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RESEARCH ARTICLE

Determination of Single Sampling Attribute Plans Based Upon Dodge-Romig Model with Application

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ABSTRACT

Bayesian sampling plans for production inspection involve using a sampling method to assess the plan's features, assuming that defect rates fluctuate randomly among different production batches. This study uses Bayesian sampling plans, specifically the beta distribution, to determine a single sampling plan's (SSP) parameter (n, c). These parameters were then compared to those from other SSP. The study was conducted at the Ala corporation for soft drinks, where 120 batches were selected to calculate the defect rate. The results showed that using Bayesian and decision-making models can lead to developing a single sampling inspection procedure that closely approximates the quality level. In addition, the decision-making model resulted in a smaller sample size and lower inspection costs than other inspection plans.

Keywords: Acceptance quality level, Bayesian sampling plans, operating characteristics, statistical quality control, average sample size

INTRODUCTION

Pepsi Company (Ala) is one of the private sector companies belonging to the food industries in the Bazian area of Sulaymaniyah Governorate. This company was established in 2006–2005 and its first production was in 2006, and the company consists of machinery and equipment (Italian and Swiss) made with high specifications as it can manufacture two types of soft drink bottles with a capacity of (1.5 L) and (330 mL), and that the allowable percentage of production damage according to international specifications is 3%, meaning that any increase in this percentage indicates a defect in the company's production stages.

Quality control is a critical aspect of production, ensuring that products meet specified standards and minimizing waste and costs. Statistical tools, such as control charts and sampling plans, play a crucial role in managing and monitoring production quality. Sampling plans, in particular, offer a precise and efficient way to assess the presence of specific characteristics in produced units by analyzing a small, randomly selected portion of the output.[1]

This study focuses on the application of Bayesian sampling plans for production inspection, a method that incorporates prior knowledge and experience to make informed decisions about product acceptance or rejection. Bayesian sampling plans have been shown to be effective in various industries, and their potential benefits in the soft drink industry are worth exploring.

The research aims to develop a Bayesian discriminant sampling plan for the Ala Pepsi Soft Drinks Company, a major producer in the Bazian area of Sulaymaniyah Governorate. By applying Bayesian theory and decision theory, this study seeks to establish a sampling procedure that closely approximates the actual quality level of the company's products. The findings of this research will contribute to the optimization of quality control practices at Ala Pepsi and potentially in other similar production settings.[2,3]

Hald[4] introduced a novel sampling methodology that uses cumulative conforming control chart count and compared it to conventional sampling techniques, illustrating its power for higher lot sizes and process averages. The mathematical frameworks for the suggested technique are built utilizing the count of cumulative conforming items and Markov chain

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modeling, which both depend on Markov modeling and negative binomial distribution. This suggested method is shown to be beneficial compared to the tables of Dodge-Romig when the findings are compared to those of the single sampling plan (SSP) based on AOQL and Lot Tolerance Percent Defective (LTPD) developed by Dodge-Romig. The optimization model makes sure that the AOQL stays within a set upper limit and that the minimal ATI is attained.

Ahmadi Yazdi and Fallahnezhad[5] focused on the creation of Dodge-Romig AOQL SSP using variables and specification limitations. The research suggests a generalization of Kapur and Wang's paradigm and is to extend their earlier work. Based on the anticipated total cost per unit, the best inspection strategy is either approval without control or 100 observations. The sample plans are designed using the quadratic quality loss function, which has proven effective in many quality control uses. The findings paper recommends additional investigation into an integrated Bayesian SSP model for variables and boundaries.

Chen[6] provided clear asymptotic formulae for the Dodge-Romig LTPD single sampling inspection plans' sample size and approval number. Numerical experiments indicate that a simple finite population adjustment of the asymptotic formulae results in a very accurate approach to the Dodge-Romig solution. The paper's major findings are that the sample size is asymptotically proportionate to the logarithm of the lot size and that the maximum permitted percent faulty in the sample merges to the tolerance fraction defective, with a difference of order l/V%.

Klůfa[7] compared LTPD single sample plans for examination by characteristics to the matching Dodge-Romig LTPD plan from an economic standpoint. The economic effectiveness of LTPD plans for variable and property inspection is investigated, and it is discovered that they are more inexpensive than the matching Dodge-Romig attribute sample plans in several cases, with a cost savings of 80%. The research also investigates the relationship between economic efficiency and lot size N, given specified characteristics. Whenever the quantity of objects in the plenty is large, the system's average fraction of defective is low, and while the cost of inspection by variables does not appear to be significantly higher as compared to the cost of inspection by characteristics, the economic efficiency of the LTPD plans for inspection by variables and attributes is higher.

Kadir and Rahi[3] utilized the beta-binomial distribution to determine the parameters for a Bayesian sampling plan and then compared it to alternative SSP. The Bayesian and decision-making models can create a sample evaluation process that closely mimics the actual level of quality. When applying the decision-making model, the sample size was smaller than other inspection plans, leading to lower inspection costs. It is critical to completely develop quality control criteria, especially in prioritizing standard and production standards.

DODGE-ROMIG MODELS

In 1994 both researchers Harold and Harry used sampling plans and these plans apply in case of filtered rectifying inspection it's an examination that examines a sample (n) that is taken from a production batch (N), the number of specified units (x) in a sample (n). If it is less or equal to several approvals ($X \leq c$) then c will decide to accept the sample and the rest quantity ($N - n$) without, and defective units in the units will be replaced with better ones. A decision is taken to refuse the sample when the number of defective units is (x) in a sample (n) is bigger than the number of acceptances, which means when ($X > c$) refuses the remaining quantity ($N - n$) and all its units are submitted to general examination for isolating purposes of effective units and exchange them with other non-defective units.

When finding the SSP for filtered examination, both researchers depended on (LTPD, and AOQL) systems, where the (LTPD) term refers to the proportion of defects allowed in the production batch, ass for the (AOQL) term refers to the highest rate of defective units in the transmitted batch and that is after implementing a filtered examination on it.[8]

Single Sampling LTPD Plans

LTPD SSPs are based on several assumptions:

Defective units occur randomly, and the production process is under the statistical control of a binomial distribution with a constant defect rate of P_1

Choose the value of LTPD to be P_2 to protect the producer from delivering unsatisfactory batches, as the probability of accepting batches with quality level ($P_2 > P_1$) P_2 is small and this probability is usually called Consumer's Risk, which is the probability of accepting a batch with a quality level worse than the LTPD, and we will symbolize it as $P(P_2)$, it can be said that the producer aims to use sampling plans where the value of $P(P_2)$ is small.

Rejected batches are completely re-inspected based on the decision to reject the sample, after which all defective units are replaced with non-defective (good) units or repaired if repair is possible for these units.

The cost of inspecting one unit in the sample is equal to the cost of inspecting one unit in the rejected lot (N-n) and is equal to one as an economic unit.

The inspection plans determined by the model aim to minimize the total inspection rate $I(P_1)$ for a product of quality (P_1). This rate is based on inspecting a quantity n in the case of acceptance and n in the case of rejection, so $I(P_1)$.

$$I(P_1) = nP(P_1) + NQ(P_1) \tag{1}$$

Where:

$P(P_1)$: Represents the probability of accepting N batches of the product of quality P1. The value of $P(P_1)$ depends on the type of sampling distribution, and according to the first assumption above, we obtain the value of $P(P_1)$.

$$P(P_1) = P(X \leq c) = \sum_{x=0}^{c} b(x,n,P_1) = B(c,n,P_1) \tag{2}$$

$Q(P_1)$ represents the probability of batch rejection, where:

$$Q(P_1) = 1 - P(P_1) \tag{3}$$

A binomial distribution is used when a random sample n is drawn from batch N or from the output of a production process whose quality rate is equal to P_1. In addition, the probability of obtaining x defective units in a sample of size n taken from a production batch of size N containing x defective units is determined from the hypergeometric distribution, since

$$P(n,x,X,N) = \frac{C_x^X C_{n-x}^{N-X}}{C_n^N} \quad a \le x \le b \tag{4}$$

$a = Max\ [0, n-(N-X)],\ b = Min\ [x, n]$

Thus, the acceptance probability of batch N containing x defective units is $P(n, c, X, N)$ since

$$P(n,x,X,N) = \sum_{x=0}^{c} P(n,x,X,N) \tag{5}$$

The cumulative probability values obtained in terms of binomial sums are a good and reasonable approximation of the probability values in terms of hypergeometric sums in the case of $X \ge n$, $N \ge 50$ $n/N \le 0.1$

Therefore, equation (5)

Can be written as follows:

$$P(n,x,X,N) = \sum_{x=0}^{c} P(n,x,X/N) \tag{6}$$

In addition to the previous distribution, the Poisson distribution can be used to calculate the acceptance probability when the focus is on the number of defects per unit of production rather than the number of defective units. If we have a production process in which the average number of defects per unit of production is equal to μ (and these defects occur randomly so that the number of defects per unit is independent of the number of defects for any other unit of production), the acceptance probability of the quality product is μ equal to:

$$P(\mu) = P_r(x \le c) = \sum_{x=0}^{c} \frac{e^{-n\mu}(n\mu)^x}{x!}$$

$$P(\mu) = G(c, n\mu) \tag{7}$$

The Poisson distribution is a good approximation to a binomial distribution when P is small and n is large, so equation (2) converges to:

$P(P_1) = B(c, n, P_1) = G(c, nP_1)$ When $(P_1 < 0.10)$

This approximation simplifies the solution for parameterizing the sampling plan because the Poisson results depend on the product (nP_1) rather than the two separate parameters P_1, n as in binomial.

Using Dodge-Romig tables

The researchers, Dodge-Romig brought out the results of the SSPs using the Hypergeometric allocation and for (LTPD) system, and the plans were set in particular tables that comprised the SSPs for eight values (LTPD ≤ 0.10) and the values are: 0.5%, 1.0%, 2.0%, 3.0%, 4.0%, 5.0%, 7.0%, and 10.0%.

These techniques have improvements over the classic Dodge-Romig table, such as greater flexibility in selecting LTPD values and attaining reduced average total inspection by adapting the plan to individual process averages and lot sizes.[9]

Utilizing Dodge-Romig tables

The singular Dodge-Romig plans for the AOQL system rely on the following assumptions[10]:

1. The production process operates under binomial control, where the constant defective rate equals P_1.
2. The examination used is of the filtering type.
3. For the producer to ensure that the quality of his product is convenient, he must pick out a value for the maximum percentage of defective units in the batch sent for the inspection (i.e., the AOQL value), Subsequently, consideration is given to the sampling plan that achieves the smallest value for the overall inspection rate $I(P_1)$ among all conjoining plans to this plan the ones that have the same (P_L) value.
4. Dodge-Romig plans for the AOQL system were based on the use of a Poisson distribution rather than a binomial when determining acceptance probabilities $P(P_1)$.

After mentioning the specific assumptions for the AOQL system, we proceed to reformulate equation (8) and make it more suitable for deriving the sampling plan parameters for the AQL system.

First, let's define P_A:

$$P_A = P_1 \frac{(N-n)}{N} P(P_1) \tag{8}$$

$$P_A = P_1 \left(\frac{N-n}{N}\right) \sum_{x=0}^{C} \frac{e^{-nP_1}(nP_1)^x}{x!} \tag{9}$$

If we assume that Pm represents the value of P_1 that maximizes the equation (9), which equals P_L, and that $m = nP_m$, we find that:

$$P_L = \left(\frac{N-n}{N}\right) \frac{m}{n} \sum_{x=0}^{c} \frac{e^{-m}(m)^x}{x!} \tag{10}$$

$$P_L = \left(\frac{1}{n} - \frac{1}{N}\right) m \sum_{x=0}^{c} \frac{e^{-m}(m)^x}{x!} \tag{11}$$

$$Y_c = \left(\frac{1}{n} - \frac{1}{N}\right)$$

Whereas:

$$Y_c = m \sum_{x=0}^{c} \frac{e^{-m}(m)^x}{x!}$$

In additional,

$$Y_c = \frac{nP_L}{\left(1 - \frac{n}{N}\right)} \tag{12}$$

The researchers Dodge-Romig calculated the values of $(m = nP_m)$, Y_c values in equation (12), for different values of c, specifically for $c = 0(1)40$, as they cleared that ion the table.[8] It is noteworthy that equation (12) represents the relationship between N, n, c, and when solved for n, we obtain:

$$n = \frac{Y_c}{(P_L + Y_c N^{-1})} \quad (13)$$

This equation targets the necessary size of the sample to examine the N batch of the product after figuring the values of P_L and c. The researchers Dodge-Romig extracted the result of sampling plans of the AOQL system and put it in a particular table, these tables obtain inspection plans for thirteen values of the value $AOQL \leq 0.10$ and the values are:

0.1%, 0.25%, 0.5%, 0.75%, 1.0%, 1.5%, 2.0%, 2.5%, 3.0%, 4.0%, 5.0%, 7.0%, 10.0%

Each table contains six plans consistent with the worth levels, and for batch sizes groups, if *N* batches were distributed from value (1) to value 100000 in 14 categories, and the value of LTPD was also determined at each level of the quality levels, with the help of these tables the parameters of the sampling plan (n, c) can be read after determining the value of P_L, the quality level, and the category that includes the batch size that produces *N*.

The use of Poisson approximation

The Poisson approximation is counted as a consistent and great approximation for the accurate solution extracted from the binomial, and that's when:

$$(P_2 \leq 0.10, \frac{P_1}{P_2} \leq 0.5, \frac{n}{N} \leq 0.10)$$

To use the Poisson approximation, we have to rewrite the equation (1) as follows:

$$I(P_1) = nP\,(P_1) + NQ\,(P_1)$$

$$= nP(P_1) + NQ\,(P_1) + nQ(P_1) - nQ(P_1)$$

$$I(P_1) = n + (N-n)\,Q\,(P_1) \quad (14)$$

$$= n + (N-n)\left[1 - \sum_{x=0}^{C} \frac{e^{-np_1}(np_1)^x}{X!}\right] \quad (15)$$

From equation (15), the parameters of the sampling plan (n, c) are determined which minimizes the value of $I(P_1)$ as claimed to the specific conditions of the consumer risk $P(P_2)$, that equals $P(P_2),) = G(c, nP_2)$.

Extracting the SSP for the dodge-romig models using Hald tables

Here is the clarification of the method that Hald suggested to determine the parameters of the sampling plan (n, c) for the filtering inspection of the LTPD system, which relies on the direct tables that Hald found, where the values of (n, c) are determined, and for each *N* as an alternative of specifying the value of (n, c) for the category that contains the batch size *N*.

This method relies on the following equation:

$$I(P_1) = n + (N-n)[1-G(c, np_1)] \quad (16)$$

Where we multiply both sides of equation (16) by P_2 to get the following equation:

$$I(P_1)\,P_2 = nP_2 + (N-n)\,P_2\,[1-G(c, np_1)] \quad (17)$$

And it's written briefly as follows:

$$R(c, m) = m + (M-m)[1-G(c, np_1)] \quad (18)$$

According to the fixed value of P_2, the parameters of the SSP (n, c) are determined by minimization of the equation (17) and that under the particular condition of the consumer risk as known as:

$$G(c, m) = 0.10,\ m = np_2$$

SSPs for AOQL System

The AOQL term refers to shortened for Average Outgoing Quality, where 1941 the researchers Dodge-Romig brought out simple sampling plans for the AOQL system, it expresses the average percentage of defective units in the transmitted batch, after doing filtered sampling on them, and the AOQL worth is defined by dividing the predicted value number of defective units in transmitted batch over the total number of units.

If the production procedure for the product of type (*P*) goes under the control of a binomial the number of defective units in the produced batch will trace the binomial distribution with () parameters, and the number of defective units *x* in the haphazard sample *n* will follow the binomial distribution with (n, p). Therefore, when the batch is accepted, the number of defective units $(Y = X - x)$ remaining quantity after pulling out the sample of size n then the binomial will follow with $(N-n, p)$ parameters and the predicted value of *Y* equals $P(N-n)$,[10,11]

In case of the batch rejection, all the batch components will be re-examined, and the defective units will be replaced with better ones, that's why *Y* of 0 possibly 1 in case of rejection, and the predicted value of *Y* can be expressed by the following equation:

$$E(Y) = E(Y|x \leq c)P(X \leq c) + E(Y|X > c)\,P_r\,(X > c)$$

$$= (N-n)PP(P) + (0)Q(P)$$

$$E(Y) = (N-n)PP(p) \quad (19)$$

We use the code (P_A) to express the value of *AOQ*, as follows:

$$P_A = AOQ = \frac{E(Y)}{N}$$

$$P_A = \left(\frac{N-n}{N}\right)P.P(P) \quad (20)$$

According to the following rule, we can derive the value of P_A where it is known that the average of the defective units in the product submitted for inspection equals *Np*, the number of defective units that are found during the inspection will lower this average, therefore the average number of defective units in the product emerging from inspection (NP_A) equals:

$$NP_A = NP - PI(P)$$

$$P_A = \frac{[NP - PI(P)]}{N} = \frac{P[N - I(P)]}{N} \quad (21)$$

Therefore:

$I(P) = n + (N–n)Q(P)$

$$P_A = \frac{[NP - P(n + (N-n)Q(P))]}{N} = \frac{(N-n)}{N} P.P(p)$$

As appeared in equation (20), but in the case of only eliminating defective units as an alternative to replacing them, the value P_A is changed to P_A^* where:

$$P_A^* = P\left(\frac{N - I(P)}{N - PI(P)}\right) \quad (22)$$

The Characteristics of AOQ:

If the value of the type P level was small the P_A will be small as well, then it will be preferred to accept the batches without inspection.

If the P was large all the batches must be thoroughly examined and as a result to repair defective units with good ones, the value of P_A will also be small.

P_A equal Zero when $P = 0$, and PA equals Zero when $P = 1$, (when the large $P = 0$ the entire batch is examined $P_A = 0$).

When P has a value that means P_A is the Greatest possible, which P_L equals:

$P_L = Max_P\, AOQ = AOQL$

This means: that the maximum average of the defective units in transmitted batches after the filtered inspection procedure is expressed as AOQL.

Determination of the Parameters of SSPs for the AOQL System

First: Usage of (Dodge-Romig) tables

Dodge-Romig single plans rely on the AOQL system on the following assumption[10]:

1. The production procedure falls under the control of a binomial and the defective average is fixed it equals P_1.
2. The used inspection is filtered type.
3. The manufacturer must be pleased with the products so that he confirms it. After the inspection, he must choose a value for the highest percentage of defective units in the transmitted batch (i.e., the *AOQL* value). He considers the sampling plan that fulfills the smallest value for the entire rate (P_1) between all the plans close to it that have the same value of (P_1).
4. Dodge-Romig plans were based on using a Poisson distribution rather than a binomial for the *AOQL* system when determining acceptance probabilities $P(P_1)$.

After mentioning propositions *AOQL* own system plans, we reframe equation (20) and make it more acceptable for the reason of deriving the sampling plan parameters for *AQL* system.

Define P_A:

$$P_A = P_1 \frac{(N-n)}{N} P(P_1)$$

$$P_A = P_1\left[\frac{N-n}{N}\right]\sum_{x=0}^{C} \frac{e^{-nP_1}(nP_1)^x}{X!} \quad (23)$$

If we presume that (P_m) represents the value of P_1 that makes the equation (23) as maximum as possible, that is, we find that $nP_m = m$ is equal to P_L:

$$P_L = \left(\frac{N-n}{N}\right)\frac{m}{n}\sum_{x=0}^{c}\frac{e^{-m}(m)^x}{X!} \quad (24)$$

$$P_L = \left(\frac{1}{n} - \frac{1}{N}\right) m \sum_{x=0}^{c}\frac{e^{-m}(m)^x}{X!} \quad (25)$$

$$Y_c = \left(\frac{1}{n} - \frac{1}{N}\right)$$

While,

$$Y_c = m\sum_{x=0}^{c}\frac{e^{-m}(m)^x}{X!}$$

Meaning that:

$$Y_c = \frac{nP_L}{\left(1 - \frac{n}{N}\right)} \quad (26)$$

The researchers, Dodge-Romig measured the ($m = np_m$) and Y_c values from the equation (26) for the other c value, in particular, $c = 0(1)40$ values, they clarified that in the table.[12]

We notice that equation (26) stands for the relationship between N, n, c, and when we solve the (n) we get:

$$n = \frac{Y_c}{(P_L + Y_c N^{-1})} \quad (27)$$

This equation (27) defines the needed sample size for batch N of product knowing the c and P_L values.

Dodge-Romig brought out the outcome of the sampling plan for the AOQL system and set them in its table, these tables involve sampling plans for thirteen values $AOQL \leq 0.10$.

Moreover, the values are: 0.1%, 0.25%, 0.5%, 0.75%, 1.0%, 1.5%, 2.0%, 2.5%, 3.0%, 4.0%, 5.0%, 7.0%, and 10.0%

Every table consists of six plans corresponding to the quality levels and category of batch sizes. If N batches were distributed from value (1) to (100,000) in 14 categories, the value of LTPD was selected at every quality level, and through these tables, the sampling plan parameters can be read (n, c) after selecting P_L value and level of quality and the category that involves the batch size producer N.

Second: Using hald table [28]

The inspections of SSPs for the (AOQL) system, which was brought out by Hald depend on minimizing the total inspection and the results of multiplying sides of equation (15) by the P_L value, which means:

$$I(P_1)\,P_L = nP_L + (NP_L – nP_L)Q(P_1) \quad (28)$$

Which can be written in this form:

$I(P_1)\ P_L = m_L + (M_L - m_L)(1 - G(c, Pm_L)$ (29)

The results of the SSPs were set which replaced the equation (29) in the tables Hald,[11,12] as the value of P, Y_C, c

IMPLEMENTING DODGE-ROMIG MODEL

The process of (Ala Pepsi 1.5 L bottle) production reveled to us that it falls under the control of the beta binomial and because of that beta distribution is considered $\bar{X}_p$ as an evaluate for the type of the product; therefore, the value of beta distribution (P = 0.005349) which is the value that is pointed in the Dodge-Romig model, and from the public facility for the fizzy drinks (Ala Pepsi) it appeared to us that the authorized value of the defective percentage (LTPD) is (P = 1%) whereas the parameters of the SSP (n, c) are extracted, which is mandatory for inspection batch N = 39388 which it represent the everyday production ratio and that's through depending on Table 1[13] that knows the value that is comparable to the size of the batch, which falls under the category (20000–50000) and quality level (0.4–0.5%) and it was found that the parameter sampling plan is (2570.19), and regarding the average total inspection $I(P)$ which is formalized to apply to this plan which is known in equation (14).

$I(P) = n + (N - n)Q(P)$

It was extracted after determining the possibility of approval of the product from the following equation:

$$I(p) = P_r(x \le 19) = \sum_{x=0}^{19} C_x^n (0.005349)^x (1 - 0.005349)^{n-x} = 0.93389$$

$Q(P_i) = 1 - 0.93389 = 0.06611$

$I(P_i) = 2570 + (39388 - 2570)\ 0.611 = 5004$ Sampling Units

Poisson Approximation was implemented after finding the sampling plan for the (LTPD) system through using Dodge-Romig tables to determine the Sampling plan with the help of Hald tables, where it relies on locating the value $M = NP_2$ = 39388 (0.01) = 39388 which falls within the column $\bar{P}$ = 0.5349 in the [Table 2],[11] then knowing the values of (c, m_c) that is comparable to the value of M from the first and second columns, and it was discovered that (c = 24, m_c = 31.58) and that's because the parameters of the SSP are ($n = m_c$, P_2 = 3158, c = 24), and to find the total inspection ratio which is connected to the SSP (3158,24), the value of $Q(P_1)$ was found in the Table 3[11] where $Q(P_1)$ = 0.0195 and therefore the total average is (The average of the accepted and rejected inspection quantity) is:

$I(P_1) = 3158 + (39388 - 3158)\ (0.0195) = 3864$

It appeared to us previously that the total average inspection in using the Poisson approximation is smaller than in the case of using the binomial.

IMPLEMENTING THE DECISION-MAKING MODEL

Based on the decision-making model, a set of Bayesian strategies was selected to assess the product by using past defective proportion data to calculate sample size (n) and acceptance criteria (c). The sampling plans for the product were listed in [Table 2] based on quality levels and batch sizes produced.

Regarding the necessary parameters for the sampling plan to analyze the daily production of the (Ala Pepsi 1.5L) product with a quality of ($\bar{X}_p = 0.005349$), quantity of (0.00617), and extracted based on the sample size (n, c) = (1495,14).

The expected dangerous value for the sampling plan was extracted (1495, 14) and it was found that the value $R\{f(p)\ n, c\}$ equals (43.210$).

IMPLEMENTING [HALD] MODEL

The defective percentages distribution $p(f)$ is a continuous distribution that can be altered at a specific point $P = P_r$ Bayesian plans for product examination will be derived from direct formulas by [Hald], with the required groups of Bayesian plans needing to be calculated beforehand:

Average cost of inspection per unit

1. $k_s = S_1 + S_2\bar{P}$

$= 0.0011 + (0.03)\ (0.005349) = 0.00126\$$

Average cost of rejection per unit

2. $k_r = R_1 + R_2\bar{P} = k_s$

Average cost of acceptance per unit

3. $k_s = A_1 + A_2\bar{P}$

$= 0 + 0.208\ (0.005349) = 0.0011\$$

Also the evaluation quantities (4, 5, 6, 7)

4. $\int_0^{p^r} (P_r - P) f(p)dp = P_r IB_{Pr}(\alpha, \beta) - \bar{P} IB_{pr}(\alpha + 1, \beta)$

$= (0.006179)\ (0.789972) - (0.005349)\ (0.734239) = 0.000954$

5. $k_m = k_r - (A_2 - A_2)\int_0^{p^r} (P_r - P) f(P)dp$

$= 0.00126 - (0.178)\ (0.0009544) = 0.0010901$

6. $\lambda_1^2 = \dfrac{p_r^{\alpha} q_r^{\beta}(A_2 - R_2)}{2B(\alpha, \beta)(k_s - k_m)}$

$\lambda_1^2 = 148.25059$

Whereas:

$P_r = 0.0061$, $A_2 = 0.208$, $R_2 = 0.03$, $\hat{\alpha} = 25$, $\hat{\beta} = 4648$

7. $\lambda_2 = \dfrac{3(\alpha+\beta)^2 - 11(\alpha+\beta) - 2 - (3\alpha - 1)}{p_r} - \dfrac{(3\beta - 1)\beta}{q_r} - \dfrac{1}{p_r q_r - 1}$

The value of λ_1, λ_2 is extracted, therefore the value on (n) needed for the inspection batch will be N = 39388 and it's the particular value of the upcoming context:

$n^* = \lambda_1 + \sqrt{N} + \lambda_2$

$n^* = (12.17582)\sqrt{39388} - 55.4084 = 2361\,Unit$

Table 1: Bayesian plans to test the product against the decision-making model and the Hald model

[Hald] model			Decision-making model			Batch size
Pn(c)	c	n	Pn(c)	c	n	
0.005998286	10	1162	0.00580307	6	669	10000
0.00610687	11	1222	0.005768515	6	701	11000
0.006049403	11	1278	0.005919349	7	733	12000
0.005994006	11	1333	0.005886681	7	763	13000
0.006107626	12	1385	0.005856515	7	791	14000
0.006056638	12	1436	0.00600874	8	819	15000
0.006008444	12	1485	0.005979344	8	846	16000
0.005962933	12	1532	0.005951307	8	872	17000
0.006079027	13	1578	0.005924596	8	897	18000
0.006035578	13	1623	0.006076854	9	922	19000
0.005993691	13	1667	0.006050899	9	946	20000
0.006110937	14	1709	0.006026232	9	969	21000
0.006070984	14	1751	0.006001765	9	992	22000
0.006033416	14	1791	0.005978548	9	1014	23000
0.00599631	14	1831	0.006130671	10	1036	24000
0.006113404	15	1870	0.006107137	10	1058	25000
0.006078104	15	1908	0.00608484	10	1079	26000
0.006044122	15	1945	0.006063756	10	1099	27000
0.006010518	15	1982	0.00621547	11	1119	28000
0.00597818	15	2018	0.006194081	11	1139	29000
0.006094842	16	2054	0.00617284	11	1159	30000
0.006064192	16	2088	0.006152794	11	1178	31000
0.006032961	16	2123	0.006132879	11	1197	32000
0.006003807	16	2156	0.00611413	11	1215	33000
0.006119773	17	2190	0.006264815	12	1233	34000
0.006091371	17	2222	0.00624578	12	1251	35000
0.006062356	17	2255	0.00622686	12	1269	36000
0.006034483	17	2287	0.006208054	12	1287	37000
0.006007724	17	2318	0.006190397	12	1304	38000
0.006123612	18	2349	0.006339673	13	1321	39000
0.006096696	18	2380	0.006321743	13	1338	40000
0.006070874	18	2410	0.006303915	13	1355	41000
0.006045269	18	2440	0.006287227	13	1371	42000
0.006020722	18	2469	0.006270627	13	1387	43000
0.005995538	18	2499	0.006254115	13	1403	44000
0.006111111	19	2527	0.006401838	14	1419	45000
0.006086596	19	2556	0.006385069	14	1435	46000
0.006063111	19	2584	0.006369427	14	1450	47000
0.006039808	19	2612	0.006352826	14	1466	48000
0.006016683	19	2640	0.006337342	14	1481	49000
0.00599455	19	2667	0.006321932	14	1496	50000

Table 2: Bayesian plans for product inspection relative to the Beta-Prior distribution extracted from the decision-making model

Quality level 0.005		Quality level 0.004		Quality level 0.003		Quality level 0.002		Quality level 0.001		Batch size
Accepted number	Sample size	Accepted number	Sample size	Accepted number	Accepted number	Sample size	Accepted number	Sample size	Accepted number	
C	n	c	n	c	n	c	N	c	n	
6	669	5	527	4	449	3	398	7	753	10000
6	701	5	553	4	471	4	418	7	790	11000
7	733	5	578	4	492	4	436	8	825	12000
7	763	5	602	4	512	4	454	8	859	13000
7	791	6	624	5	532	4	471	8	891	14000
8	819	6	646	5	550	4	488	9	923	15000
8	846	6	667	5	569	4	504	9	953	16000
8	872	6	688	5	586	5	519	9	982	17000
8	897	6	708	5	603	5	534	9	1011	18000
9	922	7	727	6	620	5	549	10	1038	19000
9	946	7	746	6	636	5	563	10	1065	20000
9	969	7	765	6	651	5	577	10	1092	21000
9	992	7	783	6	667	5	591	11	1117	22000
9	1014	7	800	6	682	5	604	11	1143	23000
10	1036	8	817	6	696	6	617	11	1167	24000
10	1058	8	834	6	711	6	630	11	1191	25000
10	1079	8	851	7	725	6	642	11	1215	26000
10	1099	8	867	7	739	6	654	12	1238	27000
11	1119	8	883	7	752	6	666	12	1261	28000
11	1139	8	899	7	766	6	678	12	1283	29000
11	1159	8	914	7	779	6	690	12	1305	30000
11	1178	9	929	7	792	6	701	13	1327	31000
11	1197	9	944	7	804	6	712	13	1348	32000
11	1215	9	959	8	817	7	724	13	1369	33000
12	1233	9	973	8	829	7	734	13	1389	34000
12	1251	9	987	8	841	7	745	13	1410	35000
12	1269	9	1001	8	853	7	756	14	1430	36000
12	1287	9	1015	8	865	7	766	14	1449	37000
12	1304	10	1029	8	876	7	776	14	1469	38000
13	1321	10	1042	8	888	7	787	14	1488	39000
13	1338	10	1055	8	899	7	797	14	1507	40000
13	1355	10	1069	8	910	7	807	15	1526	41000
13	1371	10	1082	9	921	7	816	15	1544	42000
13	1387	10	1094	9	932	8	826	15	1562	43000
13	1403	10	1107	9	943	8	836	15	1581	44000
14	1419	11	1120	9	954	8	845	15	1598	45000
14	1435	11	1132	9	964	8	854	15	1616	46000
14	1450	11	1144	9	975	8	864	16	1634	47000
14	1466	11	1156	9	985	8	873	16	1651	48000
14	1481	11	1168	9	995	8	882	16	1668	49000
14	1496	11	1180	9	1005	8	891	16	1685	50000

Table 3: Bayesian plans for the inspection of the Ala Pepsi 1.5L product that's extracted from [Hald]'s model

Quality level		Quality level		Quality level		Quality level		Quality level		Batch size
0.005		0.004		0.003		0.002		0.001		
Number of acceptances	Sample size	Number of acceptances	Sample size	Number of acceptances	Sample size	Number of acceptances	Sample size	Number of acceptances	Sample size	
c	n	c	n	C	n	c	n	c	n	
10	1162	8	798	7	671	3	588	6	529	10000
11	1222	8	839	7	707	3	620	6	557	11000
11	1278	8	879	8	741	3	650	7	584	12000
11	1333	9	917	8	773	3	679	7	611	13000
12	1385	9	954	8	804	3	706	7	636	14000
12	1436	9	989	8	835	3	733	7	660	15000
12	1485	9	1023	8	864	3	759	7	683	16000
12	1532	10	1057	9	892	3	784	7	706	17000
13	1578	10	1089	9	920	3	808	8	728	18000
13	1623	10	1120	9	946	3	832	8	750	19000
13	1667	10	1151	9	972	3	855	8	771	20000
14	1709	10	1181	9	998	3	878	8	791	21000
14	1751	10	1210	9	1022	3	900	8	811	22000
14	1791	11	1238	9	1047	3	921	8	830	23000
14	1831	11	1266	10	1070	3	942	8	849	24000
15	1870	11	1293	10	1094	3	963	8	868	25000
15	1908	11	1320	10	1116	3	983	8	886	26000
15	1945	11	1346	10	1139	3	1003	9	904	27000
15	1982	11	1372	10	1161	3	1022	9	922	28000
15	2018	12	1397	10	1182	3	1041	9	939	29000
16	2054	12	1422	10	1203	3	1060	9	956	30000
16	2088	12	1446	11	1224	3	1078	9	973	31000
16	2123	12	1470	11	1245	3	1096	9	989	32000
16	2156	12	1494	11	1265	3	1114	9	1006	33000
17	2190	12	1517	11	1285	3	1132	9	1022	34000
17	2222	13	1540	11	1304	3	1149	9	1037	35000
17	2255	13	1563	11	1323	3	1166	10	1053	36000
17	2287	13	1585	11	1342	3	1183	10	1068	37000
17	2318	13	1607	11	1361	3	1200	10	1083	38000
18	2349	13	1629	12	1380	3	1216	10	1098	39000
18	2380	13	1650	12	1398	3	1232	10	1113	40000
18	2410	13	1672	12	1416	3	1248	10	1127	41000
18	2440	13	1693	12	1434	3	1264	10	1142	42000
18	2469	14	1713	12	1452	3	1280	10	1156	43000
18	2499	14	1734	12	1469	3	1295	10	1170	44000
19	2527	14	1754	12	1486	3	1310	10	1184	45000
19	2556	14	1774	12	1503	3	1325	10	1197	46000
19	2584	14	1794	12	1520	3	1340	10	1211	47000
19	2612	14	1813	13	1537	3	1355	11	1224	48000
19	2640	14	1833	13	1553	3	1370	11	1237	49000
19	2667	14	1852	13	1570	3	1384	11	1251	50000

However, the number of is extracted from the $c^* = n^* p_r + \beta_1$ context which means:

$$\beta_1 = \hat{\beta} P_r - \hat{\alpha} q_r - \frac{1}{2}$$

Therefore, the value of the accepted unit (n = 2361) equals:

β_1 = (4648) (0.0061) – (25) (0.9939) – 0.5

β_1 = 3.0053

And added:

c^* = 2361 (0.061) + 3.0053) = 17 Unit

As a result, a SSP is necessary to assess the production mean of N=39388, as specified in [Hald]'s model (2361, 17). This plan entails inspecting a random sample size of (2361) bottles, and if the number of defective units in the sample is (17) or fewer, all units are accepted; otherwise, the batch is rejected.

The cost of quality control, determined by the sampling plan, will be derived from the smallest standard cost R_0 (N), resulting in the best sampling plan (2361.17) being implemented:

$$R_0 = \left(2n^* - \lambda_1^2 - \lambda_2\right) ds,\ ds = \frac{k_s - k_m}{(A_2 - R_2)}$$

ds = 0.000954

R_0 (N) = [2(2361)–148.2506 + 55.4084] (0.000954)

R_0 (N) = 4.4162

Therefore, the total value cost $k(p)$ is expected to control quality type which equals to:

$k(p) = R_0\ ((A_2 – R_2) + NK_m$

=4.4162 (0.178) + 39388 (0.001091)

=42.9731 $

Table 3 displays the outcomes of Bayesian strategies for assessing the Ala Pepsi 1.5L item derived from [Hald]'s model, based on the Beta distribution, categorized into quality levels $\bar{X} = 0.001(0.001)0.005$ and production batch sizes N = 10000 (1000) 50000.

CALCULATING THE VALUE OF THE DEFECTIVE FRACTION IN UNEXAMINED QUANTITIES

Based on decision-making models and the Hald model, a set of Bayesian strategies are selected to assess the product. Determining the expected value of the fraction defective in the untested quantities (N–n) is crucial. and they are going to be approved based sample's acceptance, after that we will depend on the average value of the distribution following the detective amount $E(p/x)$ when $X = c$ which means $Pn(c)$ and it appeared to us in the equation (31) that the next distribution $f(p/x)$ is also a Beta with parameters ($x + \alpha$, $\alpha + \beta + n$) therefore it is:

$$E(x) = P_n(x) = \frac{x+\alpha}{\alpha+\beta+n} \qquad (30)$$

And when $X = c$ it is:

$$P_n(c) = \frac{c+\alpha}{\alpha+\beta+n} \qquad (31)$$

The forthcoming [Table 1] shows a comparison of Bayesian designs based on [Hald]'s model and the Decision-making model, considering the quality level ($\bar{X}_p = 0.005349$), value $Pn(c)$, and all plans from both models.

Table 1 clearly shows that the expected value of the defective fraction in the quantities (N – n) that will accepted under the sample approval decision equals 0.6%, according to the Decision-Making model equals 0.6%, and according to [Hald] model that is the same value as the sells percentage that is allowed through (LTPD) that relies on the Pepsi company for fizzy drinks and the correspondences of the average value of (Pn) and (LTPD) shows the efficiency of the Bayesian plans that relies on the previous distribution of defective percentages, as it takes all available and previous information about the quality in consideration when estimating the quality in subsequent production batches, and the correspondence indicates the importance on the Bayesian plans and efficiency and the quality level of actual production $\bar{X}_p = 0.005349$, as we find that the size of the sample for different batch sizes is small compared to other sampling plans which means reducing the cost of them and therefore the reduction of the total cost.

CONCLUSION

The Beta-Binomial distribution with a rate of 0.005349 was found to be a suitable probability distribution for modeling the proportion of defective products in the Ala Pepsi factory. In this specific case, the Poisson approximation resulted in a smaller total average inspection compared to the binomial distribution, suggesting potential cost savings.

The study also demonstrated the effectiveness of the decision-making model in determining a sampling plan (n = 1495, c = 14) that resulted in smaller sample sizes and reduced inspection costs compared to the Dodge-Romig tables method. The expected value of the defective fraction in the untested quantities was consistent with the allowed LTPD, highlighting the efficiency of Bayesian plans in incorporating prior knowledge about quality.

These findings have significant implications for quality control practices in the soft drink industry, suggesting that Bayesian sampling plans can be a valuable tool for optimizing inspection processes and reducing costs. Future research could explore the generalizability of these findings to other industries and investigate the use of different prior distributions in Bayesian sampling plans.

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