^2 \beta} = \frac{2\sum_{i=1}^{n} t_i^2 - 6\sum_{i=1}^{n} t_i^2}{\beta^4}$$

$$\frac{\partial^2 lnL(\beta,\, t_1,\, t_2\; t_3, \ldots, t_n)}{\partial^2 \beta} = \frac{-4\sum_{i=1}^{n} t_i^2}{\beta^4} < 0$$

Hence, the maximum likelihood estimation for β^2 is

$$\hat{\beta}^2 = \frac{\sum_{i=1}^{n} t_i^2}{n}$$

The maximum likelihood estimation for β will be obtained as

$$\hat{\beta} = \left[\frac{\sum_{i=1}^{n} t_i^2}{n}\right]^{\frac{1}{2}} = \sqrt{\frac{\sum_{i=1}^{n} t_i^2}{n}} \qquad (20)$$

With it, a quadratic mean and β are one-to-one relationships of β^2, and it has an invariant property of maximum likelihood estimation. Then

$$R(t) = e^{-\left(\frac{t}{\hat{\beta}}\right)2} \qquad (21)$$

APPLICATION

This section includes the computations of the reliability function R(t), hazard function h(t), and the PDF of failure time f(t) of four failure car components (tires, lights, brakes, and engine) by estimated Rayleigh distribution and empirical method, with mean and variance time to failure components and the comparison between the different reliability values have made by the MAE.

Data Collection

The data about the failure car components for a sample of size (50,000) cars, which are annually inspected for 11 years (2010–2020) by one of the PVI centers obtained from the Erbil traffic directorate, as defined in the following [Tables 1-4].

Table 1 shows from 2010 to 2019, there was an increase in a number of failures of the tire from 132 to 2503, respectively, whereas the quantity of surviving tires decreased over time, with the number declining from 49,868 in 2010 to 35,457 in 2020.

The statistics in Table 2 shows that brake failures vary throughout the years, with a high in 2015 (2193 failures) and then following a decreasing trend. The survival rate, which indicates the lack of brake failures, follows the opposite trend, peaking in 2015 and then declining.

The "Light" demonstrated in Table 2 showed system data show a distinct trend from 2010 to 2020. The quantity of system failures continues to climb yearly, achieving a high of

Table 1: Number of failures and survivals of tire

Tire				
Year	**t_i**	**No. of failure**	**No. of survival**	**Total test**
2010	1	132	49868	50000
2011	2	307	49562	49868
2012	3	512	49050	49562
2013	4	748	48302	49050
2014	5	1110	47192	48302
2015	6	1329	45863	47192
2016	7	1639	44223	45863
2017	8	1674	42549	44223
2018	9	1924	40625	42549
2019	10	2665	37960	40625
2020	11	2503	35457	37960

Table 2: Number of failures and survivals of brake

Brake				
Year	**t_i**	**No. of failure**	**No. of survival**	**Total test**
2010	1	217	49783	50000
2011	2	506	49277	49783
2012	3	844	48433	49277
2013	4	1235	47198	48433
2014	5	1831	45367	47198
2015	6	2193	43174	45367
2016	7	2705	40469	43174
2017	8	2762	37707	40469
2018	9	3175	34531	37707
2019	10	4397	30134	34531
2020	11	3509	26625	30134

Table 3: Number of failures and survivals of light

Light				
Year	t_i	**No. of failure**	**No. of survival**	**Total test**
2010	1	230	49770	50000
2011	2	537	49233	49770
2012	3	895	48338	49233
2013	4	1309	47028	48338
2014	5	1942	45086	47028
2015	6	2326	42760	45086
2016	7	2869	39891	42760
2017	8	2930	36961	39891
2018	9	3368	33594	36961
2019	10	4664	28930	33594
2020	11	4131	24799	28930

Table 4: Number of failures and survivals of engine

Engine				
Year	t_i	**No. of failure**	**No. of survival**	**Total test**
2010	1	243	49757	50000
2011	2	567	49189	49757
2012	3	947	48243	49189
2013	4	1384	46858	48243
2014	5	2053	44805	46858
2015	6	2459	42346	44805
2016	7	3033	39313	42346
2017	8	3097	36216	39313
2018	9	3560	32656	36216
2019	10	4930	27727	32656
2020	11	4301	23426	27727

4,664 in 2019. In contrast, the number of survivors declined, reaching its lowest level in 2020 with 24,799 survivors. Despite such changes, the overall number of tests performed has remained largely steady, with a minor decline over time.

Table 4 indicates that the engine test data throughout time. In 2019 (t_i = 10), there were approximately 4930 failures among a total of 32,656 tests performed, suggesting a 15% failure rate. There were 27,727 surviving engines. There were 4301 failures out of 27,727 tests in 2020 (t_i = 11), with 23,426 engines surviving. In 2020, the total failure rate was about 15.5%.

The Estimated Mean and Variance Time to Failure Components Empirically

The mean and the variance time to failure of components are estimated, [Table 5] by:

$$\bar{t} = \frac{\sum f_i \; mt_i}{\sum f_i}$$

Table 5: Mean and the variance time to failure of components

	Mean	**Variance**	**Standard deviation**
Tire	8.42	6.48	2.55
Brake	8.34	6.40	2.53
Light	8.39	6.45	2.54
Engine	8.38	6.44	2.54

$$S^2 = \frac{\sum f_i (mt_i - \bar{t})^2}{\sum f_i - 1}$$

$$S = \sqrt{\frac{\sum f_i (mt_i - \bar{t})^2}{\sum f_i - 1}}$$

Where:
$\bar{t}$: Meantime
S^2: Variance
S: Standard deviation
mt_i: Midpoint of time
f_i: Number failure

Table 5 shows the statistical overview for car parts, the tire component, for example, has an average measurement (mean) of 8.42, a variation from the mean (variance) of 6.48, and an indicator of data dispersion (standard deviation) of 2.55.

Estimating the Scale Parameter of Rayleigh Distribution for Components

The scale parameter (β) values of Rayleigh distribution for the components (tire, light, break, and engine) are estimated as defined in the following [Tables 6-9]

$$\hat{\beta} = \sqrt{\frac{\sum_{i=1}^{11} f_i mt_i^2}{\sum_{i=1}^{11} f_i}}$$

Where

Table 6 displayed an estimated scale parameter of tires for the years 2010–2020, with tire sizes mt_i ranging from 1.5 to 11.5", is $\hat{\beta}_{Tire} = 8.795$.

This numerical data, as shown in Table 7, emphasizes the critical importance of braking-related parameters determining the analyzed outcomes. The predicted brake parameter ($\hat{\beta}_{Brake}$) is computed at 8.1207, offering quantitative insights into the significant influence of braking coefficients.

Table 8 shows a perceptible numerical rise in the meantime to failure from 1.5 to 11.5. Concurrently, the failure rate increases from 230 to 4131. The calculated parameter $\hat{\beta}_{Light}$, fixed at 8.7647.

Table 9 illustrates engine failure numbers over a decade, with (mt_i) reflecting the average time between failures. has an estimated parameter coefficient of the engine counted to 8.121.

Table 6: Estimated scale parameter of tire

Tire			
Year	mt_i	**No. of failure**	$f_i mt_i^2$
2010	1.5	132	297
2011	2.5	307	1918.75
2012	3.5	512	6272
2013	4.5	748	15147
2014	5.5	1110	33577.5
2015	6.5	1329	56150.25
2016	7.5	1639	92193.75
2017	8.5	1674	120946.5
2018	9.5	1924	173641
2019	10.5	2665	293816.25
2020	11.5	2503	331021.75

Table 7: Estimated scale parameter of brake

Brake			
Year	mt_i	**No. of failure**	$f_i mt_i^2$
2010	1.5	217	488.25
2011	2.5	506	3162.5
2012	3.5	844	10339
2013	4.5	1235	25008.75
2014	5.5	1831	55387.75
2015	6.5	2193	92654.25
2016	7.5	2705	152156.25
2017	8.5	2762	199554.5
2018	9.5	3175	286543.75
2019	10.5	4397	484769.25
2020	11.5	3509	464065.25

Table 8: Estimated scale parameter of light

Light			
Year	mt_i	**No. of failure**	$f_i mt_i^2$
2010	1.5	230	517.5
2011	2.5	537	3356.25
2012	3.5	895	10963.75
2013	4.5	1309	26507.25
2014	5.5	1942	58745.5
2015	6.5	2326	98273.5
2016	7.5	2869	161381.25
2017	8.5	2930	211692.5
2018	9.5	3368	303962
2019	10.5	4664	514206
2020	11.5	4131	546324.75

Table 9: Estimated scale parameter of engine

Engine			
Year	mt_i	**No. of failure**	$f_i mt_i^2$
2010	1.5	243	517.5
2011	2.5	567	3356.25
2012	3.5	947	10963.75
2013	4.5	1384	26507.25
2014	5.5	2053	58745.5
2015	6.5	2459	98273.5
2016	7.5	3033	161381.25
2017	8.5	3097	211692.5
2018	9.5	3560	303962
2019	10.5	4930	514206
2020	11.5	4301	546324.75

Determine the PDF, Failure Rates, and Reliability of Components by Estimated Rayleigh Distribution

Since the failure rates $h(t)$ of the car components are linear functions of time (t), [Figures 5-8], thus the PDF of failure $f(t)$, hazard function $h(t)$, and reliability $R(t)$ by estimated Rayleigh distribution of (tire, brake, light, and engine) components are determined by equations (14), (17), (18), as defined in the following [Tables 10-13]. The hazard function regarding the failure rate of the components over time is estimated in Figures 5-8. "Estimate hazard function of components" the chance of failing rises with time. This could be used in reliability engineering when modeling a tire component's life expectancy and determining when to do maintenance or replacements.

The hazard function regarding the failure rate of the tire component over time is estimated in Figure 5.

Table 10 illustrates a substantial decrease in performance from 2010 to 2020 using the Rayleigh distribution for tire dependability. The increasing failure rates from 0.026 in 2010 to 0.284 in 2020, along with a reduction in dependability from 0.987 to 0.209, suggest a concerning pattern of decreasing tire lifespan during the indicated time.

Over the 2010–2020 timeframe, the brake reliability data, as provided in Table 11 and examined with the Rayleigh distribution, demonstrates a disturbing pattern of increasing failure rates from 0.030 to 0.334, coupled with a reduction in dependability from 0.985 to 0.160. The PDF likewise rises, showing an increased possibility of brake failure at key time intervals.

Table 12 displayed the Rayleigh distribution's PDF, failure rate, and reliability for each year from 2010 to 2020. Notably, the PDF increases from 0.026 to 0.286, demonstrating an increasing chance of failure, yet the failure rate decreases from 0.987 to 0.207, demonstrating improved system robustness during the same time.

Table 13 summarizes the Rayleigh distribution-based reliability study of an engine from 2010 to 2020,

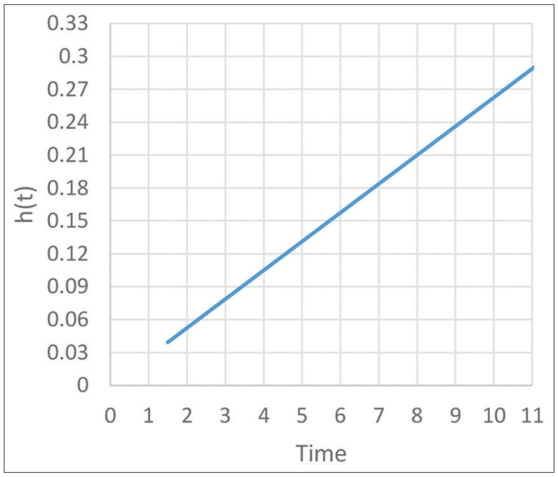

Figure 5: Estimate hazard function of tire component

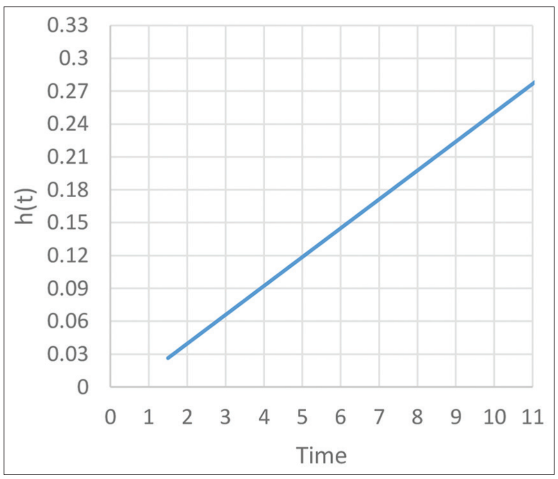

Figure 6: Estimate hazard function of brake component

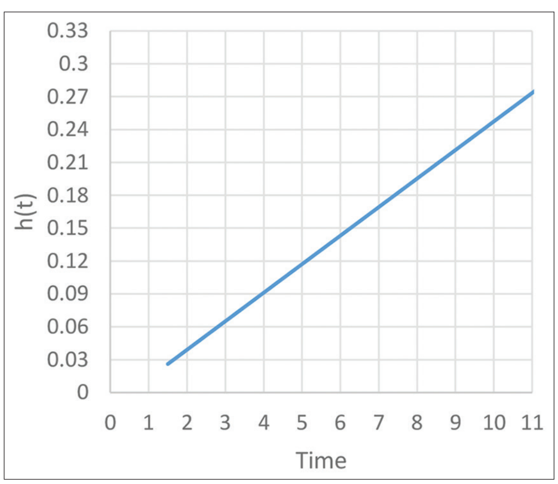

Figure 7: Estimate hazard function of light component

including characteristics such as periods (t_i), hazard rates $h(t)$, reliability rates $R(t)$, and PDFs $f(t)$. For example, in 2010, the reliability was 0.985, showing a high likelihood of failure-free operation, however in 2020, the reliability dropped to 0.160, indicating a

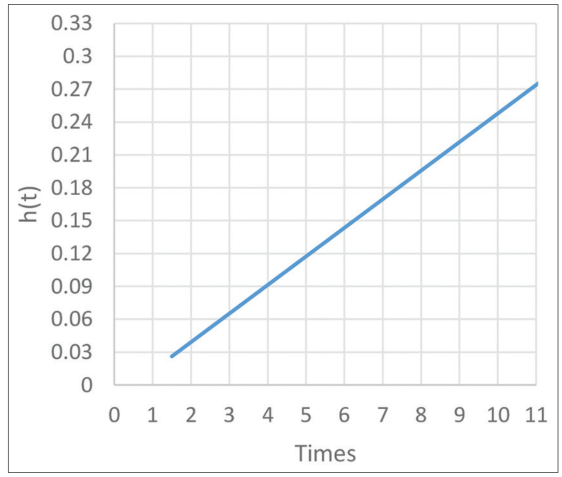

Figure 8: Estimate hazard function of engine component

Table 10: Probability density function, failure rate, and reliability by Rayleigh distribution of tire

Tire				
Year	t_i	$h(t)$	$R(t)$	$f(t)$
2010	1	0.026	0.987	0.026
2011	2	0.052	0.950	0.049
2012	3	0.078	0.890	0.069
2013	4	0.103	0.813	0.084
2014	5	0.129	0.724	0.094
2015	6	0.155	0.628	0.097
2016	7	0.181	0.531	0.096
2017	8	0.207	0.437	0.090
2018	9	0.233	0.351	0.082
2019	10	0.259	0.275	0.071
2020	11	0.284	0.209	0.060

Table 11: Probability density function, failure rate, and reliability by Rayleigh distribution of brake

Brake				
Year	t_i	$h(t)$	$R(t)$	$f(t)$
2010	1	0.030	0.985	0.030
2011	2	0.061	0.941	0.057
2012	3	0.091	0.872	0.079
2013	4	0.121	0.785	0.095
2014	5	0.152	0.684	0.104
2015	6	0.182	0.579	0.105
2016	7	0.212	0.476	0.101
2017	8	0.243	0.379	0.092
2018	9	0.273	0.293	0.080
2019	10	0.303	0.220	0.067
2020	11	0.334	0.160	0.053

significant fall in the engine's dependability all over the investigated time.

Table 12: Probability density function, failure rate, and reliability by Rayleigh distribution of light

Light				
Year	t_i	*h(t)*	*R(t)*	*f(t)*
2010	1	0.026	0.987	0.026
2011	2	0.052	0.949	0.049
2012	3	0.078	0.889	0.069
2013	4	0.104	0.812	0.085
2014	5	0.130	0.722	0.094
2015	6	0.156	0.626	0.098
2016	7	0.182	0.528	0.096
2017	8	0.208	0.435	0.091
2018	9	0.234	0.348	0.082
2019	10	0.260	0.272	0.071
2020	11	0.286	0.207	0.059

Table 13: Probability density function, failure rate, and reliability by Rayleigh distribution of engine

Engine				
Year	t_i	*h(t)*	*R(t)*	*f(t)*
2010	1	0.030	0.985	0.030
2011	2	0.061	0.941	0.057
2012	3	0.091	0.872	0.079
2013	4	0.121	0.785	0.095
2014	5	0.152	0.685	0.104
2015	6	0.182	0.579	0.105
2016	7	0.212	0.476	0.101
2017	8	0.243	0.379	0.092
2018	9	0.273	0.293	0.080
2019	10	0.303	0.220	0.067
2020	11	0.334	0.160	0.053

Mean and Variance Time to Failure by Estimated Rayleigh Distribution for Components

The mean E(t) and the variance (σ_T^2) of failure components by estimated Rayleigh distribution are calculated by equation (15) and equation (16), as shown in the following in [Table 14].

According to Table 14, the car's component with the greatest mean is "Tire" (mean = 2.6276), whereas the component with the smallest variance is "Brake" (variance = 4.9807), indicating a more closely packed distribution around its mean.

Estimate PDF, Failure Rate, and Reliability by Empirical Method for Components

The PDF of failure $\hat{f}(t)$, hazard function $\hat{h}(t)$, and reliability $\hat{R}(t)$ by empirical method for the components (tire, brake, light, and engine) are estimated by equation (7), equation (9), and equations (11), likewise demonstrated in [Tables 15-18].

Table 14: Mean and the variance time to failure of components

Components	Mean	Variance
Tire	2.6276	5.6552
Brake	2.5248	4.9807
Light	2.6230	5.6247
Engine	2.5249	4.9811

Table 15: Estimate probability density function, failure rate, and reliability by empirical method for tire

Tire				
Year	t_i	$\hat{R}(t)$	$\hat{f}(t)$	$\hat{h}(t)$
2010	1	0.997	0.006	0.006
2011	2	0.994	0.010	0.010
2012	3	0.990	0.015	0.015
2013	4	0.985	0.023	0.023
2014	5	0.977	0.028	0.028
2015	6	0.972	0.035	0.036
2016	7	0.964	0.037	0.038
2017	8	0.962	0.044	0.045
2018	9	0.955	0.063	0.066
2019	10	0.934	0.062	0.066
2020	11	0.934	--	--

Table 16: Estimate probability density function, failure rate, and reliability by empirical method for brake

Brake				
Year	t_i	$\hat{R}(t)$	$\hat{f}(t)$	$\hat{h}(t)$
2010	1	0.996	0.010	0.010
2011	2	0.990	0.017	0.017
2012	3	0.983	0.025	0.025
2013	4	0.975	0.038	0.039
2014	5	0.961	0.046	0.048
2015	6	0.952	0.060	0.063
2016	7	0.937	0.064	0.068
2017	8	0.932	0.078	0.084
2018	9	0.916	0.117	0.127
2019	10	0.873	0.102	0.116
2020	11	0.884	--	--

Based on Table 15 supplied, the tire component's dependability shows a distinct pattern over time. Estimated reliability was high in 2010, at 0.997, indicating great effectiveness of operations. However, dependability has since declined, with scores falling to 0.972 in 2015, 0.955 in 2018, and even lower to 0.934 in both 2019 and 2020.

Table 17: Estimate probability density function, failure rate, and reliability by empirical method for light

Light				
Year	t_i	$\hat{R}(t)$	$\hat{f}(t)$	$\hat{h}(t)$
2010	1	0.995	0.011	0.011
2011	2	0.989	0.018	0.018
2012	3	0.982	0.027	0.027
2013	4	0.973	0.040	0.041
2014	5	0.959	0.049	0.052
2015	6	0.948	0.064	0.067
2016	7	0.933	0.069	0.073
2017	8	0.927	0.084	0.091
2018	9	0.909	0.126	0.139
2019	10	0.861	0.123	0.143
2020	11	0.857	--	--

Table 18: Estimate probability density function, failure rate, and reliability by empirical method for engine

Engine				
Year	t_i	$\hat{R}(t)$	$\hat{f}(t)$	$\hat{h}(t)$
2010	1	0.995	0.011	0.011
2011	2	0.989	0.019	0.019
2012	3	0.981	0.028	0.029
2013	4	0.971	0.043	0.044
2014	5	0.956	0.052	0.055
2015	6	0.945	0.068	0.072
2016	7	0.928	0.073	0.079
2017	8	0.921	0.091	0.098
2018	9	0.902	0.136	0.151
2019	10	0.849	0.132	0.155
2020	11	0.845	--	--

Table 16 indicates the dependability data for the braking system over time. The dependability has steadily reduced from 0.996 in 2010 to 0.884 in 2020, indicating a possible performance degradation. It is worth noting, however, that particular information on the failure rate and hazard rate for 2020 is missing from Table 16, preventing an exhaustive evaluation for that year.

The reliability calculation of the light system from 2010 to 2020 is shown in Table 17, with estimated reliability (R [t]) dropping from 0.995 in 2010 to 0.857 in 2020. However, the absence of missing data in the PDF and the rate of failure columns for 2020 (NaN) emphasizes the requirement for additional data.

Table 18 includes empirical estimates for an engine's PDF, failure rate, and reliability from 2010 to 2020. The estimations, which are based on observable data, are critical in reliability engineering for analyzing the engine's efficiency and probable failure features.

Table 19: Mean absolute error for tire

Tire			
Year	$\hat{R}(t)$	$R(t)$	$\lvert R-\hat{R}\rvert$
2010	0.997	0.987	0.01024
2011	0.994	0.950	0.04429
2012	0.990	0.890	0.09953
2013	0.985	0.813	0.17154
2014	0.977	0.724	0.25314
2015	0.972	0.628	0.34389
2016	0.964	0.531	0.43351
2017	0.962	0.437	0.52487
2018	0.955	0.351	0.60383
2019	0.934	0.275	0.65986
2020	0.9341	0.209	0.72482

Table 20: Mean absolute error for bake

Brake			
Year	$\hat{R}(t)$	$R(t)$	$\lvert R-\hat{R}\rvert$
2010	0.996	0.985	0.00875
2011	0.990	0.941	0.04117
2012	0.983	0.872	0.09468
2013	0.975	0.785	0.16457
2014	0.961	0.684	0.24182
2015	0.952	0.579	0.32934
2016	0.937	0.476	0.41298
2017	0.932	0.379	0.50141
2018	0.916	0.293	0.57181
2019	0.873	0.220	0.60485
2020	0.884	0.160	0.68047

MAE for Components

The comparison between the reliability values which are found by the estimated Rayleigh distribution R(t) and empirical method $\hat{R}(t)$ of (tire, brake, light, and engine) components has made by *MAE* method, as illustrated in [Tables 19-22] where:

$$MAE = \frac{\sum \lvert R-\hat{R}\rvert}{n}$$

Between the years 2010 and 2020, Table 19 showed both predicted and real tire ratings decreased, with a notable rise in absolute errors, notably in 2019. The MAE $MAE_{Tire} = 0.3517$ is an average measure of the model's precision, showing the total level of departure from expected and real scores.

Table 20 shows a significant reduction in brake ratings from 2010 to 2020, shown in both projected

$\hat{R}(t)$ and real R(t) values. The absolute errors, including the MAE $MAE_{Brake} = 0.3659$, showed a significant difference

Table 21: Mean absolute error for light

Light			
Year	$\hat{R}(t)$	$R(t)$	$\|R-\hat{R}\|$
2010	0.995	0.987	0.00833
2011	0.989	0.949	0.03994
2012	0.982	0.889	0.09236
2013	0.973	0.812	0.16093
2014	0.959	0.722	0.23650
2015	0.948	0.626	0.32255
2016	0.933	0.528	0.40449
2017	0.927	0.435	0.49192
2018	0.909	0.348	0.56052
2019	0.861	0.272	0.58916
2020	0.857	0.207	0.65023

Table 22: Mean absolute error for engine

Engine			
Year	$\hat{R}(t)$	$R(t)$	$\|R-\hat{R}\|$
2010	0.995	0.985	0.00806
2011	0.989	0.941	0.03943
2012	0.981	0.872	0.09155
2013	0.971	0.785	0.15963
2014	0.956	0.685	0.23442
2015	0.945	0.579	0.31978
2016	0.928	0.476	0.40060
2017	0.921	0.379	0.48718
2018	0.902	0.293	0.55398
2019	0.849	0.220	0.57760
2020	0.845	0.160	0.63852

between predicted and real brake ratings during the selected years. In addition, Tables 21 and 22 displayed the predicted $\hat{R}(t)$ and real R(t) rating for light and engine, respectively, from 2010 to 2020, with appropriate absolute errors. The MAEs MAE_{Engine} and MAE_{Engine} are 0.3235 and 0.355, indicating the average absolute errors in predicted and real ratings throughout the years of interest. Both tables show an ongoing decrease in ratings and a rise in absolute errors throughout the duration.

CONCLUSION

According to the results of the application, the following conclusions are found:

The number of failure components increased annually, except in 2020, since the number of arrived customers to the system had lessened, because of COVID-19.

The reliability values of components defined by estimated Rayleigh distribution are more decreasing with time than reliability values defined by empirical method.

The mean and variance times of failure components which are determined by estimated Rayleigh distribution and empirical method prepared in the same ascending order as follows brakes, engine, lights, and tires successively.

MAE between $R(t)$ by Rayleigh distribution and $\hat{R}(t)$ values by Empirical method of car components sorting from minimum to maximum value is (light < tire < engine < brake).

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