Optimizing Gravity-Fed Sewer Systems using GRG and PGSL: A Path to Cost-Effective Design

Teerapat Keawsriyong
School of Engineering and Technology, Walailak University, Thailand | Center of Excellence in Sustainable Disaster Management (CESDM), Walailak University, Thailand
teerapat.ke@mail.wu.ac.th

Warit Wipulanusart
School of Engineering and Technology, Walailak University, Thailand | Center of Excellence in Sustainable Disaster Management (CESDM), Walailak University, Thailand
wwarit@engr.tu.ac.th

Satjapan Leelatanon
School of Engineering and Technology, Walailak University, Thailand | Center of Excellence in Sustainable Disaster Management (CESDM), Walailak University, Thailand
lsatjapa@wu.ac.th

Nukul Suksuwan
School of Engineering and Technology, Walailak University, Thailand | Center of Excellence in Sustainable Disaster Management (CESDM), Walailak University, Thailand
snukul@wu.ac.th

Jakkarin Weekaew
School of Engineering and Technology, Walailak University, Thailand | Center of Excellence in Sustainable Disaster Management (CESDM), Walailak University, Thailand
wjakkari@mail.wu.ac.th

Quoc Bao Pham
Faculty of Natural Sciences, Institute of Earth Sciences, University of Silesia, Poland
quoc_bao.pham@us.edu.pl

Pakorn Ditthakit
School of Engineering and Technology, Walailak University, Thailand | Center of Excellence in Sustainable Disaster Management (CESDM), Walailak University, Thailand
dpakorn@mail.wu.ac.th (corresponding author)

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ABSTRACT

In this paper, Generalized Reduced Gradient (GRG) and Probabilistic Global Search Lausanne (PGSL) optimization algorithms are employed to enhance sewer network design, focusing on link length, path, diameter, and cost. The former are compared with linear and dynamic programming, with the results indicating PGSL as the most cost-efficient, achieving optimal lengths of 70.00 m for Link I, 48.97 m for Link II, and 76.41 m for Link III, with paths of 1-2, 1-2, and 1-3, respectively, and a total cost of $15,688.17. In comparison, other algorithms incurred higher costs, while the optimal diameters remained

consistent across all methods, ensuring structural integrity. The minor variations in lengths and paths reflect network design robustness. The importance of selecting the right optimization algorithm based on cost, length, path, and diameter is emphasized. PGSL is introduced for the first time in this context, demonstrating superior cost-effectiveness and significant implications for sewer network optimization. The findings provide valuable insights to engineers and planners, promoting more efficient and sustainable infrastructure development.

Keywords-cost-efficiency analysis; GRG algorithm; infrastructure design; PGSL algorithm; sewer network optimization

I. INTRODUCTION

Optimization techniques have been successfully applied to various engineering problems, including water leakage reduction [1], joint cost optimization [2], and composite beam design [3]. A gravity-fed sewer system relies on gravity to transport wastewater, providing cost-efficient and energy-saving solutions critical to urban and rural water infrastructure [4]. These systems support urban sanitation, public health, and environmental quality while minimizing maintenance, operational costs, and urban flooding risks [5]. Proper design balances hydraulic efficiency, cost-effectiveness, and environmental sustainability, considering hydraulic constraints, economic factors, topography, and environmental impact [4, 6, 7]. Such designs align with sustainable development, particularly Goal 6 of the UN SDGs, by reducing resource use and improving wastewater quality [8]. Optimization methods for gravity-fed sewer systems include traditional techniques, like linear, dynamic, and nonlinear programming, which struggle with large-scale problems [4]. Researchers have successfully applied Genetic Algorithms (GA) and Tabu Search (TS) for global optimization [5]. Newer methods, such as Particle Swarm Optimization (PSO) and Simulated Annealing (SA), address hydraulic constraints effectively [9, 10]. Hybrid approaches, which combine GA with cellular automata or quadratic programming, improve efficiency and solution quality [11, 12]. Ant Colony Algorithms (ACA) outperform GA and PSO in convergence and speed [13, 14]. Other methods, including Differential Evolution (DE) [15], Non-dominated Sorting Genetic Algorithm II (NSGA2) [16], and Harmony Search (HS) [17], demonstrate significant cost savings and performance enhancements. Combining GA with Heuristic Programming (HP) [18] and TS with SA [19] further improves global optimization capabilities. Despite the wide GRG and PGSL method utilization in water science and engineering, including infiltration rate estimation [20], rating curve development [21], Muskingum flood routing [22], and irrigation canal design [23], the former's application in sewer network design remains unexplored. Thus, the current study investigates GRG and PGSL potential for sewer network optimization using a benchmark case introduced in [7].

II. THEORETICAL METHODS USED

A. Gravity-Fed Sewer Systems: Theoretical Fundamentals

1) Hydraulic Principles in Gravity-Fed Sewer Systems

- Manning's equation:

The velocity (m/s) of wastewater flow in pipes is calculated using Manning's equation considering the pipe roughness coefficient (n), slope (S_o), and hydraulic radius (R):

$$V = \frac{1}{n} R^{\frac{2}{3}} S_o^{\frac{1}{2}} \quad (1)$$

- Self-Cleansing Velocity:

The system must maintain a minimum flow velocity to prevent sediment accumulation in the pipes. The optimal velocity range is typically between 0.6 and 3.0 m/s. Velocities below these values may lead to solid particle generation, while excessively high velocities could cause pipe erosion.

- Slope:

The pipe slope (S_o) must be designed to ensure a continuous wastewater flow. Determining the appropriate slope allows wastewater to flow effectively without the need for additional energy from pumps.

2) Pipe Size in Sewer Systems

Pipe size is a major factor, directly affecting a sewer system's cost and effectiveness. Hence, choosing the right pipe size helps the system operate efficiently and economically, while wrong size selection can lead to efficiency problems, excessive costs, or environmental impacts.

- Pipe Diameter Selection

The pipe diameter (d) is determined by the maximum wastewater volume the system must handle. Oversized pipes mitigate the risk of overflow and decrease flow velocity, which can cause sedimentation within the pipe. However, this increases unnecessary material and installation costs. On the other hand, undersized pipes reduce initial costs, but are more susceptible to overflows and future repair and maintenance cost increase.

- Limitations in installation in narrow spaces

Space is very limited in urban areas. So, pipe size selection must be carefully performed. Sometimes it is necessary to use smaller-size pipes because of possible interference with buildings, roads, and underground cables. Moreover, adopting flexible joints with strong materials may prevent damage in earthquake areas, while trenchless technology reduces disruption and can be safer. Therefore, flexible design and soil-structure interaction are important parameters for safer underground pipelines [24].

3) Cost Factors in Gravity-Fed Sewer Systems

There are several cost considerations when designing gravity drainage systems to ensure project cost-effectiveness:

- Excavation Cost:

Installation depth and topography directly influence excavation costs. Installing the pipe deeper will result in higher material and labor costs. If the site is rocky or soft, specialist equipment and highly skilled labor may be required.

- Pipe Cost:

The used materials, such as PVC, steel, or concrete, as well as pipe length and diameter, determine pipe cost.

- Manhole Cost:

Manholes are installed at appropriate locations, such as connection points or direction changes. The cost increases with manhole depth.

B. Optimization Techniques

1) Generalized Reduced Gradient

The GRG method [25] has been proven highly effective in solving various optimization problems. However, its efficiency is not always highlighted [25]. Widely applied in areas such as portfolio optimization and energy efficiency control systems, GRG is particularly valued for its ability to ensure control system effectiveness. By utilizing gradient descent, GRG effectively tracks and optimizes target objectives while managing constraints, such as normality conditions or size limitations, like pipe dimensions. Its precision makes it ideal for applications demanding accuracy. GRG computational efficiency has been further enhanced, enabling solutions for large-scale problems, while the utilization of methods like GA or PSO improves solution quality [25]. These innovations solidify GRG's role as a versatile and reliable tool for tackling complex optimization challenges across diverse domains.

2) Probabilistic Global Search Lausanne

Initially developed in [26], PGSL is a direct search algorithm based on global space sampling using a Probability Density Function (PDF). PGSL is also based on the observation that sampling in an appropriate area can lead to efficient solution detection without the need for special operators. A key assumption of PGSL is that points with better performance tend to be close to those already identified as "good" [27]. Testing PGSL on a sample problem with a multivariate, non-linear target function has shown that it outperforms GA and various SA forms [26, 27]. PGSL starts by randomly selecting a population of initial solutions and ranks the results according to the target function value, intending to continuously improve the search process toward the best solution. The former consists of four main steps: sampling cycle, probability updating cycle, focusing cycle, and sub-domain cycle [27]. The strengths of PGSL include:

- Exploration Capability: It is well suited for problems involving large search spaces and many constraints.
- Avoidance of Local Minima: The probabilistic approach reduces the risk of getting stuck in suboptimal solutions.
- Flexibility: PGSL efficiently deals with problems with a large number of variables and complex constraints [28].

Combining robust global exploration with focused local refinement, PGSL is a flexible and powerful tool for addressing optimization challenges across domains.

III. DEVELOPMENT OF MATHEMATICAL OPTIMIZATION MODEL

Designing a gravity-fed sewer system is a complex process that requires high accuracy to reduce costs and maintain long-term operational efficiency. This research focuses on using optimization techniques to find the optimal solution by employing nonlinear GRG and PGSL methods capable of handling problems with complex constraints and multiple variables. The research methodology consists of four main steps:

A. Defining the Problem and Objective Function

The key components of a gravity-fed sewer system which influence cost and efficiency are:

- Excavation Cost: Soil excavation to lay the pipes must consider soil depth and volume. The costs depend on the soil type and used equipment. The excavation cost calculation equation (C_e) is:

$$C_{ei} = C_e * V_e \tag{2}$$

where C_e is the cost per unit of excavation volume and V_e represents the excavated soil volume (depending on depth and cross-sectional area).

- Pipe Cost (C_p): It is determined by the pipe size and length:

$$C_{pi} = C_p * L \tag{3}$$

where C_p is the cst per unit of pipe length and L is the length of the pipe between two points.

- Manhole Cost: Manholes are essential for maintaining and inspecting the system. Cost (C_m) is determined by the manhole depth:

$$C_{mi} = C_m * Y \tag{4}$$

where C_m is the cost per unit of depth and Y is the manhole depth.

- Total Objective Function: the total cost function (T) for optimization is expressed as:

$$T = \sum_{i=0}^{n}(C_{pi} + C_{ei} + C_{mi}) \tag{5}$$

B. Constraints:

This mathematical model is developed to support optimization based on the total cost function and various constraints:

- Flow Velocity Constraints: Flow velocity must be within an appropriate range to ensure that the particles in the wastewater do not accumulate in the pipes and do not cause pipe material erosion.

$$V_{min} \leq V \leq V_{max} \tag{6}$$

- Slope Constraints: The slope (S_o) must be within a range that supports the minimum and maximum flow velocity:

$$S_{min} \leq S \leq S_{max} \quad (7)$$

- Discharge Constraints: The wastewater flow rate (Q) must not exceed the pipe's capacity:

$$Q_{design} \leq Q_{capacity} \quad (8)$$

This study employed the GRG Solver, available as a plugin in Microsoft Excel, and the PGSL tool, implemented using Excel VBA (version 4).

C. Case Study

To demonstrate the efficiency of the proposed metaheuristic optimization approaches, the Goodwin Avenue sewer design problem is used as a benchmark study case, as presented in [7, 29]. The sewer line connects four manholes, M1, M2, M3, and M4, through three sewer links, as displayed in Table I.

TABLE I. DATA RELATED TO GOODWIN AVENUE SEWER LINE

U/S-D/S manholes	Sewer link	Sewer length (m)	Peak inflow (m^3/s)
$M_1 – M_2$	1	70.104	1.0423
$M_2 – M_3$	2	48.768	1.1896
$M_3 – M_4$	3	76.505	1.3312

Table I and Figure 1 demonstrate that for each link, there are 3 possible elevations for the upstream (u/s) and downstream (d/s) ends. This generates a total of nine possible paths. The difference between each elevation level is 0.3048 m. The design ensures that the sewer line meets hydraulic requirements, including: 1) minimum cover depth: 1.0668 m (3.5 ft) over the sewer pipe, 2) self-cleaning velocity: maintained within the range of 0.6-3 m/s, and 3) elevation options: each sewer link has three potential elevations for both u/s and d/s ends.

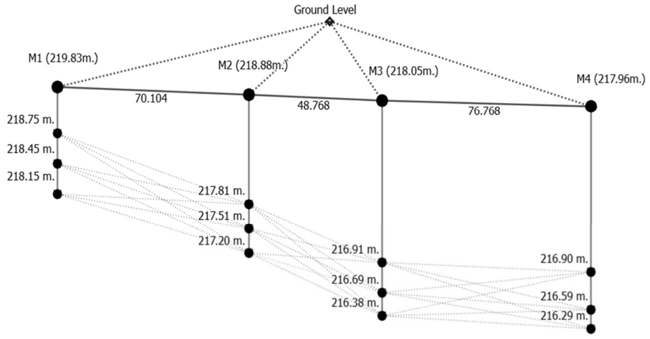

Fig. 1. Schematic diagram illustrating elevations and possible paths of sewer links.

The permissible slopes are evaluated for each path to satisfy the self-cleansing velocity constraint while avoiding adverse slopes where downstream crown elevations exceed upstream levels. The costs of the sewer system's three main components—pipe, excavation, and manhole—are displayed in Table II [7]. The cost types for the cost components are calculated for each feasible path using the combination of pipe size, excavation depth, and manhole requirements to determine the most cost-effective solution. The unit cost of available commercial pipe sizes is given in Table III [7]. The Manning's coefficient value is assumed to be 0.014 for all pipe sizes.

TABLE II. TOTAL COST OF SEWER SYSTEM [7]

Cost type	Parameter	Value
Excavation cost	Per unit volume	\$6.00/yd³ (1 yd³ = 0.764555 m³)
Manhole cost	Per unit depth	\$100/ft depth (1 ft = 0.3048 m)
Excavation width	Average	1.524 m. (5 ft)
Pipe cost	Based on diameter (Table III)	Ranges \$11.155 - \$129.43/m.

TABLE III. COST OF DIFFERENT PIPE SIZES OF SEWER LINE [7]

Pipe size (cm)	Cost (\$/m)
30.48	11.155
38.10	14.600
45.72	19.357
53.34	24.279
68.58	36.254
76.20	46.588
91.44	62.500
106.68	82.021
121.92	127.461

IV. RESULTS AND DISCUSSION

A. Results from Generalized Reduced Gradient and Probabilistic Global Search Lausanne

The optimization of gravity-fed sewer systems using nonlinear GRG and PGSL revealed performance variations, particularly in terms of cost efficiency and structural output consistency. PGSL demonstrated its superiority by achieving the lowest total cost of \$15,688.17, reflecting a saving of \$1,142.75 (6.79%) compared to nonlinear GRG. This cost reduction highlights PGSL's ability to explore broader solution spaces and balance multiple cost components, such as excavation and pipe materials. In contrast, GRG, while faster, resulted in a higher total cost of \$16,830.92 due to its tendency to converge to local optima and its limited capacity to fully explore global cost-saving opportunities. Both algorithms produced optimal pipe diameters, such as 68.58 cm and 91.44 cm, which adhered to hydraulic constraints, ensuring sufficient capacity and flow velocity. Additionally, the selected paths, including logical transport routes, like links 1-2 and 1-3, maintained overall system design effectiveness. Although nonlinear GRG can reduce costs to some extent and deliver quick and accurate results for less complex systems, it struggles to overcome multi-dimensional constraints and often fails to avoid local minima. In contrast, PGSL consistently demonstrated better performance by employing a dynamic search process that adjusts probability values during each iteration, enabling it to avoid local minima and find the global optimum. As a result, PGSL proves to be a more effective approach for the optimization of gravity-fed sewer systems, offering significant cost savings and complex constraint handling. A comparison of the best designs for gravity-fed sewer systems using GRG and PGSL is provided in Figure 2.

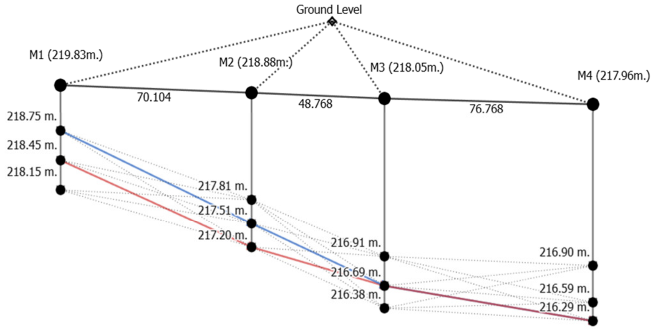

Fig. 2. Comparison of the best gravity-fed sewer systems obtained from GRG and PGSL.

In Figure 3, GRG and PGSL cost reduction trends during optimization are compared. GRG steadily reduces costs initially, but slows as it nears a local optimum, showcasing its efficiency in early-stage cost reduction. PGSL, however, achieves faster cost reduction using a random search strategy, avoiding local minima and reaching the global optimum more effectively. Although GRG is faster and suitable for time-sensitive scenarios, PGSL requires more computational time due to its iterative and probabilistic approach. Despite this, PGSL delivers superior cost optimization, demonstrating its effectiveness in achieving high-quality solutions.

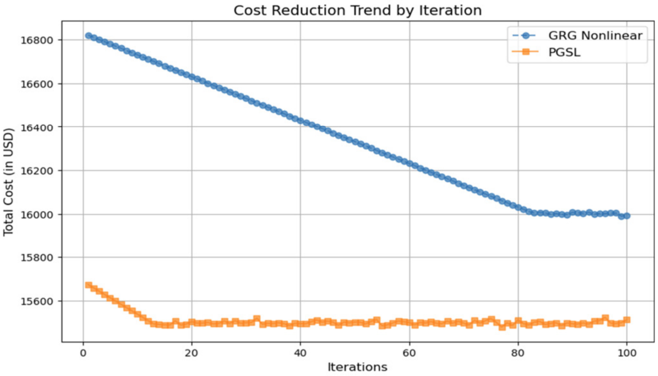

Fig. 3. Cost reduction with respect to iterations for GRG and PGSL.

B. *Comparing Model Performance of Four Mathematical Optimization Models*

Table IV displays the optimal cost. The comparison of four mathematical optimization models revealed that PGSL is the most cost-effective, with the lowest total cost of $15,688.17. This is slightly lower than that of the linear programming model, the cost of which was estimated at $15,712.25. Meanwhile, the dynamic programming model and GRG exhibited the highest costs, namely $16,882.35 and $16,830.92, respectively. In terms of optimal length, PGSL produced the shortest results for Links I and II (70.0040 m and 48.9728 m, respectively) and had similar results with other algorithms for Link III (76.4050 m). Regarding optimal paths, PGSL and the linear programming model selected the paths 1-2 for Links I and II, while GRG and the dynamic programming model chose different paths for Link II. However, all algorithms demonstrated consistent path selection, 1-3 for Link III. Regarding the optimal diameter, all algorithms displayed identical values of 68.58 cm for Links I and II and 91.44 cm for Link III, ensuring network structural stability. PGSL stands out as the most cost-effective algorithm in terms of cost and optimal length, while other algorithms maintain consistency in ensuring sewer network structural integrity.

TABLE IV. OPTIMAL DESIGN OF GRAVITY-FED SEWER SYSTEM USING VARIOUS OPTIMIZATION ALGORITHMS

Methods	Link no.	Length (m)	Path	Diameter (cm)	Cost ($)	Ref.
LP	I	70.115	1-2	68.58	15,712.25	[8]
	II	49.086	1-2	68.58		
	III	76.508	1-3	91.44		
DP	I	70.104	1-2	68.58	16,882.35	[29]
	II	49.073	2-3	68.58		
	III	76.104	1-3	91.44		
GRG	I	70.104	2-3	76.20	16,830.92	Present study
	II	49.073	2-2	76.20		
	III	76.505	1-3	91.44		
PGSL	I	70.004	1-2	68.58	15,688.17	Present study
	II	48.973	1-2	68.58		
	III	76.405	1-3	91.44		

V. CONCLUSION

This study presents a novel evaluation of optimization algorithms for sewer network design, emphasizing cost minimization and parameter optimization, including length, path, and diameter. Unlike previous studies that primarily relied on linear and dynamic programming, this research introduces a comparative analysis of Probabilistic Global Search Lausanne (PGSL) and Generalized Reduced Gradient (GRG) methods to assess their effectiveness. The findings demonstrate that the PGSL model outperforms existing approaches by achieving the most cost-efficient design, with optimal lengths of 70.0040 m, 48.9728 m, and 76.4050 m for Links I, II, and III, and corresponding paths of 1-2, 1-2, and 1-3, at a total cost of $15,688.17. Although alternative algorithms resulted in higher costs, they maintained consistent diameters, ensuring structural integrity. The key contribution of this study lies in providing a systematic framework for selecting optimization algorithms based on cost-effectiveness, performance, and design criteria. Valuable insights were derived for urban planners and engineers, offering practical guidelines for optimizing sewer network design.

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