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RESEARCH ARTICLE

Adopting Discrete Wavelet Transformation with Mean Time Between Failure to Design a New Quality Control Chart

Hawkar Q. Birdawod[1], Dlshad M. Saleh[2], Dashty I. Jamil[3], AbdulRahim K. Rahi[4]

[1]Department of Business Administration, College of Administration and Financial Sciences, Cihan University-Erbil, Kurdistan Region, Iraq, [2]Department of Statistics and Informatics, College of Administration and Economics, Salahaddin University-Erbil, Kurdistan Region - F.R. Iraq, [3]Department of Accounting, Cihan University-Erbil, Kurdistan Region, Iraq, [4]Department of Business Administration College of Administration and Economics, Al-Ameen University, Baghdad, Iraq

ABSTRACT

This study's goal was to design a new individual observation chart for quality control by utilizing the discrete wavelet transformation (DWT) approach to calculate the mean time between reliability failures. The new chart's effectiveness was demonstrated by comparing it to a classical chart. This aspect is crucial in addition to the conclusion that the production process is under control for both charts, with the new chart exhibiting a reduced standard deviation. The study's main focus was on a new area for device monitoring and defect detection, which involves halting production and eliminating any potential causes of defects. One of the main elements that aid in achieving the goals of predictive maintenance is the individual control chart based on the mean time between failures with DWT of the operating time between failures.

Keywords: Quality control, individual observation chart, reliability, mean times between failure, discrete wavelet transformation

INTRODUCTION

Quality science is considered one of the fastest developing sciences in recent decades, as it has evolved from merely inspecting products to a diverse group of overlapping specialties and systems interconnected with each other and with various other specialties in the industry.[1] Standard specifications have evolved from mere product specifications to specifications for quality systems associated with all facility activities. The use of statistical methods can help understand variables and thus help facilities solve problems and improve efficiency and returns.[2] These methods also facilitate better use of available data to help make the right decision. The importance of the topic of reliability in our practical life comes to know cases of equipment and machinery systems failure or failure, which reduces the cost of their production and maintenance. It is also important in protecting and averting danger to human life by evaluating the performance and efficiency of these systems. One of the technical and important methods that lead to improving quality is the use of statistical methods in the field of quality control.

The specific goal of this research is to develop a new individual observation control chart for mean time between failure (MTBF) by incorporating discrete wavelet transform (DWT) for signal processing and to evaluate its effectiveness compared to classical charting methods. Adherence to these specifications will lead to improving the quality of production and reducing damage, in addition to protecting the consumer and the producer from commercial fraud and reducing costs. To discover faults early, DWT is frequently used to analyze signals from electronic components or equipment. It allows for the detection of irregularities or abrupt shifts that can point to possible problems by breaking down signals into their component frequencies.[2] DWT can be used on vibration or acoustic data in rotating machinery to detect anomalous frequency patterns linked to shaft misalignment, gear problems, or bearing wear. The capacity to identify minute alterations at various scales (including short-term and long-term problems). Wavelet thresholding improves diagnostic accuracy by reducing noise.

Corresponding Author:
Hawkar Q. Birdawod, Department of Business Administration, College of Administration and Financial Sciences, Cihan University-Erbil, Kurdistan Region, Iraq.
E-mail: hawkar.birdawod@cihanuniversity.edu.iq

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BASIC CONCEPTS OF STATISTICAL QUALITY CONTROL

Control

The process of monitoring and inspecting a production process to keep conforming to the standards to produce a high percentage of acceptance quality.[3]

Quality

When consumers are choosing between competing goods and services, quality has emerged as one of the most crucial deciding factors.[4,5]

Quality Control

The use of methods and procedures to attain, maintain, and enhance the quality of a good or service is known as quality control. It entails incorporating the following associated methods and exercises:[3,6]

1. An explanation of the requirements
2. A product or service that is designed to satisfy the requirements
3. Installation or production that satisfies the standard in its entirety
4. Inspection to assess if the specifications are being followed
5. Check if the specification has been revised.

Quality Characteristics

Two major categories can be used to classify quality attributes:

1. Measurable characteristics: These are traits that may be quantified and represented numerically on certain continuous scales of measurement; control charts of this kind are known as variable charts.
2. Unmeasurable characteristics: These are traits that are not measurable on a continuous scale or even a quantitative scale. Attribute charts are the control charts used to measure these traits.[6]

Quality Control Charts

The process is statistically controlled, a quality control chart, also known as a process chart, is a graph that displays the average for the data (output) or the product falling within the typical or typical range of variation. In 1924, Walter A. Shewhart of Bell Telephone Laboratories created the first quality control chart, which he and his colleague later improved. In 1931, he released a comprehensive explanation of control charts.

Shewhart control charts consist of three parallel lines which are:[7]

(T) represents the center line (also known as the target line) of the control chart, which is the mean or overall average of the quality feature being monitored.

The largest allowable deviation from the mean for a process in a state of control is known as the upper control limit (UCL).

Mathematically expressed as:

$$UCL = T + 3\sigma$$

The smallest allowable deviation from the mean for a process in a state of control is known as the lower control limit (LCL).

Mathematically expressed as:

$$LCL = T\text{-}3\sigma$$

$$MTBF = E(T) = \int_0^\infty t f_T(t)dt = \int_0^\infty R_T(t)dt$$

RELIABILITY

It is known that the quality of the product may change with the age of the product, that is, its efficiency decreases over time. Accordingly, one of the aspects of product acceptance depends on its ability to perform satisfactorily for a period of time.[8] This aspect is known as product reliability, that is, its capacity to remain appropriate for the task at hand or to satisfy the demands of the customer. Therefore, the product's long-term quality continuity is what determines its dependability.[9]

The reliability function is expressed by the following mathematical equation:

$$R(t) = 1 - \int_0^t f(t)dt = 1 - F(t)$$

Where:
t: Random variable representing time
R(t): Reliability function
f(t): Probability Density Function (PDF)
F(t): Cumulative Density Function.

MTBF

The predicted amount of time the system should operate before failing is known as the MTBFs.[10]

Exponential Distribution

Based on the assumption that the time between failures follows an exponential distribution, a classical individual control chart for MTBF can be constructed. The control limits are derived using natural logarithms (ln) to transform the data for symmetry, corresponding to a standard Type I error rate (false alarm rate) $\alpha = 0.0027$ (equivalent to 3-sigma limits for normally distributed data).

Many important industrial uses in terms of measuring the lifespan of electrical goods such as the lifespan of bulbs and others, in addition to determining the time required until electronic devices fail to perform their functions.[11] The exponential distribution is also used primarily in reliability theory, which is the theory that determines the extent of the system's ability to work and perform its function during the time period T without failure. If we assume that there is a device whose lifespan or life time is expressed by the continuous random variable T, and thus takes values in the continuous period from the smallest value (zero to infinity), then the probability that this system will work without failure is called the reliability function as follows R = P(T>0). A random variable (t) is said to have an exponential distribution if the PDF of this variable is:

$t \geq 0 \; f(t;\lambda) = \lambda e^{-\lambda t}$

$= 0$ elsewhere

and

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i} = \frac{1}{\bar{t}} \Rightarrow \frac{1}{\hat{\lambda}} = \bar{t} = \mathrm{E}(t) = MTBF$$

Classical Individual Values Chart Based on MTBF

It is employed when it is more cost-effective to take a single production line observation each time period to regulate the product's quality. The overall average of the qualitative characteristics of all production line observations is represented by the target line for this section of the graph.

Where:
α: false alarm in quality control charts equal to (0.0027).

$$LCL = MTBF * \ln\left(\frac{2}{2-\alpha}\right)$$

$$CL = MTBF * \ln(2)$$

$$UCL = MTBF * \ln\left(\frac{2}{\alpha}\right)$$

FUNDAMENTAL IDEAS FOR ACHIEVING A DWT OF DATA

DWT

A mathematical method for breaking down signals into their time-frequency components is called the DWT. DWT is very helpful for evaluating non-stationary signals (signals whose frequency content fluctuates over time), as opposed to the Fourier Transform, which only analyzes data in the frequency domain. This is because DWT records both time and frequency information.

In many fields, including science, engineering, mathematics, and computer science, the DWT is a broadly applicable signal processing algorithm. DWT uses scaled and shifted versions of a compact supported basis function (mother wavelet) to break down a signal.

Given a vector of a signal *X* consisting of 2^j observation where *j* is an integer. The DWT of *X* is

$$W = wX$$

Where *W* is an $n * 1$ vector comprising both discrete scaling and wavelet coefficients. The vector of wavelet coefficients can by organized into $j + 1$ vectors.

$$W = \left[W1, W2, \ldots, Wj_0, Vj_0\right]^T$$

Where W_j is a length $N_j = N/2^j$ vector of wavelet coefficients (Details) associated with changes on a scale of length $\lambda_j = 2^{j-1}$ symboled as CD, and V_{jo} is a length $N_{j0} = N/2^j$ vector of scaling coefficients (approximation or smoothing) associated with average on a scale of length $\lambda_{J_0} = 2^{J_0}$ symboled as CA, and *w* is an orthonormal *N*N* matrix associated with the orthonormal wavelet basis chosen.

Following each DWT, the approximation coefficients are separated into bands using the same filter as previously. This result in the details being appended with the most recent decomposition details, and at each level, the inverse transform may rebuild the de-noised signal.

$$X = Ww^T = \sum_{j=1}^{j_0} W_j^T W_j + V_{j0}^T V_{j0}$$

Universal Thresholding Method

The threshold by splitting the wavelet coefficient into two sets, one representing the signal and the other representing the noise, thresholding is the most straightforward technique for non-linear wavelet denoising. The wavelet coefficient thresholds can be applied according to many principles, and there are numerous approaches to selecting a threshold value, including:

$$\eta^U = \tilde{\sigma}_{(MAD)}\sqrt{2\log N}$$

Soft Thresholding Rule

$$Wn^{(st)} = sign\{Wn\}(|Wn| - \eta)$$

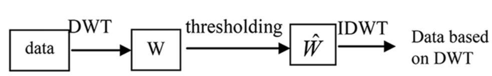
Soft thresholding of the wavelet coefficient, which was also suggested by Donoho and Johnstone, is the other common method for wavelet denoising.[12] It is defined as follows:

Daubechies (Db) Wave

Ingrid Db is the originator of the Db wavelets, a family of orthogonal wavelets renowned for their smoothness and compact support. Because they can analyze signals with little overlapping effects and maintain crucial properties, such as crisp edges, they are frequently utilized in signal and image processing.[13]

The basic outline of the wavelet shrinkage is pioneered by Donoho and Johnstone as follows:

1. The DWT converts the data into a new representation known as wavelet coefficients. The orthogonal matrix W multiplies them
2. The thresholding rule is used to alter the wavelet coefficients. The fundamental idea of wave shrink is to reduce the number of coefficients
3. To get an approximation of the signal, the changed coefficients are subjected to the inverse discrete wavelet transformation.



A new Individual Values Chart Based on MTBF with DWT

To design the new chart using the soft threshold rule and small wave (db2), and to estimate the threshold level using the universal approach, a new chart will be created and contrasted with the classical ones, as shown in Figure 1 below.

Consequently, we can determine the new control chart's based on MTBF with DWT, control limits (LCL & UCL), and central line by using the following formula:

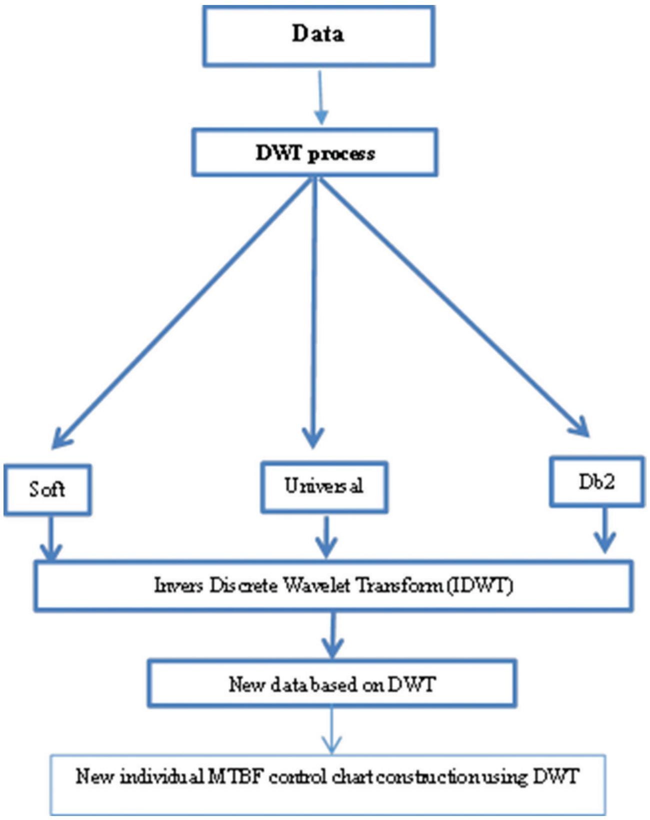


Figure 1: Block diagram to transform data and construction individual control chart

$$LCL = MTBF_{DWI} * \ln\left(\frac{2}{2-\alpha}\right)$$

$$CL = MTBF_{DWI} * \ln(2)$$

$$UCL = MTBF_{DWI} * \ln\left(\frac{2}{\alpha}\right)$$

Applied Part

The primary objective of proposing and developing a new technique is to offer a substitute for addressing some of the issues and shortcomings of the existing approaches; hence, the new method must be used to determine its suitability for reality and, consequently, its accuracy and efficiency.[11] Based on this idea, we will create and use individual observations charts for the exponential distribution using the DWT and MTBFs in this study on actual data to determine the effectiveness, precision, and appropriateness of this chart. The data were gathered from the Babylon tire factory to provide a clear understanding of how to create and use the individual observations chart for the exponential distribution using the operational times of one failure and another.[14] Where this data represent the operating times (hours) between one failure and another through the times that were recorded in the internal statements of the factory.

Figure 2 shows that all points are within the control limits, and based on the chart's initial formation (chart formation for the first time), we can see that the data in Table 1 are suitable for creating this chart. This indicates that the same party from which we obtained the data can use this chart going forward for control and monitoring purposes.[10]

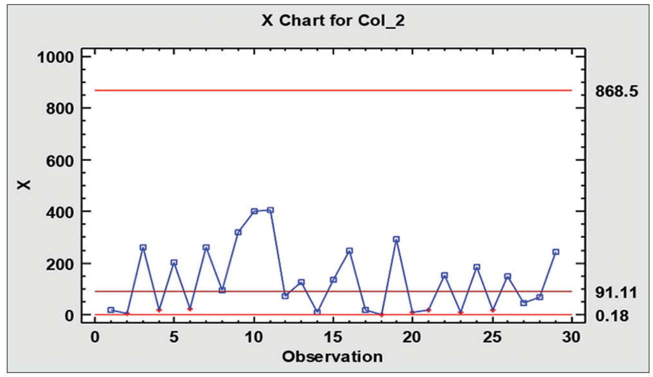


Figure 2: Individual observation chart of the exponential distribution using the mean time between failure

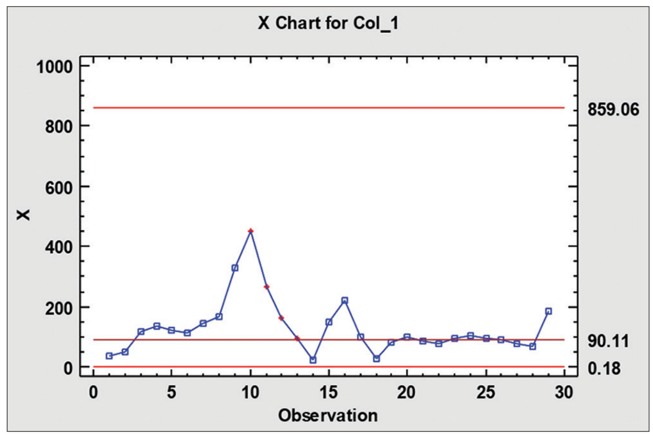


Figure 3: Individual observation chart of the exponential distribution using the mean time between failure with discrete wavelet transform

In addition to demonstrating that all points fall inside the control boundaries, Figure 3 also demonstrates that the values in Table 1 are appropriate for the creation of this chart based on the chart's initial formation. This means that the same person who provided the data can use this chart in the future for monitoring and control.[15]

DISCUSSION

To move the qualities closer to the target line, DWT is helpful approach. In other words, this chart converts irregular cases to regular cases. Both charts based on MTBF under controlled conditions, where no observations fall beyond the control limits, is the manufacturing process. One of the main elements that aids in achieving the goals of predictive maintenance is the individual control chart based on MTBF with DWT of the operating time between failures. This focuses on a new area of predictive maintenance that is based on the device and control to be a tool for predicting maintenance activities. The MTBF control chart based on DWT is an alternative chart to the MTBF control chart. Furthermore, the use of DWT is beneficial for reducing standard division. The process has deteriorated if the point drops below the LCL, in which case appropriate action needs to be performed. The failure rate may have

Table 1: The operating times (hours) between failure and another

S. No.	MTBF	MTBF with DWT
1	18.75	35.234
2	4	50.001
3	259.5	119.74
4	19	135.79
5	203.5	120.6
6	24	113.78
7	261	146.11
8	96	167.96
9	321	327.43
10	402.5	450.02
11	404	264.9
12	72	162.22
13	127.5	96.494
14	10.5	20.866
15	135	148.35
16	247	221.41
17	17	97.816
18	2.5	26.914
19	292.5	81.002
20	8.5	101.6
21	19.25	85.507
22	152	79.245
23	11.5	95.27
24	183.25	105.32
25	17.5	96.351
26	147	92.478
27	44.5	78.219
28	66.5	66.744
29	245	183.03
Mean	131.45	130.01
Standard division	126.3144	91.26084

MTBF: Mean time between failure, DWT: Discrete wavelet transform

dropped, increasing the interval between failures, if the point is above the UCL. This is a significant sign in the process of improvement. The management of the factory should act to determine the cause and keep it up to date if this occurs.

CONCLUSION

Integrating DWT with MTBF offers a robust approach to quality control chart design. This hybrid method enhances the sensitivity and accuracy of detecting process shifts, particularly in complex and non-stationary manufacturing environments. By leveraging the signal decomposition capabilities of DWT and the reliability insights from MTBF, the proposed control chart provides a more effective tool for monitoring and improving product quality and system performance.

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