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Content knowledge of undergraduate students in solving the algebra's problem

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Abstract: Content knowledge (CK) is essential in mathematics education, particularly for understanding algebraic concepts. This study explores the algebraic content knowledge of undergraduate mathematics education students. Using a descriptive qualitative approach, the research involved third-year students from a private university in East Nusa Tenggara, Indonesia. Two validated instruments, the Diagnostic Test (TD) and the Content Knowledge Test (TCK), were employed to assess students' understanding. The instruments showed strong validity scores, with 4.6 for the TD and 4.0 for the TCK (on a 5-point scale), and reliability scores of 0.9993 for the TD and 0.9983 for the TCK. Data were analyzed using two frameworks: the Direct Translation Approach (DTA) and the Meaning-Based Approach (MBA). Findings revealed varied levels of CK: some students demonstrated procedural fluency and the ability to relate symbolic representations to contextual meaning (MBA), while others exhibited difficulties in applying concepts flexibly, leading to illogical reasoning (DTA). These outcomes highlight the need to strengthen conceptual understanding and reflective thinking among pre-service teachers. The study recommends instructional strategies that integrate symbols, context, and meaning to enhance students' ability to solve algebraic problems effectively. Such approaches are crucial for preparing future mathematics teachers capable of connecting mathematical reasoning to real-world contexts.

Keywords: *Algebra, Pedagogical content knowledge, Problem solving.*

1. Introduction

Pedagogical Content Knowledge (PCK) refers to the integration of Content Knowledge (CK) and Pedagogical Knowledge (PK), both of which are essential for effective teaching and learning [1-7]. CK encompasses the understanding of concepts, theories, rules, and procedures related to the subject matter, while PK pertains to instructional strategies that help students learn and apply this knowledge to solve mathematical problems. This conceptual framework, introduced [7] emphasizes that teachers must not only understand what to teach but also why a concept works the way it does. As such, they are expected to deliver instruction and address mathematical problems in ways that align with students' developmental levels, especially in the domain of algebra.

Algebra is not only a component of higher education curricula but is also taught from lower secondary school onwards [3]. At the secondary level, algebraic content includes topics such as algebraic expressions, operations, and relationships within contextual problems. Algebra learning is crucial for developing students' critical thinking skills. However, in Indonesia, fewer than 5% of secondary school students can solve contextual problems related to real-life situations [8]. One major factor contributing to this issue is the lack of content mastery in algebra among teachers. Previous studies indicate that many teachers struggle to grasp fundamental algebraic concepts [8].

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A systematic review of 237 mathematics PCK studies conducted over the past decade identified algebra as one of the most frequently examined topics, highlighting 83 learning obstacles. These include both conceptual and procedural difficulties commonly encountered by students, as well as a lack of appropriate pedagogical strategies to address them [9]. The review recommends the use of manipulative tools and visualization-based approaches to strengthen teachers' PCK.

The problem of limited algebraic content mastery is not confined to in-service teachers; it also affects prospective teachers. A study Rupasinghe [10] revealed that lower secondary mathematics teachers in Sri Lanka still show low levels of PCK, particularly in interpreting algebraic symbols and addressing student misconceptions. Similar patterns have been observed in Indonesia, where pre-service mathematics teachers often make fundamental errors in algebraic operations. For example, some students misinterpret expressions within parentheses during multiplication operations due to a poor understanding of structural components [2].

Observations during microteaching sessions which serve as a prerequisite for teaching internships, reveal that many students incorrectly factor expressions such as $a^2 - 25b^2$, by providing flawed solutions like $(a - 25b)(a - 25b)$, indicating a lack of understanding of the difference of squares. This finding echoes the results of a recent meta-analysis that identified conceptual misunderstandings in algebra as a common issue among pre-service teachers across various regions [11]. Additionally, studies have shown that a significant portion of mathematics education undergraduates in Jombang are unable to solve mathematical problems conceptually [8].

To address this gap, it is important to assess students' conceptual understanding through the Content Knowledge Test (TCK), which evaluates their ability to solve algebraic problems [12]. The TCK serves as a foundational assessment for undergraduate students before they begin teaching internships in junior secondary schools, allowing educators to infer the depth and diversity of students' mathematical understanding even when it is not directly observable.

The TCK used in this study consisted of three algebra-based problems designed to explore students' problem-solving abilities. Problem-solving [13] involves applying concepts and skills to achieve specific goals. Successful problem-solving requires multiple cognitive processes, including understanding the problem, organizing information, identifying relationships, selecting strategies, monitoring progress, and interpreting mathematical meaning.

According Pape [14] students' problem-solving strategies can be categorized into five approaches: Direct Translation Approach Proficient (DTA-p), Direct Translation Approach Non-Proficient (DTA-np), Direct Translation Approach Limited Context (DTA-lc), Meaning-Based Approach Full Context (MBA-fc), and Meaning-Based Approach Justification (MBA-j). Therefore, this study aims to explore the content knowledge of undergraduate students in solving algebraic problems.

2. Theoretical Framework

Content Knowledge (CK) is one of the foundational components of effective mathematics instruction, particularly within the context of teacher education. This concept was first comprehensively introduced by Shulman [7] who distinguished between content knowledge and pedagogical content knowledge. CK refers to the knowledge necessary to understand the curriculum and integrate it into instruction, particularly in relation to specific subject matter aligned with grade-level expectations [15].

In mathematics, content knowledge encompasses the understanding of concepts, rules, theorems, and properties. It includes knowledge of fundamental mathematical concepts, procedural fluency, the interrelation among concepts, and various mathematical representations Hill, et al. [5]. Hill, et al. [16] further categorize content knowledge into two essential domains: Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK). CCK refers to the mathematical knowledge that is generally accessible to all individuals, whereas SCK is specific to teachers and pertains to instructional contexts for instance, the ability to identify and explain students' mathematical errors.

Therefore, the content knowledge possessed by undergraduate pre-service teachers becomes a critical prerequisite for conducting reflective and adaptive teaching practices. During instruction, pre-service teachers are required to demonstrate both conceptual and procedural knowledge, which enables them to solve problems effectively and provide accurate explanations to students [16].

A problem is a theoretical or practical difficulty that stimulates curiosity [17]. In the context of mathematics education, a problem refers to a question or task drawn from everyday life that requires non-routine procedures to solve typically involving more than one mathematical operation, systematic problem-solving steps, in-depth analysis, and the application of different procedures to arrive at a solution [18]. Problems used in mathematics instruction are often contextualized within daily experiences and necessitate non-routine strategies for resolution. Therefore, problem-solving is regarded as a central component of mathematics learning [17]. Problem-solving approaches can be categorized into two major types [14]: The Direct Translation Approach (DTA) and the Meaning-Based Approach (MBA). The Direct Translation Approach is characterized by a limited ability among students to transform the information presented in a problem into mathematical expressions. This includes a lack of context integration, insufficient use of relevant mathematical concepts, and difficulty in connecting various pieces of information within the problem. DTA is further divided into three subcategories: DTA-proficient (DTA-p): Students in this category utilize the given information automatically and efficiently in mathematical computations, typically relying on their initial reading of the problem without needing to reread it. DTA-non-proficient (DTA-np): Students exhibit low competence in reading comprehension, problem understanding, solution planning, and mathematical computation. DTA-limited context (DTA-lc): Students apply the given data directly into computations but provide minimal contextual explanation related to the problem. In contrast, the Meaning-Based Approach (MBA) is marked by three key features: documenting given information, utilizing the context of the problem, and offering logical justifications for the mathematical operations employed. MBA is divided into two subcategories: MBA-full context (MBA-fc): Students carefully read each sentence of the problem and fully consider the context in their problem-solving process. MBA-justification (MBA-j): Students approach each problem using a structured schema, incorporating logical reasoning and contextual interpretation. Several studies have highlighted the lack of content knowledge among undergraduate students when engaging in conventional instructional settings, particularly in solving algebraic problems [3]. This gap in content understanding underscores the necessity of equipping pre-service teachers with both conceptual and procedural knowledge to enhance their problem-solving competence in mathematics education

Algebra is a branch of mathematics concerned with the use of symbols and rules to achieve specific objectives [19]. The nature of algebra at the university level differs significantly from the algebra taught in school settings. At the school level, algebra primarily involves the understanding of variables and the operations associated with them [20]. Algebraic problems frequently require generalization, symbolic manipulation, as well as numerical and graphical representations [21]. These problems can be broadly classified into three categories: (1) routine problems, which emphasize the application of specific algorithms; (2) non-routine problems, which demand flexible and strategic problem-solving approaches; and (3) contextual problems, which incorporate real-world situations. Solving algebraic problems necessitates mastery of fundamental concepts, along with the capacity for abstract and symbolic reasoning—skills that often present challenges for undergraduate students [22]. Students with strong content knowledge tend to recognize mathematical structures within algebraic problems and employ appropriate solution strategies. In contrast, limited conceptual understanding often results in systematic errors, such as mistakes in symbolic manipulation, misapplication of operational rules, or misconceptions regarding the meaning of variables [23].

As such, the algebraic content knowledge of undergraduate students has become a focal point in studies examining the readiness of prospective mathematics teachers to meet the demands of curriculum reform and problem-based instruction. Zazkis and Leikin [23] noted that prospective teachers frequently exhibit a gap between procedural fluency and conceptual understanding when solving algebraic tasks. They may be able to perform correct procedures, yet struggle to articulate the rationale behind their steps. Further evidence from Stephens, et al. [24] suggests that many students demonstrate difficulties in flexibly interpreting algebraic representations. For example, students often struggle to transition between symbolic and graphical forms or fail to grasp the role of parameters in linear and quadratic functions. These challenges indicate that their existing content knowledge is insufficient to support conceptual problem-solving in algebra.

Moreover, research has shown that students' abilities in solving algebraic problems are strongly influenced by their prior learning experiences, the quality of instruction received, and the extent to which they are given opportunities to reflect on their mathematical thinking. Therefore, it is essential to explore the nature of undergraduate students' content knowledge in algebra as a foundation for explicitly developing this knowledge in teacher education programs. Such development can be fostered through problem-solving tasks, reflective discussions, and in-depth exploration of core algebraic concepts

3. Research Methodology

This study employed a descriptive qualitative approach to explore the content knowledge abilities of third-year undergraduate students in mathematics education. Participants were selected from a private university in East Nusa Tenggara, Indonesia, specifically from the 2019 cohort: Class A (10 students) and Class B (15 students). For the purpose of this research, Class A comprising 10 female students was chosen due to their active participation and communicative learning behavior observed during the sixth semester.

Two primary instruments were utilized in the study: Diagnostic Test (TD):Consisted of five open-ended questions covering foundational mathematical topics such as numbers, sets, algebra, and linear equations. Content Knowledge Test (TCK): Included three open-ended questions focused specifically on algebraic concepts.

Both instruments were validated by three assistant professors in mathematics education using a five-point Likert scale [25]. The validation process produced strong results, with average validity scores of 4.6 for the Diagnostic Test (TD) and 4.0 for the Content Knowledge Test (TCK) (on a maximum scale of 5.0), indicating a high level of content and construct validity. Reliability coefficients were also high: 0.9993 for the TD and 0.9983 for the TCK.

Students were allotted a total of 100 minutes to complete both tests. All 10 students from Class A completed the TD and TCK. Their responses to the TCK were analyzed using two theoretical frameworks: the Direct Translation Approach (DTA) and the Meaning-Based Approach (MBA) [14]. These were further broken down into subcategories: DTA Subcategories: DTA-Proficient (DTA-p), DTA-Non-Proficient (DTA-np), DTA-Limited Context (DTA-lc). MBA Subcategories: MBA-Full Context (MBA-fc) and MBA-Justification (MBA-j) [14]. The empirical testing results are summarized in Table 1.

Table 1.
Empirical Test Results of the Diagnostic Test (TD) and the Content Knowledge Test (TCK).

Validation Aspect	TD	TCK
Difficulty Level	0.69	0.69
Discriminating Power	0.58	0.60
Empirical Validity	0.99	0.99
Reliability	1.00	0.99

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Table 2.
Recapitulation of Diagnostic Test (TD) and Content Knowledge Test (TCK).

No	Initials	Gender	TD 1	TD2	TD3	TD4	TD5	TD Score	TCK 1	TCK 2	TCK 3	TCK Score
1	MP	F	14	12	16	16	22	80	5	6	9	67
2	YB	F	16	12	16	20	16	80	4	10	9	77
3	KW	F	12	12	10	20	20	74	8	7	10	83
4	EM	F	10	12	18	20	8	68	7	9	10	87
5	MS	F	14	8	12	10	16	60	3	7	10	67
6	MY	F	10	12	20	20	28	90	10	10	9	97
7	MA	F	16	12	20	18	12	78	3	5	9	57
8	MM	F	20	12	18	18	26	94	7	6	10	77
9	TR	F	16	12	16	18	6	68	5	9	8	73
10	FA	F	10	12	18	18	4	66	2	9	8	63

Based on the results in Table 2, three students were purposively selected for interviews according to the following criteria: (i) Their TCK responses demonstrated distinctive patterns or unique characteristics; (ii) Their answers addressed at least three out of four problem-solving components: *Networking*, *Extraction*, *Monitoring*, and *Strategy*; and (iii) They voluntarily agreed to participate in interviews conducted outside of class hours. The interviews were conducted in a calm and supportive environment to encourage thoughtful and reflective responses from participants.

3.1. Content Knowledge Test (TCK)

3.1.1. Problem 1 (40 Minutes)

In 1963, Indonesia first launched two rockets from the Sanden Beach, Bantul, Yogyakarta students of Gajah Mada University (UGM). The rockets were Gama II A and II B with 454 kg of solid fuel, shaped as below the rocket is launched from the earth to the moon and is scheduled to enter the mesosphere layer (the third layer of air from the earth as a constituent of the atmosphere) at 7:00 am. If the rocket is launched at $40\ km/h$, it will enter the mesosphere 10 minutes late from 7 am. If the rocket is launched at $60\ km/h$, it will enter the mesosphere layer 20 minutes before 7 am. What is the distance between the earth and the mesosphere layer?

3.1.2. Problem 2 (30 Minutes)

Below are 3 wooden block arrangements that have different heights and are composed of two different shapes, namely a block and a tube:

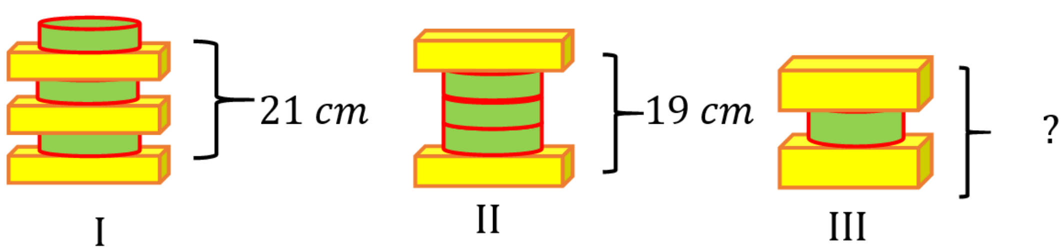

Figure 1.
Three wooden block.

What is the height of the third wooden block arrangement? Explain how you got the result with a rational argument!

3.1.3. Problem 3 (30 Minutes)

Amin and Dimas each have a lucky number. Amin's lucky number is 3 more than Dimas's lucky number. If the difference between the squares of their lucky numbers is 45, what is Dimas's lucky

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number?"Question: Explain the steps of the problem solving process above for junior high students to easily understand?

4. Results

4.1. Case One: Subject S01 and S02

Subject S01 provided a comprehensive response to Question 1. The student systematically identified the distance between the Earth and the mesosphere layer by integrating the information given in the problem and representing it mathematically. S01 successfully connected this representation to fundamental concepts, such as the relationship between speed, distance, and time. The student correctly applied time unit conversions and clearly wrote the relevant speed formula on the answer sheet.

Formula:

$$S = t \times v$$

$S = 40 \times \left(t + \frac{1}{6}\right)$ → Speed = 40 km/h

$S = 40t + \frac{40}{6} = \frac{40t+20}{3}$ . . . (1)

Speed = 60 km/h

$$S = 60 \times \left(t - \frac{2}{6}\right)$$

$S = 60t - 20$ . . . (2)

Eliminate both equations:

Equation (1): $\frac{40t+20}{3}$ → ×3 → $120t + 20$

Equation (2): $60t - 20$ → ×2 → $120t - 40$

From both:

$S = 60$ km

Figure 2.
Response sheets of S01.

Subject S02 also provided a complete response to Question 1 but demonstrated a limited understanding of the problem. Although the student applied the speed-distance-time relationship, there was a misunderstanding regarding the conversion of time units, as shown in the student's answer sheet (Figure 3).

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a. $S = V \times t$

$= 40 \times 07.00$ hours

$= 40 \times 70$ minutes

$= 2{,}800$

b. $S = V \times t$

$= 60 \times 06.40$ hours

$= 60 \times 40$ minutes

$= 2{,}400$

Figure 3.
Response sheets of S02.

Based on Figure 2 and Figure 3, the answer sheets of S01 and S02 demonstrate their ability to represent content knowledge in solving the problem. However, to clarify certain aspects that remained unclear such as S01's process of converting time units and S02's justification of the correctness of the answer the researcher conducted follow-up interviews with both students. The following are excerpts from the interviews with S01 and S02 regarding question number 1

R Please explain where you got the value 1/6. What does it represent?

S01 I converted the rocket's arrival time, which is less than 10 minutes. Since 1 hour = 60 minutes, then 10/60 = 1/6 hour.

S02 Because 1 hour = 60 minutes and the time was more than 10 minutes, I used 70 minutes.

R How can you prove that your answer is correct?

S02 Because I used the formula for speed, s = v × t

In solving the first problem, the two subjects demonstrated different approaches in interpreting and representing time information algebraically. This analysis focuses on how they converted time units and applied algebraic concepts in solving for speed.

Subject S01 demonstrated a sound conceptual understanding of time unit conversion from minutes to hours. By recognizing that 10 minutes equals $\frac{10}{60} = \frac{1}{6}$ hour, S01 ensured consistency in units within the formula $s = v \times t$ particularly when speed is given in km/h. This representation indicates an appropriate functional understanding of algebra. Conversely, Subject S02 exhibited a misconception by assuming the time duration was 70 minutes without clear justification. This response highlights a failure to apply proper unit conversion and a disregard for consistency in mathematical representation. Although S02 referenced the speed formula, there was no elaboration

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on how values were substituted or calculated, indicating a procedural but not conceptual application of algebra. Thus, S01 falls under the MBA-j category, while S02 is classified as DTA-lc.

4.2. Case Two: Subject S04 and S06

S04 was not able to provide a complete answer to question number 2 due to the omission of key information from the problem. Although S04 correctly applied the difference of squares formula, the response did not fully address the question, indicating a lack of understanding of the given information. Furthermore, S04 did not include a final conclusion in the answer (See Figure 4).

Given: Amin's luck number is 3 more than Dimas's.

Find: What is Dimas's luck number?

Answer: → Since the difference is involved, it means subtra : : ı (–

Use:

$$a^2 - b^2 = c^2$$

$$a^2 = 9^2 = 81$$

$$b^2 = 6^2 = 36$$

$$c^2 = 45$$

Figure 4.
Response sheets of S04.

S06 completed question number 2 adequately, although not all of the information from the problem was fully written. S06 was able to use the difference of squares formula and algebraic operations to determine the lucky number, and provided an accurate answer and conclusion (See Figure 5).

Suppose: Amin's luck number = x

Dimas's luck number = y

Then:

$$x^2 - y^2 = 45$$

$$(y+3)^2 - y^2 = 45$$

$$y^2 + 6y + 9 - y^2 = 45$$

$$6y = 36$$

$$y = 6$$

Conclusion: Dimas's luck number is 6

Figure 5.
Response sheets of S06.

Based on Figures 4 and 5, the answer sheets of S04 and S06 reflect their content knowledge in solving the problem. However, to clarify certain aspects such as the meaning of the equations they constructed and the justification of their answers the researcher conducted follow-up interviews with both participants. The following are excerpts from the interviews with S04 and S06 regarding question number 2.

R Can you explain the meaning of the equation you constructed?

S06 The equation $a^2 - b^2 = c^2$ represents the difference in "lucky numbers" between Amin and Dimas, which differ by three units and are represented by $a, b,$ and c.

S04 I assigned x to Amin and y to Dimas. Since Amin's lucky number is three more than Dimas', I wrote $x = y + 3$, and the difference of their squared lucky numbers is 45, thus $x^2 - y^2 = 45$ This problem required students to represent the relationship between two quantities using an algebraic equation involving the difference of squares. Symbolic representation and equation structure were key indicators of their understanding of advanced algebraic concepts, particularly the identity $a^2 - b^2 = (a-b)(a+b)$

S04 showed clear and consistent symbolic representation. By assigning $x = y + 3$ and constructing the equation $x^2 - y^2 = 45$, S04 demonstrated an understanding of algebraic structure and successfully translated verbal information into logical symbolic form. Although the identity was not explicitly stated, the combined use of linear and quadratic relations indicates a solid grasp of mathematical modeling. In contrast, S06 showed confusion in constructing a suitable equation. The equation $a^2 - b^2 = c^2$ reflects a misunderstanding, likely confusing the identity of the difference of squares with the Pythagorean theorem $a^2 + b^2 = c^2$. This indicates that S06 lacks clarity in distinguishing between algebraic identities and their appropriate contexts. Therefore, S04 is categorized under DTA-np, while S06 falls under the MBA-j category.

4.3. Case Three: Subject S09 and S10

S09 did not answer question number 3 completely. The student's lack of understanding of the information in the problem is evident from the work shown. S09 concluded that the length of the block is 21 cm and the cylinder is 19 cm, whereas the problem clearly states that each block has a different height. Each stack of blocks consists of a square and a cylinder. S09 used subtraction to determine the height of the third block but failed to connect the information from the problem with the appropriate rules, formulas, and procedures, resulting in an incorrect solution.

Suppose:

Square = 21 cm

Cylinder = 19 cm

21 – 19 = 2

Therefore, the height of the third wooden block is 2 cm.

Figure 6.
Response sheets of S09.

S10 response to question number 3 is not fully complete due to missing information. S10 applied the substitution and elimination methods to determine the height of the third block. The first equation was formed from three cylinders and three blocks, and the second equation from three cylinders and two blocks. These two equations were then eliminated to find the value of y, which was subsequently substituted into the third equation involving one cylinder and two blocks. Through this process, S10 was able to arrive at the correct answer and draw an accurate conclusion about the height of the third block.

1. Let x = cylinder, y = cuboid

$$\Rightarrow 3x + 3y = 21 \quad (1)$$

$$\Rightarrow 3x + 2y = 19 \quad (2)$$

From (1) – (2):

$$y = 2 \quad (3)$$

Substitute into (2):

$$3x + 2(2) = 19 \Rightarrow 3x + 4 = 19 \Rightarrow x = 5 \quad (4)$$

Check:

$$x + 2y = 5 + 4 = \boxed{9}$$

Figure 7.
Response sheets of S10.

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Based on Figures 6 and 7, the answer sheets of S09 and S10 reflect their content knowledge in solving the problem. However, to clarify several aspects that remained unclear such as their interpretation of the problem statement, the structure of each block arrangement, the reasoning behind their answers, and the justification of their solutions the researcher conducted follow-up interviews with both students. The following are excerpts from the interviews with S09 and S10 regarding question number 3

R How did you solve this problem?

S09 The height of the first stack of blocks (squares) is 21 cm, while the second stack (cylinders) is 19 cm. Since the third stack consists of both shapes, I subtracted: 21 − 19 = 2 cm

R Were all blocks in the first stack squares, and all in the second stack cylinders?

S09 No. According to the diagram, the first stack has 3 squares and 3 cylinders, while the second stack has 2 squares and 3 cylinders

R How are you confident that your answer for question 3 is correct?

S10 I represented the square as x and the cylinder as y. Since the first stack consists of 3 squares and 3 cylinders, I wrote $3x + 3y = 21$ and the second stack as $2x + 3y = 19$. I used the elimination method to find x, and substitution to find y. Then I substituted into the target expression $2x + y = 2(2) + 5 = 9.$ So, the height of the third stack is 9 cm.

R Can you prove that your answer is correct?

S10 Yes. Substituting $x = 2$ and $y = 5$ into the first equation gives $3(2) + 3(5) = 6 + 15 = 21$, which matches the total height. Therefore, the values are correct.

This problem evaluated students' ability to model a concrete situation into a system of linear equations and to use elimination and substitution to solve it. The visual context required interpretation and algebraic translation. S09 relied on a direct comparison of total heights without using symbolic representation. While he recognized the composition of each stack, his approach remained intuitive and lacked systematic mathematical formulation. Without defining variables or constructing equations, his response cannot be algebraically validated. In contrast, S10 demonstrated stronger algebraic reasoning. He correctly formulated the system of equations, applied algebraic techniques systematically, and validated his solution by substitution. These steps illustrate a more advanced level of algebraic literacy, particularly in mathematical modeling. Therefore, S10 is categorized as MBA-j, while S09 falls under DTA-np.

5. Discussion

An analysis of three algebra cases revealed significant variations in the pre-service teachers' ability to represent, manipulate, and interpret symbolic information. Each problem context required students to accurately connect contextual information with algebraic representations. However, there were fundamental differences in students' conceptual approaches and problem-solving strategies. During the problem-solving process, students frequently identified misconceptions related to the mathematical concepts under investigation [26]. These differences reflect a broad range of mastery in Content Knowledge (CK) among the participants.

An analysis of the three algebra cases revealed significant variations in undergraduate students' ability to represent, manipulate, and interpret symbolic information. Each problem context required students to accurately connect contextual information with algebraic representations. However, fundamental differences were observed in their conceptual approaches and problem-solving strategies. During the problem-solving process, students frequently demonstrated misconceptions related to the mathematical concepts being assessed [26]. These differences reflect a wide range of Content Knowledge (CK) mastery among the participants.

Strong CK enables undergraduate students to present content through varied representations, anticipate student errors, and offer meaningful conceptual explanations. A recent meta-analysis [11] demonstrated a significant correlation between high CK levels and improved student understanding in mathematics. Undergraduate students with strong CK are also more flexible in employing pedagogical strategies tailored to students' needs. This finding is supported by the present study: students with a strong conceptual foundation (such as S01 and S10) were able to accurately apply mathematical concepts in tasks involving unit conversion, formula application, and linear system modeling. For instance, in the context of time conversion and use of the velocity formula $s = v.t$, S01 showed a clear understanding of converting 10 minutes to hours and the importance of consistent units in algebraic operations. Similarly, S10 demonstrated proficiency in modeling real-world contexts through systems of linear equations, using elimination and substitution to reach a validated solution.

Conversely, students with a predominantly procedural understanding (e.g., S02 and S06) struggled to relate mathematical symbols to contextual meaning, indicating weaker CK. S02, for example, failed to convert time units properly and merely cited the formula without showing conceptual insight. This illustrates a common issue—students being able to perform procedures without understanding their underlying logic. S06 also misapplied the difference of squares identity, confusing it with the Pythagorean theorem, further highlighting the impact of rote learning and a lack of deep conceptual engagement [27, 28]. These findings support the argument made by Salazar and Yilmaz [29] who emphasized that effective CK involves the ability to translate between symbols, context, and meaning especially in algebra and geometry.

Furthermore, systematic reviews [11] indicate that the development of CK in teacher education remains uneven. Instructional designs that overemphasize procedural tasks often fail to cultivate deeper conceptual understanding. In this study, S09 relied on direct numerical comparisons without developing an algebraic model. Although S09 recognized the block arrangement's components, the solution lacked formulaic representation and systematic execution, reflecting an intuitive rather than structured algebraic approach [30-34].

In contrast, S04, when tasked with modeling relationships using the difference of squares identity, was able to construct a symbolic representation from verbal information. This showed a clear understanding of variable structure and algebraic modeling, even if a final conclusion was missing. These contrasting cases underscore a major challenge in CK development bridging the gap between procedural fluency and conceptual application. As Pincheira and Alsina [34] observed, many pre-service teachers can perform algorithms but struggle to explain the reasoning behind their steps. This issue was mirrored in our findings, where some students could recognize numerical elements in problems but failed to build comprehensive mathematical models.

This study contributes new insight by applying refined categories MBA-j, DTA-lc, and DTA-np to characterize undergraduate students thinking in solving contextual algebra problems. These categories provide a more nuanced classification than traditional dichotomies between "conceptual" and "procedural" understanding. Few previous studies have examined all three dimensions (conceptual, procedural, justificatory) simultaneously within the context of algebra education. Our study shows that the success or failure in each dimension directly affects the coherence and completeness of students' problem-solving processes.

Additionally, this study employed data triangulation through written responses and interview excerpts, providing deeper insight into students' reasoning beyond surface-level answers. This method offers a valuable contribution to in-situ analysis of mathematical thinking. Bruna [35] in a study on teacher education programs at Pontificia Universidad Católica de Chile, demonstrated that using academic portfolios in advanced mathematics courses improved pre-service teachers' engagement and conceptual understanding. The portfolio process including reflection, documentation, and negotiated learning contracts was shown to strengthen mathematical understanding and better prepare teachers for classroom practice. These findings support the methodological design of this study, which integrates reflective written and verbal data to explore students' cognitive processes in greater depth.

Overall, these results suggest that mathematics teacher education should place greater emphasis on developing students' abilities in contextual interpretation, symbolic representation, and solution validation. This provides a foundation for designing context-based modules that foster reflective thinking and mathematical justification skills essential for future educators.

6. Conclusion

The ability to represent contextual information algebraically serves as a key distinguishing factor between students categorized under the Meaning-Based Approach (MBA) and those under the Direct Translation Approach (DTA). Subjects S01, S04, and S10 exhibited characteristics of students with stronger algebraic understanding (MBA). This was demonstrated through their ability to identify variable relationships, construct symbolic representations, apply appropriate procedures, and validate their results.

In contrast, subjects S02, S06, and S09 although capable of recognizing important problem elements struggled to construct coherent algebraic models and apply them consistently. Their problem-solving approaches tended to rely on procedural routines or intuitive reasoning, often lacking sufficient validation.

These contrasting approaches underscore the need to emphasize the development of conceptual understanding in algebra, particularly in aspects such as unit conversion, equation structure, and real-world mathematical modeling. The study reveals that undergraduate students display a wide range of understandings and strategies when solving contextual algebra problems. These variations are evident in three key areas: (1) The ability to convert units and maintain consistency in symbolic representation; (2) The ability to construct algebraic models from verbal or visual contexts; (3) The ability to validate and justify mathematical solutions.

Undergraduate students categorized under MBA-j demonstrated strong conceptual understanding, accurately built systems of equations, and engaged in solution validation. In contrast, those in the DTA-lc and DTA-np categories predominantly relied on procedural strategies with limited integration of contextual understanding or justification.

Therefore, algebra instruction in teacher education programs must place greater emphasis on developing meaningful symbolic representations and fostering reflective algebraic thinking. These findings support the theoretical perspective that robust algebraic competence cannot be cultivated without integrating context, symbolism, and justification.

7. Implications

Contextual Problem-Based Learning Design: Algebra instruction for prospective undergraduate students should prioritize learning experiences grounded in real world problems that require unit conversion, visual interpretation, and mathematical modeling. Such approaches enhance students' ability to link abstract concepts to practical applications, fostering deeper and more meaningful algebraic understanding. Emphasis on Justification and Answer Validation Processes: Instructors must guide students not only toward correct answers but also through rigorous validation techniques, such as substitution, logical proof, and reflective evaluation. These processes are critical in training future undergraduate students to foster mathematical reasoning and critical thinking among their students. Development of Algebraic Modeling Support Programs: There is a need to design dedicated modules or supplementary programs that explicitly train students in formulating algebraic equations from verbal and visual contexts. These should include targeted practice in translating real-world situations into mathematical models, thereby strengthening students' modeling competencies. Diagnostic Assessment Based on Understanding Categories: The categories MBA-j, DTA-lc, and DTA-np offer a useful framework for diagnostic assessments that can help identify students' dominant modes of thinking. These assessments will allow teacher educators to tailor instructional interventions that address specific conceptual or procedural weaknesses in algebraic reasoning.

Institutional Review Board Statement:

This study was reviewed and approved by the Promoter and Co-promoter of Universitas Negeri Surabaya, Indonesia (Letter No. B/4065/UN38.3/LT.02.02/2023). All participants provided written informed consent prior to their participation, in accordance with the ethical standards set by institutional and national research committees.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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