Using the lower bound set by the universal modal to investigate the status of partial objects and count nouns

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Abstract. Prior research has demonstrated that when given objects (e.g., forks) broken into pieces, children deviate from adults by counting each discrete object-piece as on par with a whole. A recent proposal ties this behavior to the vagueness and context-sensitivity inherent to count noun semantics. The present study leverages the universal modal *have to* in order to investigate how a linguistic context, one which sets lower bounds on numerals in its scope, regulates nominal application. Our results show that for children, who prefer the 'exact' reading of numerals, the partial object not only serves to meet the lower bound, but also exceeds a numerical upper bound. Adults, on the other hand, do not consider the partial object as meeting the lower bound induced by the modal. Because we cannot determine the explanation for this finding with our current design, we plan to adapt it to use the existential modal *allowed to.*

Keywords. partial objects; modals; numerals; gradability; context-sensitivity; count noun semantics

1. Introduction. Count nouns such as *ball* and *fork* are among the first words children produce. Yet, children show a surprising, non-adult-like willingness to apply such words not just to whole balls and forks, but also to their discrete parts. In a seminal study, Shipley & Shepperson (1990) gave children sets of whole objects and object parts ("partial objects") with specific instructions of what to count. When shown a set as in Figure 1, with four whole forks and two broken fork-pieces, and asked to "count the forks", children tended to count 6, as if treating the partial objects on par with the wholes.

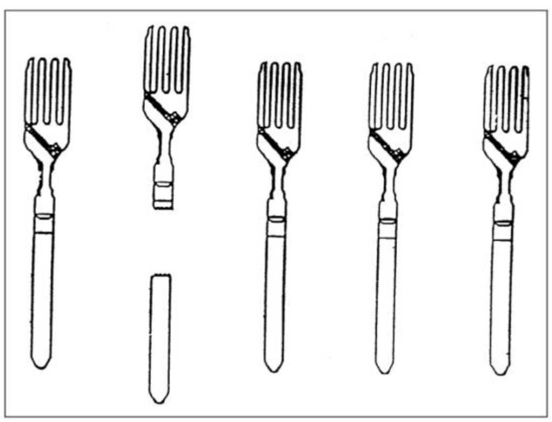

Figure 1: Example counting prompt from Shipley & Shepperson (1990)

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Adults either counted 4, ignoring the partial objects entirely, or 5, combining the two fork-parts into an imagined whole and counting it as 1. The authors took these findings to indicate that children have a conceptual bias to treat discrete physical objects as “default units” for counting. Since then, many have replicated and extended Shipley and Shepperson’s original findings, and proposed different accounts of children’s part-counting behavior. The details of these theories vary substantially: some argue that the problem lies in children's still-developing conceptual knowledge (Wagner & Carey 2003), others suggest it stems from the interaction between cognitive defaults and developing noun semantics (Brooks et al. 2011), and some point to children’s more limited access to lexical alternatives to refer to parts of objects or degraded objects (Srinivasan et al. 2013). Despite these differences, most of these theories converge on the idea that children's behavior fundamentally differs from how adults handle partial objects.

In contrast, a more recent account by Syrett & Aravind (2022) proposed that children’s performance may be consistent with an adult noun semantics at its core. Count nouns, for both adults and children, have meanings that depend on context to determine what counts as a unit (Krifka 1989, Rothstein 2010), and in certain contexts, either a whole or a partial object could fall under the extension of the count noun. This idea gains support from anecdotal examples of adults using count nouns to label partial objects similarly as they would whole objects—for example, when presenting findings from an archeological dig, a shard of pottery could be described as a ‘plate’ *tout court*. What differentiates adults from children, they argue, is adults’ more sophisticated ability to integrate context-specific information to restrict the noun’s application on a case-by-case basis.

To test their *context-sensitivity* hypothesis, Syrett & Aravind presented the participants with a task in which they had to determine whether a partial object counted as an instance of a count noun like *ball*, given the presence or absence of a speaker goal related to the object. Children, but not adults, were influenced by the degree of contextual support in deciding whether a partial object was a suitable referent for a count noun. When children were told, for example, that someone intended to play tennis with a ball, they were less likely to accept a partial object as a referent for “a ball.”

These results demonstrate that there are limits to children’s application of count nouns to partial objects, modulated by contextual information. But whereas Syrett & Aravind’s task probed object reference (and therefore, category membership as indicated by the count noun and what falls under its extension), much of the prior work was focused on counting and quantification of sets of objects. Our goal in this paper is to test the context-sensitivity account in a task that calls upon participants to quantify objects, without overtly counting them. To achieve this, we leverage the bounding conditions induced by modals and their effect on the interpretation of numerical expressions to determine the status of partial objects relative to wholes. If children malleably treat partial objects as either parts of wholes or wholes depending on the context, then in a quantification task where the goal is to meet and exceed a lower bound, children should allow partial objects to serve this purpose. Adults, however, should continue to recognize partial objects as such, and not allow them to meet the lower bound. Previewing our results, we find that the adult-child difference in how partial objects are treated re-emerges in this task.

2. Bounding conditions on numerals in the scope of modals. Depending on the environment in which they appear, numerals can receive different interpretations: an ‘exact’, ‘at least’ or ‘at most’ reading. In a discourse context such as (1), the numeral *three* is naturally understood to mean ‘*exactly* three’—as in, providing both upper and lower limits on the number of mistakes made. If

B made exactly three mistakes, answering A's question with any other numeral would be misleading, or even untruthful.

(1) A: How many mistakes did you make?
B: I made three mistakes.

In a somewhat different context, such as (2), the numeral is understood as providing a lower bound meaning '*at least* three'. B's response can be understood as felicitous and truthful even if they, in fact, have four children.

(2) A: People with three children get a tax break. Do you have three children?
B: Yes, I have three children.

The surrounding *linguistic* context can highlight different readings of numerals. In the scope of a universal modal such as *have to*, numerals are most naturally understood as having a lower-bounded, 'at least', reading. Thus, (3) is understood as implying that anyone with three or more children will receive the tax break. Numerals in the scope of an existential modal like *allowed to*, in contrast, typically receive an upper-bounded reading. Thus, (4) implies that anyone who makes three or fewer mistakes will pass the test.

(3) You have to have three children to receive the tax break.
(4) You are allowed to make three mistakes and still pass the test.

Prior developmental work suggests that children can access these different interpretations of numerals, despite sometimes showing less flexibility than adults in their interpretations. In unembedded contexts, children have been shown to prefer an 'exact' interpretation of numerals (e.g., Huang & Snedeker 2009, Huang et al. 2013, Papafragou & Musolino 2003). When the numeral is in the scope of modals, however, children more readily access the upper- and lower-bounded readings.

Musolino (2004) used a Truth-Value Judgment Task to test four- and five-year-olds' understanding of the interaction between numerals and modals by pairing stories with various modal statements that induced either a lower bound in the 'at least' condition, as in (5), or an upper bound in the 'at most' condition, as in (6). In both cases, children were asked, if the Troll won the coin.

(5) Goofy said that the Troll had to put two hoops on the pole in order to win the coin.
(6) Goofy said the Troll could miss two hoops and still win the coin.

Initially, given the contexts above, while children were able to access the upper-bounded readings of numerals in the 'at most' condition, they struggled with lower-bounded readings in the 'at least' condition and favored 'exact' readings. However, in a second experiment, which featured simplified scenarios and modal statements such as (7), children much more readily accessed lower-bounded readings of numerals.

(7) Let's see if Goofy can help the Troll. The Troll needs two cookies. Does Goofy have two cookies?

Kennedy & Syrett (2022) expanded on Musolino's experiments by adding a condition missing from Musolino's design: one in which the upper bound, induced by the existential modal *allowed to*, was *exceeded*. Across trials, and within participants, they manipulated the quantity of objects so that a character took less than 2, exactly 2, or greater than 2 objects or measurements of substances. Their scenarios were paired with modal statements, such as (8).

(8) You are allowed to use two lemons.

Children successfully rejected the action when the upper bound was exceeded, and were also significantly less likely to accept actions that did not meet the upper limit.

These studies provide a good starting point for our experiment, which assesses children's and adults' treatment of partial objects and the role of language in mediating how such objects are quantified. Here, we focus on the universal modal *have to*, which induces a lower bound on numerals in its scope (as in (3)), to investigate how a linguistic context affects children's or adults' numerical judgments. We ask whether partial objects serve to satisfy the numerical lower bound in a modal statement such as (3). If they can be both flexibly treated as category members denoted by the count noun in the right contexts and also counted as one unit, partial objects should count towards meeting the limit. Our question is whether this is the case for both children and adults, in a task focused on quantification.

3. Experiment. Sample size, procedures, and analyses for this experiment were pre-registered at https://osf.io/phyds.

3.1. PARTICIPANTS. 40 English-acquiring children (4;6-5;6, M=4;11) and 21 English-speaking adults (N=40 preregistered, in prog.) participated in the experiment. An additional 10 children were tested but excluded from the full sample for failing comprehension checks (4), not completing the experiment (3), inattentiveness (1), less than 50% home exposure to English (1), or experimenter error (1). All children were recruited from a database of families interested in participating in research with the MIT Language Acquisition Lab, and Zoom-tested by a live experimenter. Four additional adults were tested as well, but excluded from the full sample for failing comprehension checks (2) or not completing the experiment (2). All adults were undergraduate students from Rutgers University who received extra credit in a Linguistics or Cognitive Science course for their participation, and took an asynchronous variant of the child experiment, in which video clips of each trial were inserted into a self-paced Qualtrics survey.

3.2. MATERIALS. Participants were introduced to a game in which characters had to satisfy a rule to receive a reward. Across trials, object kind and the distribution of whole and partial objects in the sets shown to participants varied, while the numeral in the instruction sentences was always 'three'. Participants gave a star when they found the numerical conditions to be met, and a calculator (or 'counting machine') otherwise. See Figure 2.

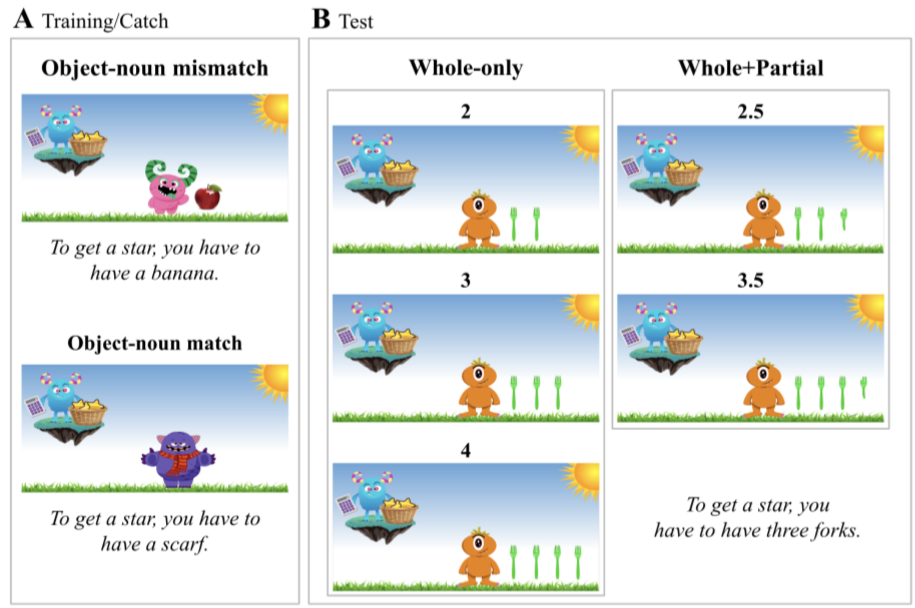

Figure 2: (**A**) During training, participants were shown a trial in which the pictured object did not match the noun in the modal statement, and one in which it did. Catch items were designed similarly. (**B**) Test trials split by set type (Whole-only and Whole+Partial), all for which participants heard the same pre-recorded modal statement.

All participants saw two types of trials based on the nature of the object sets on display. The ‘Whole-only’ set type included trials with 2, 3, or 4 identical whole objects. The ‘Whole+Partial’ set type consisted of object sets with either 2 or 3 identical whole objects, plus a single partial object. The partial object stimuli were created by removing portions from the images of the whole objects, but were identical in all other respects. Altogether, there were object sets with the following five cardinalities: 2, 2.5, 3, 3.5, 4. Each object set was paired with a modal statement such as (9), in which we used count nouns denoting everyday objects (e.g., balls, cups, forks).

(9) To get a star, you *have to* have three forks.

3.3. PROCEDURES. All participants saw trials in both the Whole-only and Whole+Partial set types in a pseudorandomized order. The experimental session began with an introduction to Zoryn, the protagonist, and her friends, a group of aliens visiting Earth. Participants were invited to play a counting game with them, in which they had to listen to a rule provided by Zoryn and then determine whether the set of objects in a friend’s possession was rule-compliant. If they followed the rule, they should be rewarded with a star; if not, they should receive a calculator to count better the next time.

To ensure that participants understood the task, they saw two training trials prior to the experimental phase. The instruction sentences for these trials involved the universal modal, but without a numeral (e.g., “To get a star, you have to have a banana”). In one trial, the friend had an object that matched the noun in the rule, and in the other, they had one that did not.

The test phase consisted of 18 total trials: 2 per cardinality for the Whole-only trials (6 total), 4 per cardinality for the Whole+Partial trials (8 total), and 4 catch items. These catch items followed a similar structure as the training trials, and served as both task-comprehension and attention checks. For each trial, we coded whether participants judged a set as compliant with the modal statement, with a choice of star indicating 'Yes' (coded as 1) and a choice of calculator indicating 'No' (coded as 0).

3.4. PREDICTIONS. For the Whole-only trial types (2-object, 3-object, and 4-object), we expect that all participants, regardless of their interpretation of the numeral, will agree on 2-object and 3-object trials, but vary on 4-object trials based on how they interpret the phrase *have to have three N*. In 2-object trials, participants should respond 'No', as the lower bound is not met. On the other hand, in 3-object trials, the lower bound is met, so participants are expected to respond 'Yes'. In 4-object trials, participants who prefer an 'at least' reading of the numeral should respond 'Yes' (since the lower bound is met and exceeded). Thus, for the same reason, those who prefer an 'exact' reading should say 'No', as not only is the lower bound met, but the upper bound is *exceeded.* This pattern might be observed with children, who have previously been shown to most readily access 'exact' readings of numerals (Huang, Spelke, & Snedeker, 2013; Papafragou & Musolino, 2003).

For the Whole+Partial trials, expectations vary based on two factors: how participants quantify partial objects based on context, and their preferred reading of the numeral. If partial objects can count as 1 unit as the context demands, then sets with 2 whole objects and 1 partial object could be treated as on par with a set containing 3 wholes, leading to a 'Yes' response in the 2.5-trials. On the other hand, if the partial objects do *not* count as having a cardinality of 1, 2.5-trials should yield 'No' responses, in contrast to 3-trials. As for the numeral, if participants access a lower bounded reading, 3.5-trials should be accepted irrespective of their treatment of the partial object. If they access only an 'exact' reading, 3.5-trials may yield 'No' responses if the partial object counts as an instance of the noun, pushing the set beyond the limit. See Figure 3.

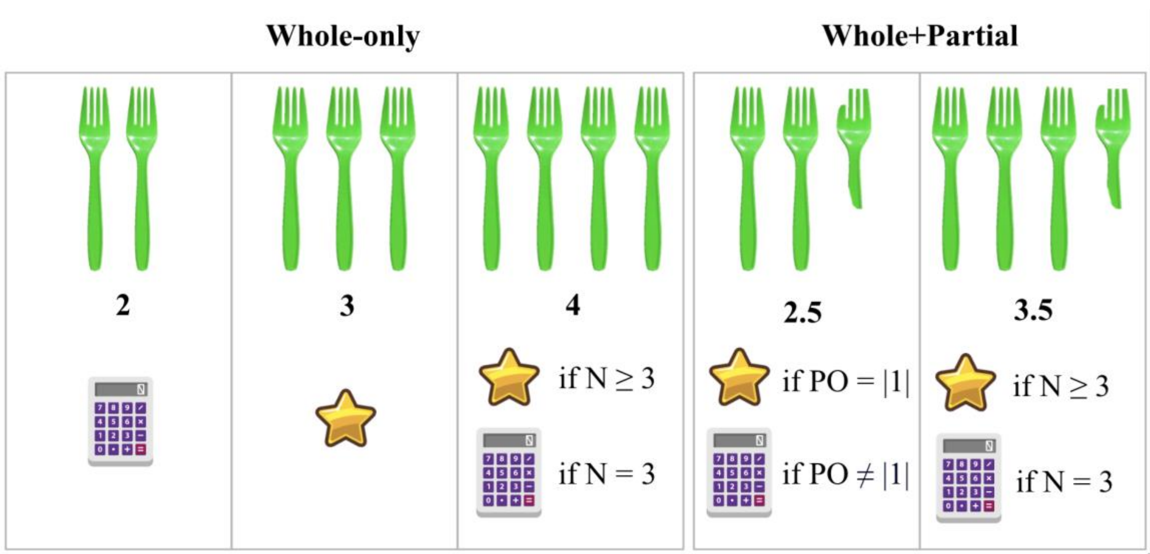

Figure 3: Predictions for behavioral responses from participants in response to the alien's selection of object quantities, given the rule *You have to have three forks*, where a star corresponds to a 'Yes'-response (coded as '1'), and a calculator corresponds to a 'No'-response (coded as '0').

3.5. ANALYSES. All analyses were performed using R Statistical Software (v4.3.1; R Core Team 2023). Our primary question was how the rates of accepting a set as rule-compliant vary based on set composition, and whether this differs across adult and child populations. To test these questions, we fit separate logistic mixed effects models for the Whole-only and Whole+Partial set types. For each, we predicted the probability of responding 'Yes' as a function of set type and age group, with random intercepts for participants[1]. The model syntax was as follows (where 'SetType' refers to the number of objects in the set): YesResponse ~ SetType * AgeGroup + (1|Participant). For the Whole-only model, set type had three levels (2, 3, 4). For the Whole+Partials model, set type had two levels (2.5, 3.5). Age group had two levels (adults, children) in both models. All factors were treatment-coded. To test for main effects and interactions, we used log-likelihood chi-squared tests to compare models with and without the relevant effect.

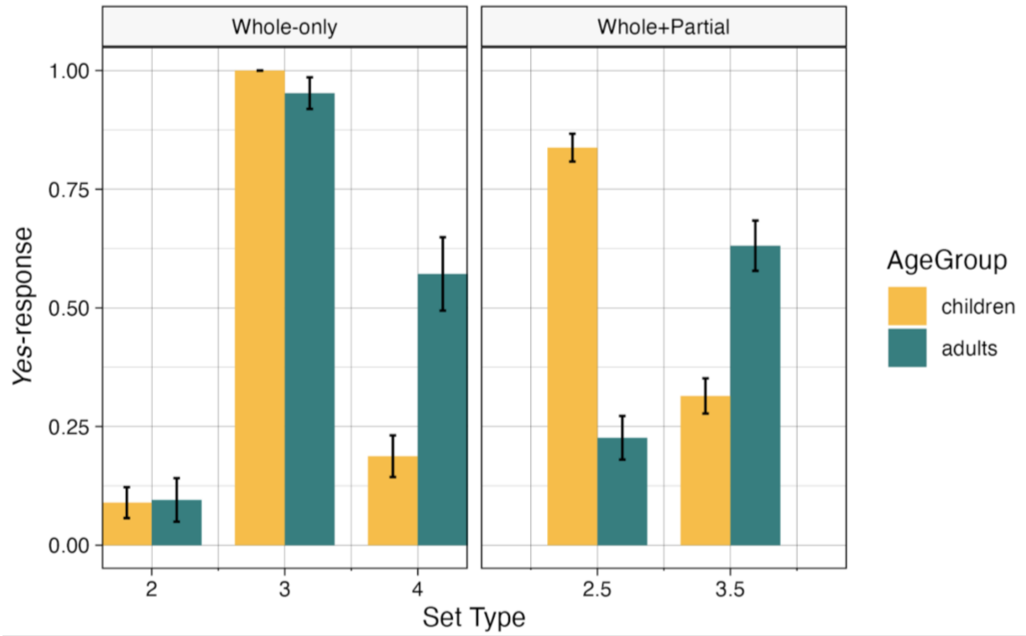

Figure 4: Mean 'Yes'-responses (+/- 1 SEM) split by set type and age group

For the Whole-only trial types (Fig.4, left panel), model comparisons revealed that Age Group ($\chi2(1)$=4.1, p = 0.04), Set Type ($\chi2(2)$ = 48.2, p < .001), and their interaction ($\chi2(2)$ = 9.3, p = 0.001) significantly improved model fit. The Age Group effect is driven by adults being more likely to respond 'Yes' than children. Participants in both groups were also more likely to respond 'Yes' to 3-object trials—corresponding to an 'exact' reading of the numeral in the rule. Post-hoc tests exploring the interaction reveal that while both populations were alike in their judgments of 2-object and 3-object trials, they diverge in their responses to 4-object trials: adults were significantly more likely than children to respond 'Yes' to 4-object trials (β=-2.5, SE= 0.7, z=-3.5 p <.001), suggesting that they were able to access the 'at least' reading, which children resisted.

In the Whole+Partial trial types (Fig. 4, right panel), children and adults also differed in their judgments. Model comparisons revealed a significant interaction of Age Group and Set Type ($\chi2(1)$= 73.7, p<0.001). This interaction was driven by the fact that children's and adults' 'Yes'-responses patterned in opposite directions for 2.5-object and 3.5-object trials: children were significantly *more* likely than adults to accept sets of 2.5 objects as rule-compliant and meeting the lower bound of 3 (β=3.9, SE=0.6, z=6.3, p<0.001). At the same time, they were significantly *less* likely than adults to do so for sets of 3.5 objects (β=-1.8, SE=0.6, z=-3.4, p=0.001). In other words, children saw a set containing 2 wholes and 1 partial object as satisfying the bounds

[1] Using more complex random effect structures led to problems with model convergence.

conditions set by "have to have *three* N", whereas a set containing 3 wholes and 1 partial object exceeded it.

4. Discussion. In our study, we used a quantification task probing numerical judgments to better understand children's and adults' treatment of partial objects. We asked whether participants' willingness to treat a partial fork as falling under the extension of a count noun like *fork*, and counting it as one fork-unit, depended on contextual factors as previously proposed (e.g., Syrett & Aravind 2022). If so, can information contained in the sentence itself modulate whether a partial object counts as 1? To address these questions, we capitalized on the interaction of numerals and the universal modal *have to*, which favors a lower-bounded, or 'at least', reading of a numeral in its scope. Specifically, we asked, when given a rule such as "You have to have three forks" paired with a set of 2 whole and 1 partial forks, are participants willing to count the partial fork as 1 if doing so serves to meet the lower bound conditions set by the modal?

Our results answer this question in interestingly different ways for our two populations. Adults were able to access the lower-bounded, 'at least' interpretation of the numeral, as evidenced by their acceptance of sets containing 4 whole objects. However, they rarely accepted sets containing two wholes and one partial object. This result indicates that for adults, the partial object was treated on par with a whole, so as to satisfy the lower bound.

In order to gain further insight into how adults (and children) quantify partial objects in light of an interaction between a modal and a numeral, we are currently conducting a complementary experiment that features the existential modal *allowed to,* which induces an upper bound on a numeral in its scope, as in (10) below and (4) above.

(10) Employees are allowed to take three snacks from the break room.

In these examples, the numeral *three* receives an upper-bounded, 'at most', interpretation. While taking up to three snacks (or making up to three mistakes) would be acceptable, more than three would exceed the upper bound and be unacceptable relative to the conditions imposed by the modal. This experiment will allow us to determine if a partial object can serve to exceed the upper bound and incur a penalty, even while for adults, it does not serve to meet a lower bound.

Children diverged from adults in having a strong preference for the 'exact' reading of the numeral, demonstrated by their low acceptance of sets of 4. Likewise, with sets containing three whole objects and one partial object, children responded 'No'. Thus, for them, the partial object not only served to satisfy the numerical lower bound, but also served to push the cardinality of a set beyond the upper bound. They also accepted sets with two whole objects and one partial object as meeting the lower bound almost as often as they did sets of three whole objects. This pattern highlights a second, critical divergence from adults: for children, a partial object *does* count as one instance of the relevant count noun, thereby meeting the lower bound of three. This finding is in line with earlier work showing that in tasks involving counting and quantification, children treat partial objects on par with wholes.

Our results raise key questions about the source of the current results, especially against the backdrop of previous tasks. In many quantification-focused tasks, children have consistently treated partial objects as wholes. And yet, in Syrett & Aravind (2022), they resisted this treatment when given contextual factors (there, a speaker-articulated goal depending on the object performing a function dependent on its shape). It may be that when the focus is on quantification of objects, children treat partial objects as wholes, in the absence of a more fine-grained scalar system allowing for partial objects to be measured as fractional portions (which adults have). As a result, children gravitate to quantities of 0 and 1, without a middle ground—even as their linguistic production highlights halves, pieces, and broken parts (as they did in their justifications for us here). It may instead be that children were sensitive to the context of this experiment, which privileged a requirement to meet lower bounds, and their sensitivity to this contextual goal influenced their treatment of partial objects as wholes satisfying this lower bound. Both explanations allow for the possibility that for both children and adults, noun application and object individuation may diverge; that is, the question of whether an object can serve as a referent for a noun and how to count or measure that entity may not always align. We find the comparison between count noun application and nominal reference on the one hand, and object individuation and quantification on the other, to be an exciting avenue to pursue.

Although adults did not treat the partial object as meeting the lower bound, we cannot say that they excluded it from their counts entirely. There is, in fact, suggestive evidence that this is not the case. Had adults ignored the partial object altogether, we would expect their treatment of the 3.5 set to be comparable to their treatment of the 3 set: the 3.5 set contains 3 whole objects and an irrelevant object that doesn't count. But numerically, adults' Yes-responses to the 3.5 set were comparable to their treatment of the 4 set, and lower than their Yes-responses to the 3 set. In other words, for adults, the 3.5 set does not seem to satisfy, the 'exact' reading of the numeral.

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